Problem 5: SVM (18 Points)

Part A (2 Points)

Consider the two arrangements of + and - points in drawings A and B, in which the arrangement in B is produced by rotating the arrangement in A around the center. Then select true or false for the following assertions:

Given a radial-basis kernel, $\kappa(v_1, v_2) = e^{\frac{-\|v_1 - v_2\|^2}{0.5}}$, the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- **True** Vectors do not reference origin, so situations same under rotation and translation.
- False

Given a polynomial kernel, $\kappa(v_1, v_2) = (v_1 \cdot v_2)^2$, where the vectors are drawn from the origin, the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- True
- False Vectors are from origin, so situations are not rotationally symmetric.



Part B (4 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagrams. Sketch the -1 or +1 lines (gutters of the widest street) and circle the points corresponding to support vectors assuming a polynomial kernel, $\kappa(v_1, v_2) = (v_1 \cdot v_2)^1$





0,0







Part C.1(2 Points)

Assume that a support vector machine is to learn to separate the + and - points in the following diagram. Sketch the <u>decision boundary</u> (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel,

 $\kappa(v_1, v_2) = e^{\frac{-\|v_1 - v_2\|^2}{2\sigma^2}}$, and a <u>very large sigma</u>.





Two are support vetors

Part C.2(2 Points)

Assume that a support vector machine is to learn to separate the + and - points in the following diagram. Sketch the <u>decision boundary</u> (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel,

 $\kappa(v_1, v_2) = e^{\frac{-\|v_1 - v_2\|^2}{2\sigma^2}}, \text{ and a } \underline{very \ small \ sigma}.$





All are support vectors

0,0

Part D (8 Points)

On the separate sheet, there are nine colored diagrams, labeled A through I, representing graphs of SVMs trained to separate pluses (+) from minuses (-). Indicate which diagram results from using which kernel function by writing the letter of the diagram next to the corresponding kernel.

<u>Note that the points are the same in diagrams A, B, C, and D. They are also the same in E, F, and G.</u>

$\boldsymbol{\kappa}(\boldsymbol{v}_1,\boldsymbol{v}_2) = \left(\boldsymbol{v}_1\cdot\boldsymbol{v}_2\right)^1$	A, E, H	$\kappa(v_1, v_2) = e^{\frac{-\ v_1 - v_2\ ^2}{0.5}}$	C, G
$\kappa(v_1, v_2) = (v_1 \cdot v_2)^2$	В	$\kappa(v_1, v_2) = e^{\frac{-\ v_1 - v_2\ ^2}{0.22}}$	D, F
$\kappa(v_1, v_2) = (v_1 \cdot v_1)^2$	Nonsense	$\kappa(v_1, v_2) = e^{\frac{-\ v_1 - v_2\ ^2}{0.08}}$	Ι







D.







G.



