Problem 5: SVM (18 Points)

Part A (2 Points)

Consider the two arrangements of + and – points in drawings A and B, in which the arrangement in B is produced by rotating the arrangement in A around the center. Then select true or false for the following assertions:

Given a radial-basis kernel, \( \kappa(v_1, v_2) = e^{-\|v_1-v_2\|^2} \), the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- True  Vectors do not reference origin, so situations same under rotation and translation.
- False

Given a polynomial kernel, \( \kappa(v_1, v_2) = (v_1 \cdot v_2)^2 \), where the vectors are drawn from the origin, the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- True
- False  Vectors are from origin, so situations are not rotationally symmetric.
Part B (4 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagrams. Sketch the -1 or +1 lines (gutters of the widest street) and circle the points corresponding to support vectors assuming a polynomial kernel, $\kappa(v_1, v_2) = (v_1 \cdot v_2)^d$.
Part C.1 (2 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagram. Sketch the decision boundary (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel,

\[ \kappa(v_1, v_2) = e^{-\frac{||v_1 - v_2||^2}{2\sigma^2}}, \]

and a very large sigma.

![Decision Boundary Sketch]

Two are support vectors

Part C.2 (2 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagram. Sketch the decision boundary (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel,

\[ \kappa(v_1, v_2) = e^{-\frac{||v_1 - v_2||^2}{2\sigma^2}}, \]

and a very small sigma.

![Decision Boundary Sketch]

All are support vectors
**Part D (8 Points)**

On the separate sheet, there are nine colored diagrams, labeled A through I, representing graphs of SVMs trained to separate pluses (+) from minuses (-). Indicate which diagram results from using which kernel function by writing the letter of the diagram next to the corresponding kernel.

*Note that the points are the same in diagrams A, B, C, and D. They are also the same in E, F, and G.*

<table>
<thead>
<tr>
<th>$\kappa(v_i, v_j) = (v_i \cdot v_j)^2$</th>
<th>A, E, H</th>
<th>$\kappa(v_i, v_j) = e^{-|v_i - v_j|^2}$</th>
<th>C, G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa(v_i, v_j) = (v_i \cdot v_j)^2$</td>
<td>B</td>
<td>$\kappa(v_i, v_j) = e^{-|v_i - v_j|^2}$</td>
<td>D, F</td>
</tr>
<tr>
<td>$\kappa(v_i, v_j) = (v_i \cdot v_j)^2$</td>
<td>Nonsense</td>
<td>$\kappa(v_i, v_j) = e^{-|v_i - v_j|^2}$</td>
<td>I</td>
</tr>
</tbody>
</table>