APPLICATION

Mathematics Toolkits Use Problem Reduction to Solve Calculus Problems

Because problem reduction is such an obvious, yet powerful, problem-solving method, it is among the first to be put to use. One application of problem reduction, the SAINT program, pioneered the use of artificial intelligence in mathematical problem solving.

Basically, SAINT was a program that integrated symbolic expressions in one variable. For example, when given

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$$

as the integrand, SAINT produced the dazzling result:

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{1}{3}\tan^3(\arcsin x) - \tan(\arcsin x) + \arcsin x.$$

SAINTlike programs perform variable substitutions and other algebraic manipulations to transform difficult integrands into one or more simpler integrands, thus exhibiting problem reduction, with the hope of eventually producing integrands so simple that the answers are given directly in integral tables.

To do the work, SAINT reduced the integrand to a function of sines and cosines by a substitution:

$$\int \frac{\sin^4 y}{\cos^4 y}\,dy.$$

Next, SAINT reduced the integrand to a function of the tangent of y using a trigonometric identity:

$$\int \tan^4 y\,dy.$$

SAINT reduced the integrand to a rational function with another substitution:

$$\int \frac{z^4}{1+z^2}\,dz.$$

Then, SAINT divided out the two polynomials, producing the following:

$$\int z^2 - 1 + \frac{1}{1+z^2}\,dz.$$

The first two of the integrand expressions are easy to integrate:

$$\int z^2\,dz = \frac{z^3}{3} \qquad \int -1\,dz = -\int 1\,dz = -z$$

Finally, another reduction takes care of the third expression:

$$\int \frac{1}{1+z^2}\,dz = \int dw = w$$

Thus, the original integrand is reduced to three separate integrands, all readily handled by reference to an integral table. The results—$z^3/3$, z, and w—produce the desired result once the variable substitutions are reversed.