Massachvsetts Institvte of Technology

Department of Electrical Engineering and Computer Science

6.313 Contemporary Computer Design

Problem Set #2 Fall, 1998, SOLUTION

Problem 1. 8 points

1a) The wires are in *series*, so the resistance is double that of a single wire. When calculating the area of a circle, make sure you use the radius, not the diameter! Answer = 0.168 Ohms/m.

b) The single capacitance is equivalent to two capacitances (from the wires to a virtual ground plane) which are in *series*. The total capacitance is half that of each virtual capacitor. Since $h >> w$ and $K = 1$, $C = 1/2 * 3/4 * \epsilon_0 = 3.3pF/m$.

This is the dominant capacitance in the circuit. Some of you thought that this capacitance and the capacitance to ground were the same since using the $w' = w + 0.75h$ equation gives you the same number for both. But, using $w' = w + 0.75h$ is only an *approximation*. In reality, capacitance always falls off with height.

c) Inductance is messy. Probably the best thing to do is note that $LC = 1/c^2$ and solve for L once you have C.

d) $Z = \sqrt{L/C}$

e) Just consider what happens when you hold down the telegraph key. In steady state, the circuit is modelled as three resistances in series (wire - solenoid - wire); the total current works out to about 0.4mA, so the solenoid won't close.

NOTE: you need to use R - the resistance of the wire - **NOT** Z. Characteristic impedance is very different from resistance. It gives the relation between V and I at a given point for a wave travelling along a perfect transmission line. It does *not* give you the voltage difference between two points.

f) The LC delay is about $5 * 10^6/c = 1.6 * 10^{-2}s$. If we treat R and C as lumped we get a crude approximation of 14s for the RC delay. Therefore, the RC delay is more significant and the baud rate is 1/14 bits/s or so.

NOTE: Make sure you understand the different types of delays and the effect that they have. LC delay affects the time needed for a wave to reach a destination point. RC delay affects the amount of time needed for that point to charge up to a given voltage.

g) Since delay is proportional to l^2, a factor of n speedup can be achieved with (n-1) evenly spaced repeaters.

Problem 2. 9 points

1a) Two important comments about this problem. First, you can't just assume that Z = 50; you have to prove it. Second, don't forget about the second reflection!! In general, a wave will reflect at B, then reflect again at A. At 10ns, the voltage at A is the forward wave plus the reflected wave *plus* the second reflection. To help you think about this, here's a chart showing the situation at A, B and the midpoint M of the transmission line; '1' denotes the original wave, '2' the first reflection (off of B) and '3' the second reflection (off of A):

time	A	M	B
0ns	1	0	0
2.5	1	1	0
5	1	1	1+2
7.5	1	1+2	1+2
10	1+2+3	1+2	1+2
12.5	1+2+3	1+2+3	1+2
etc..			

In the same way that B jumps instantaneously from 0 to 1+2, A jumps from 1 to 1+2+3. Make sure you understand this chart; if you don't, please talk to me about it.

Now in this problem it turns out that Z=R, so the second reflection is in fact 0. This means that you end up with the right answer even if you forget about the second reflection. However, the *correct* sequence of steps to solve this problem is as follows:

i) Since the voltage is divided in half, $Z/(R+Z) = 1/2 ==> Z = R$

ii) Since R = Z, the second reflection is zero

iii) Therefore at 10ns the voltage at A is the original step + the first reflection, so the first reflection is -1V => the coefficient of reflection at B is -1 ==> B is a short circuit.

Other info:
$delay = 10/2 = 5ns$;
$v = c/\sqrt{MK} = c/2$;
$l = delay * v = .75m$

Now that we *know* that R=Z and the coefficient of reflection at A is 0, we can ignore the second reflection for b, c and d.

b) Let R be the termination resistance. The coefficient of reflection is 0.5. So, $0.5 = (R - Z)/(R + Z) \;\; ==> \;\; R = 3Z = 150ohms$.

c) Since the voltage drops over time, less current flows backwards over time and more current flows forwards over time. This is what we would expect from an inductor. When the wave first hits, the inductor looks like an open circuit. However, we do not see a reflection of +1 which is what we would expect if there was only an inductor. Hence, we need a resistor in parallel with the inductor. When the wave first hits the reflection is $0 ==> R = Z = 50ohm$. So, $L = 2ns * (R||Z) = 50nH$.

NOTE: For both (c) and (d), you need to take Z into account when computing L, C! Remeber that the inductor/capacitor sees the transmission line as a resistor of value Z; the resistance used to compute the time constant is therefore $R + Z$ or $R||Z$ depending on the circuit. If this seems fishy to you, you can convince yourself by actually deriving the differential equation using KCL at the end of the line. In fact, I recommend this; it's a good exercise.

The waveform at the end is 0 from 0ns to 5ns, then it steps to 1V and falls back towards 0 with a 2ns time constant.

d) This time the voltage increases over time, so we need a capacitor. A capacitor looks like a closed circuit when the wave hits, so to produce the 0.5 reflection we need a series resistance of 150ohms (as in b). $C = t/(R + Z) = 10pF$

Problem 3. 8 points 3a,b) This problem is easy if you use the right formulas:

$sqrt(L/C) = Z = 50; 1/sqrt(LC) = v = c/3$

$L = Z/v = 150/c = 5 * 10^{-7}H/m \; ; \; C = 1/Zv = 3/50c = 200pF/m$

$Ctot = 2pF/cm * 18cm = 36pF$

These formulas should be suggested by the fact that you are trying to find L, C given Z, v. You don't need h, w, e0, u0 etc.. Don't confuse capacitance with capacitance/unit length!!

c) $C = K \; e_0 \; wl/h = 1.38pF$ (using K = 3.9). You get a slightly different answer if you use K=4 or K=9; all of these are fine. We are not in the $w << h$ regime, so $w' = w + 3/4h$ is not completely right. Don't use K=1, though, unless the insulator is air or foam.

d) Total capacitance/SIMM = 8 + 1.38 = 9.38pF.

Total capacitance = 36 + 6 * 9.38 = 92.48pF or 5.14pF/cm.

e) $Z = \sqrt{L/C} = 31.1 ohm$. Change R to 31.1 ohm.

f) $v = 1/\sqrt{LC} = c/5$

Problem 4. 4 points

4) Signal A is being driven across a circuit board to a pin through a 60ohm transmission line. When the signal gets to the end of the circuit board, it expects to see either another 60 ohm transmission line or a 60 termination. If we connect two 120 ohm twisted pairs to the pin, A sees $120||120 = 60 ohms$. Then one twisted pair is connected to B and the other to C. Each with a 60 ohm resistor at the end. The 60 ohm resistor in series with the 60 ohm resistor already existing at B and C form a 120 ohm termination