# Cameras, lenses, and sensors

Reading: Chapter 1, Forsyth & Ponce
Optional: Section 2.1, 2.3, Horn.

6.801/6.866
Profs. Bill Freeman and Trevor Darrell
Sept. 10, 2002

# Today's lecture

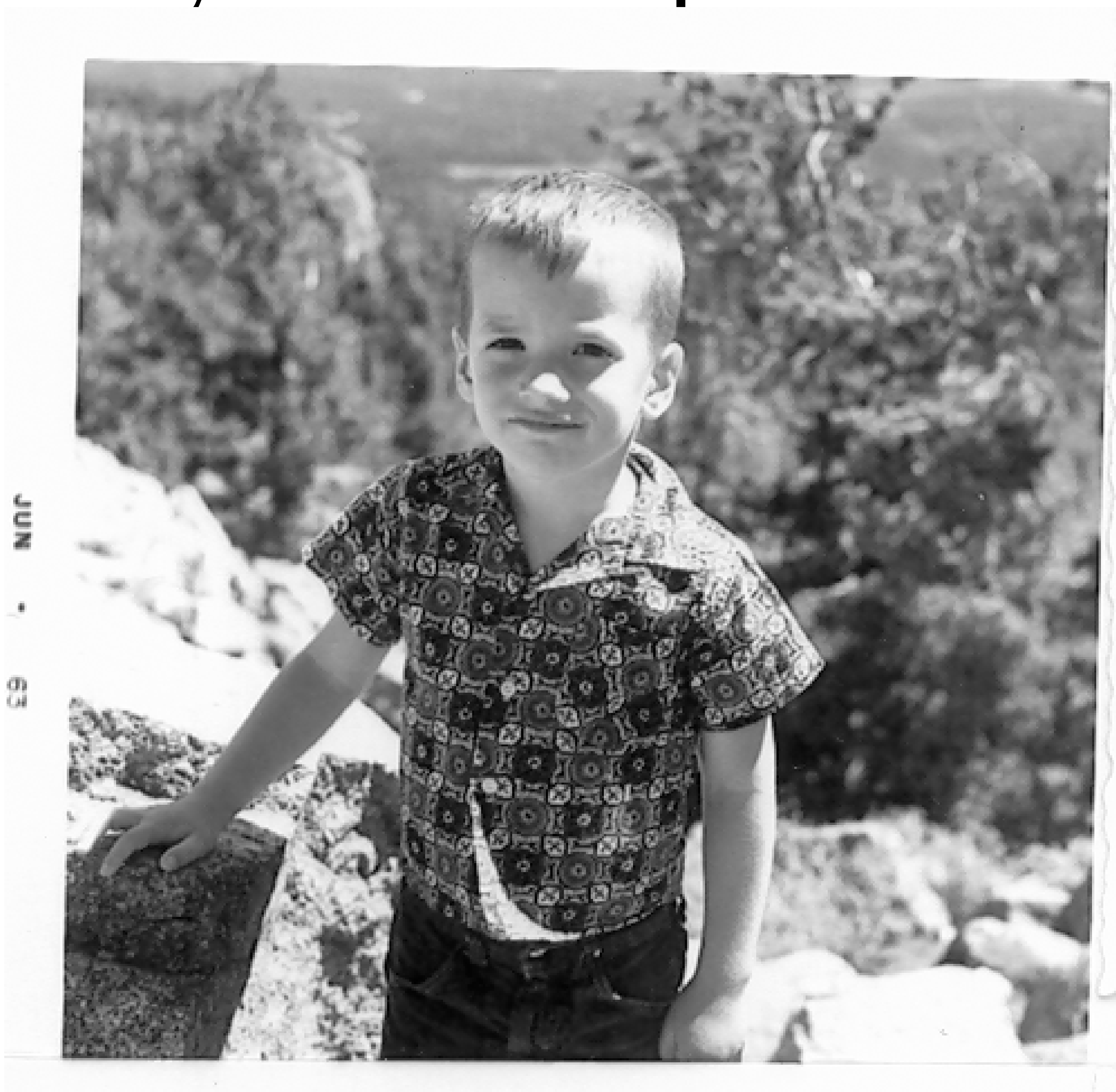
## 6.801/6.866 Machine Vision

### Syllabus

| # | Date | Description | Assignments | Materials |
|---|---|---|---|---|
| 1 | 9/5 | Course Introduction | Pset #0 (not collected) | Freeman Slides<br>Darrell Slides<br>Matlab Tutorial Diary |
| 2 | 9/10 | Cameras, Lenses, and Sensors | | |
| 3 | 9/12 | Radiometry and Shading Models | Pset #1 Assigned | |

How many people would also want to take machine learning?

# 7-year old's question

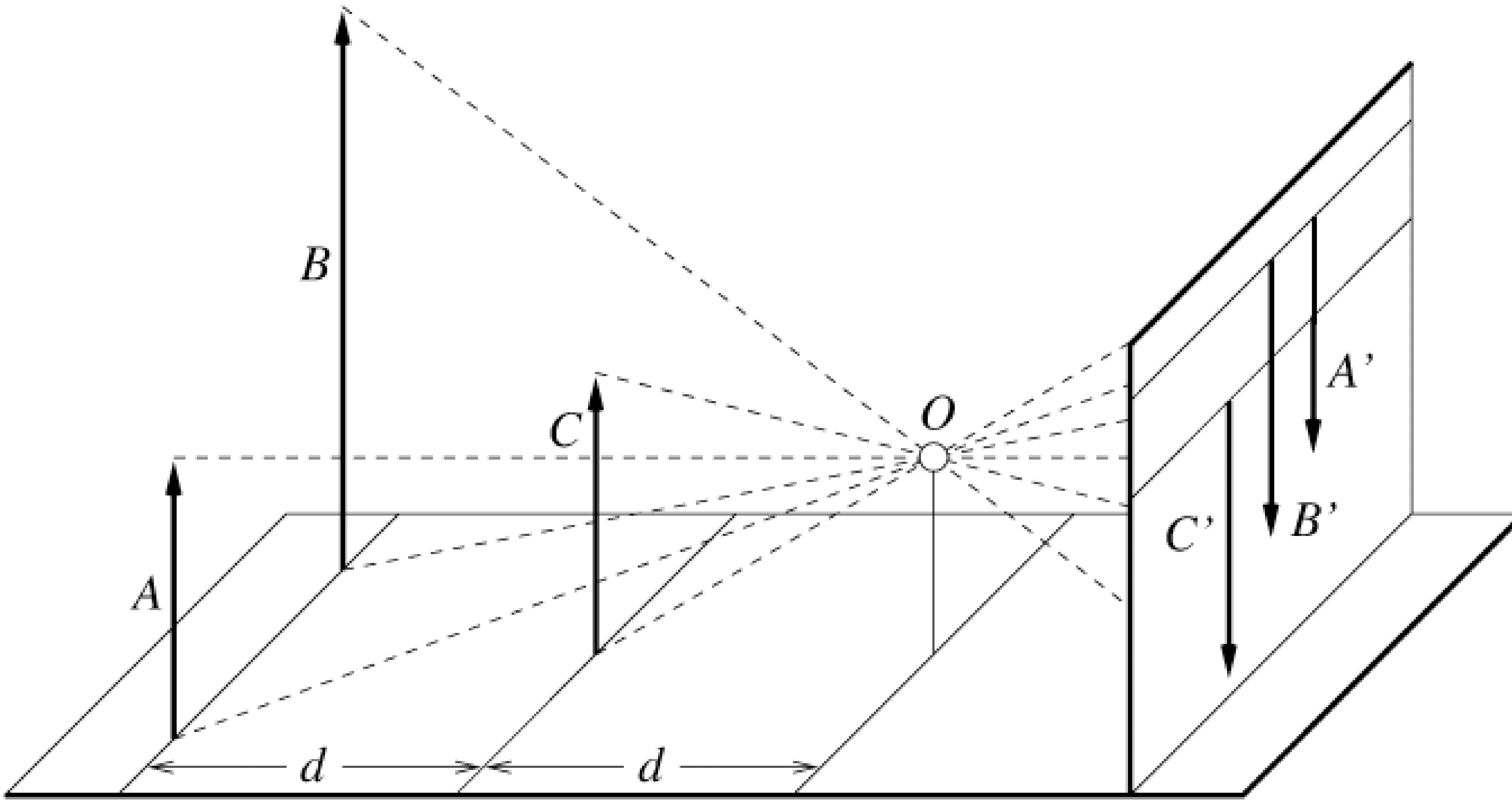



Why is there no image on a white piece of paper?

# Pinhole cameras

- Geometry

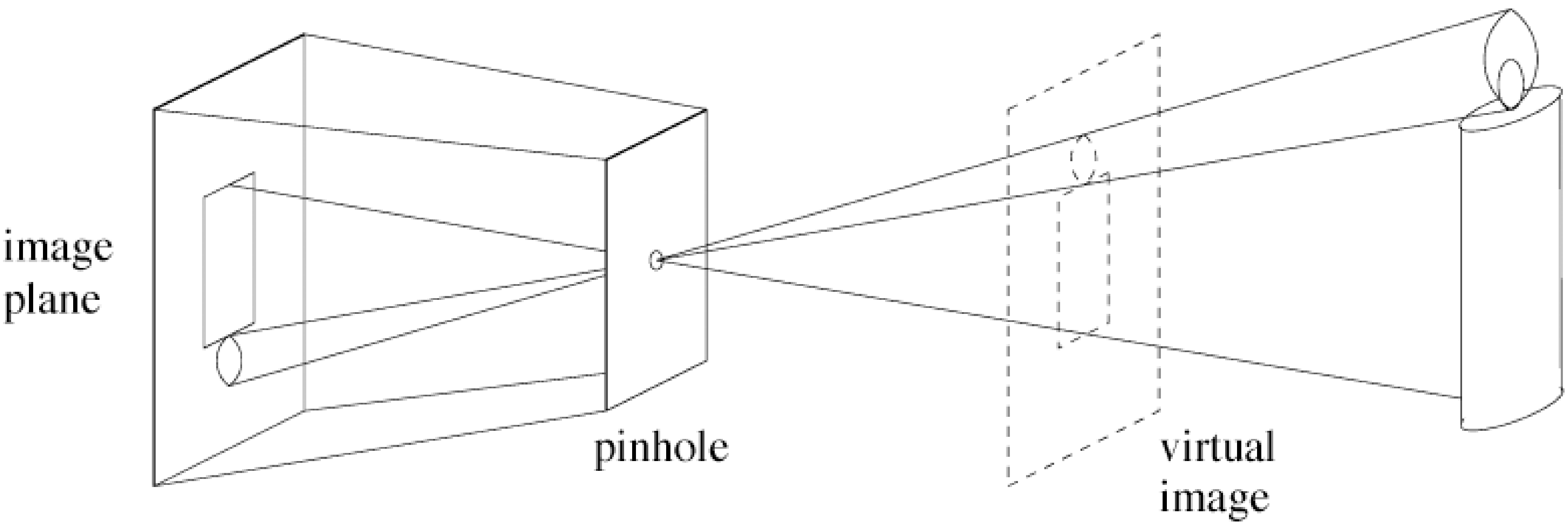
# Distant objects are smaller




Forsyth&Ponce

# Virtual image, perspective projection

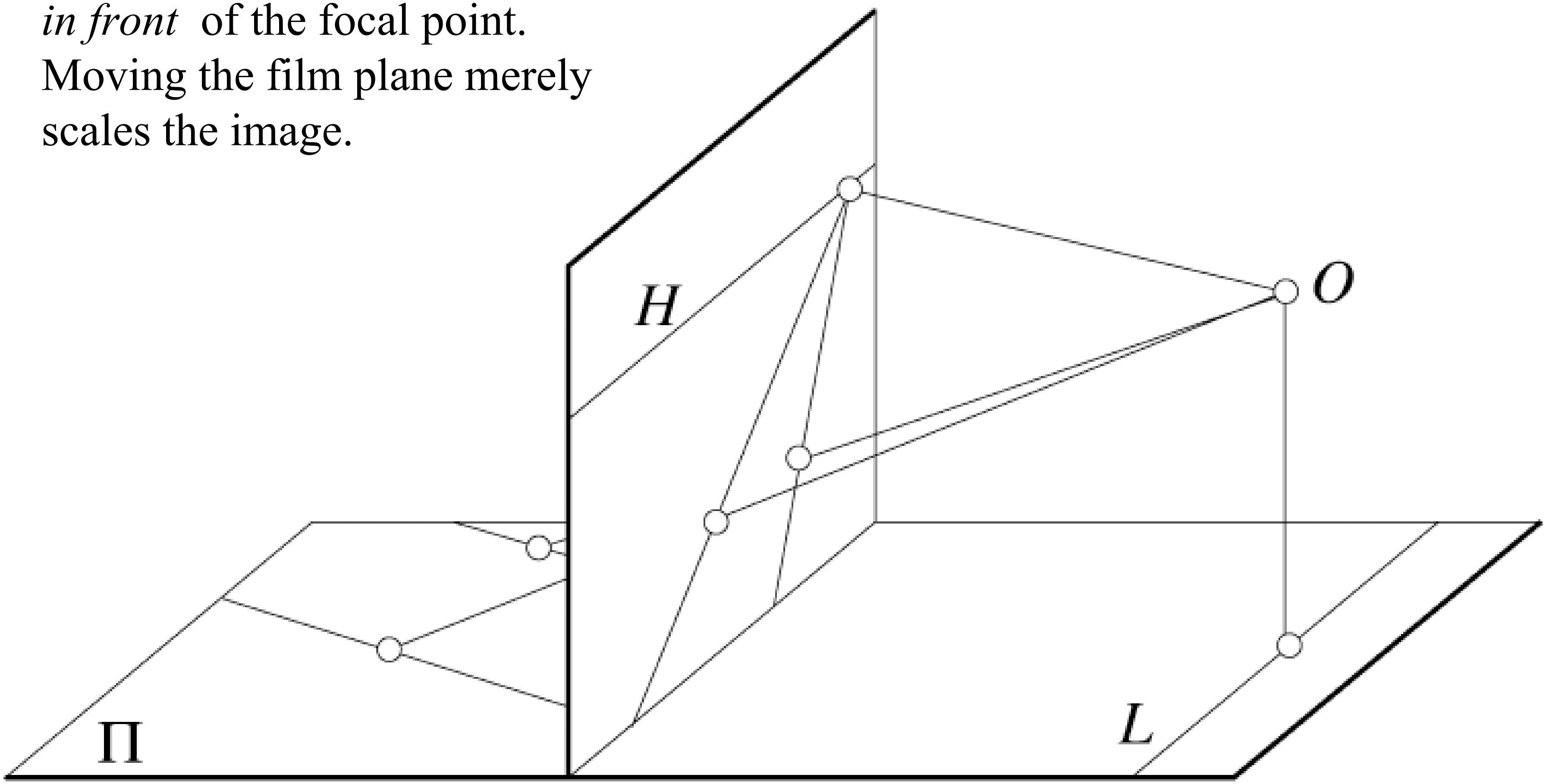
- Abstract camera model - box with a small hole in it




Forsyth&Ponce

# Parallel lines meet

Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.



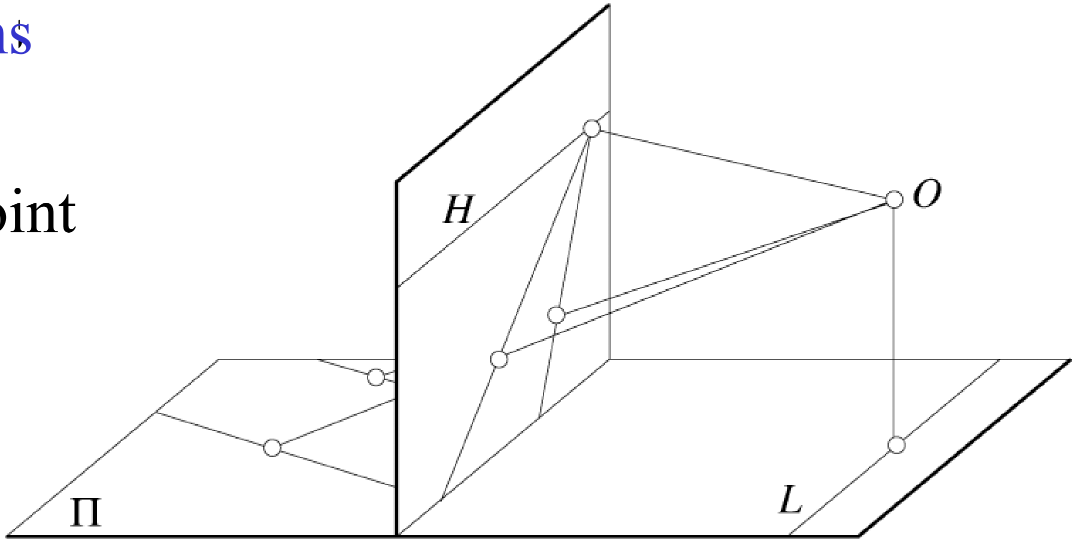

Forsyth&Ponce

# Vanishing points

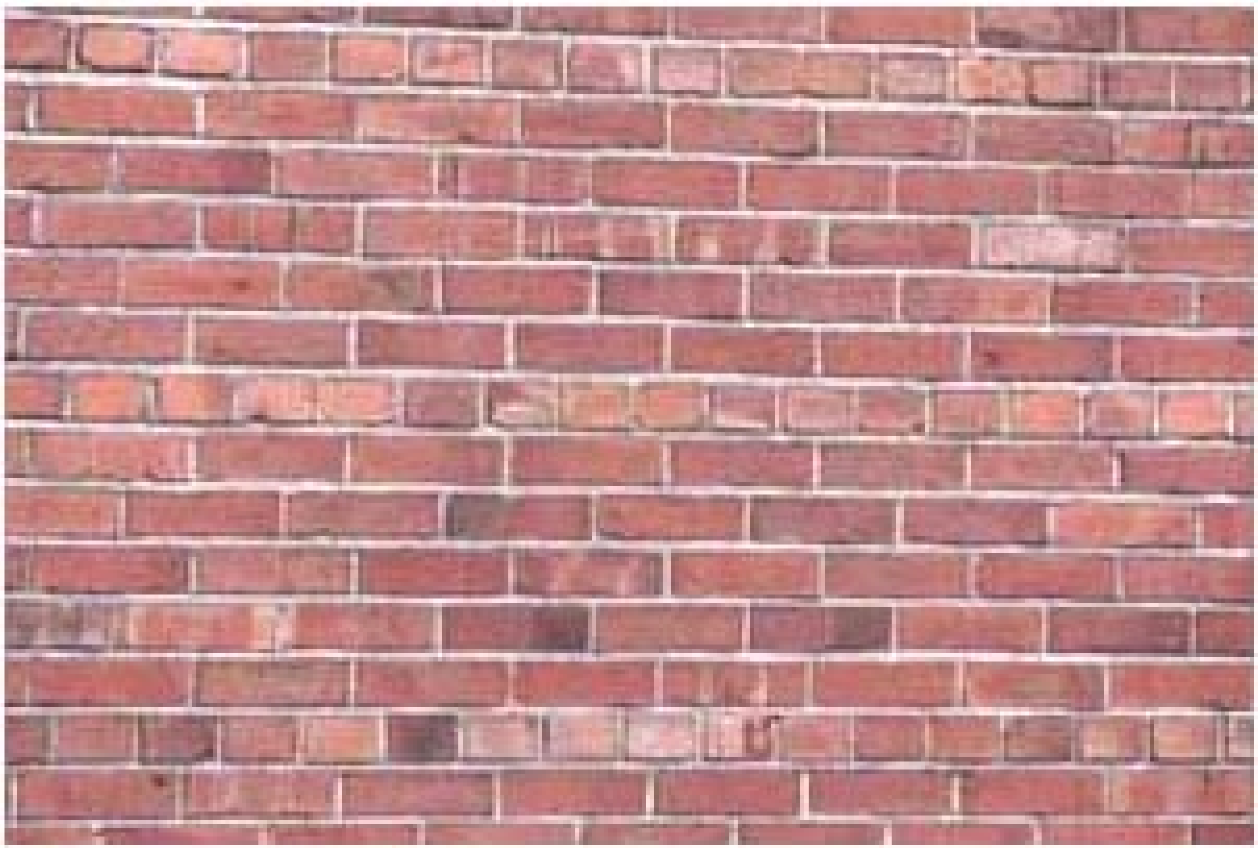
- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
- We show this on the board…

# Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to the whole image or a half-plane
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line

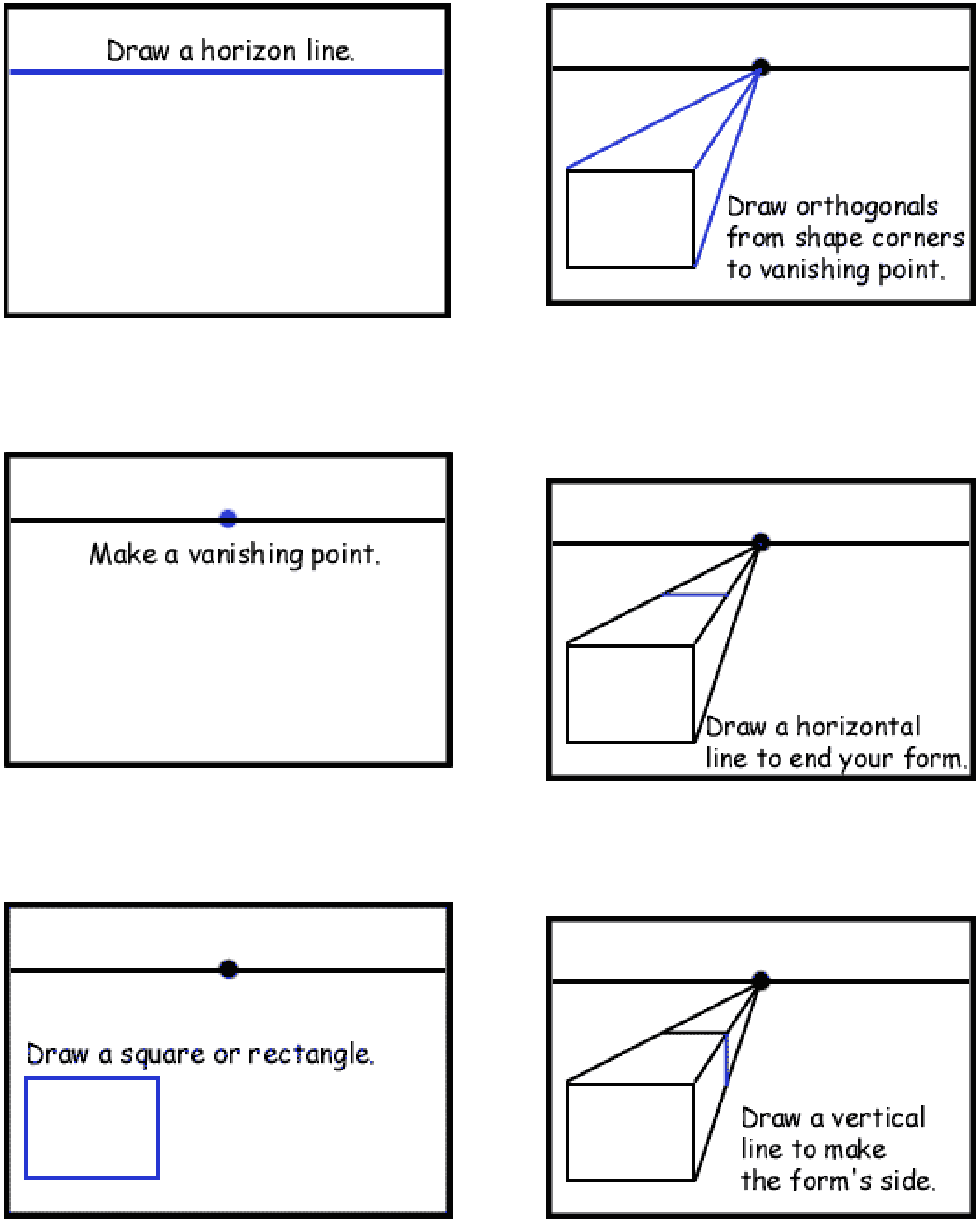


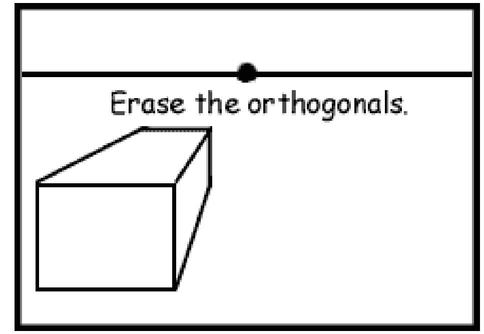
# What if you photograph a brick wall head-on?



# Now we learn how to draw

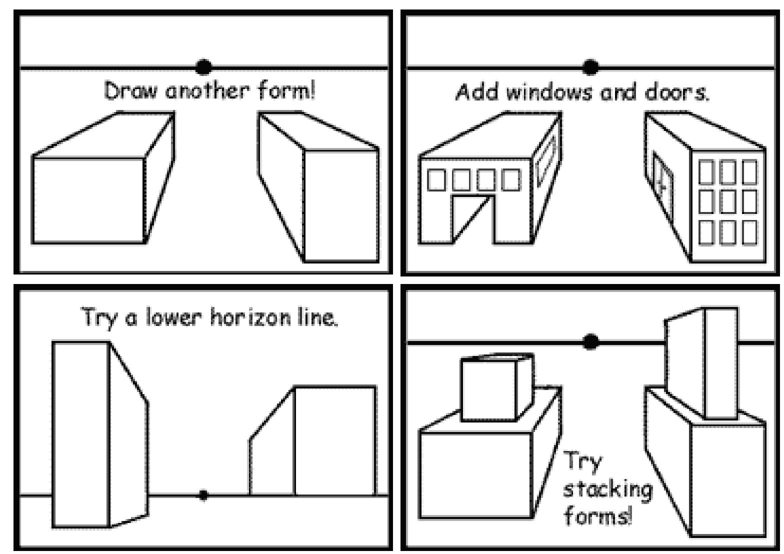
- One-point perspective
- Two-point perspective



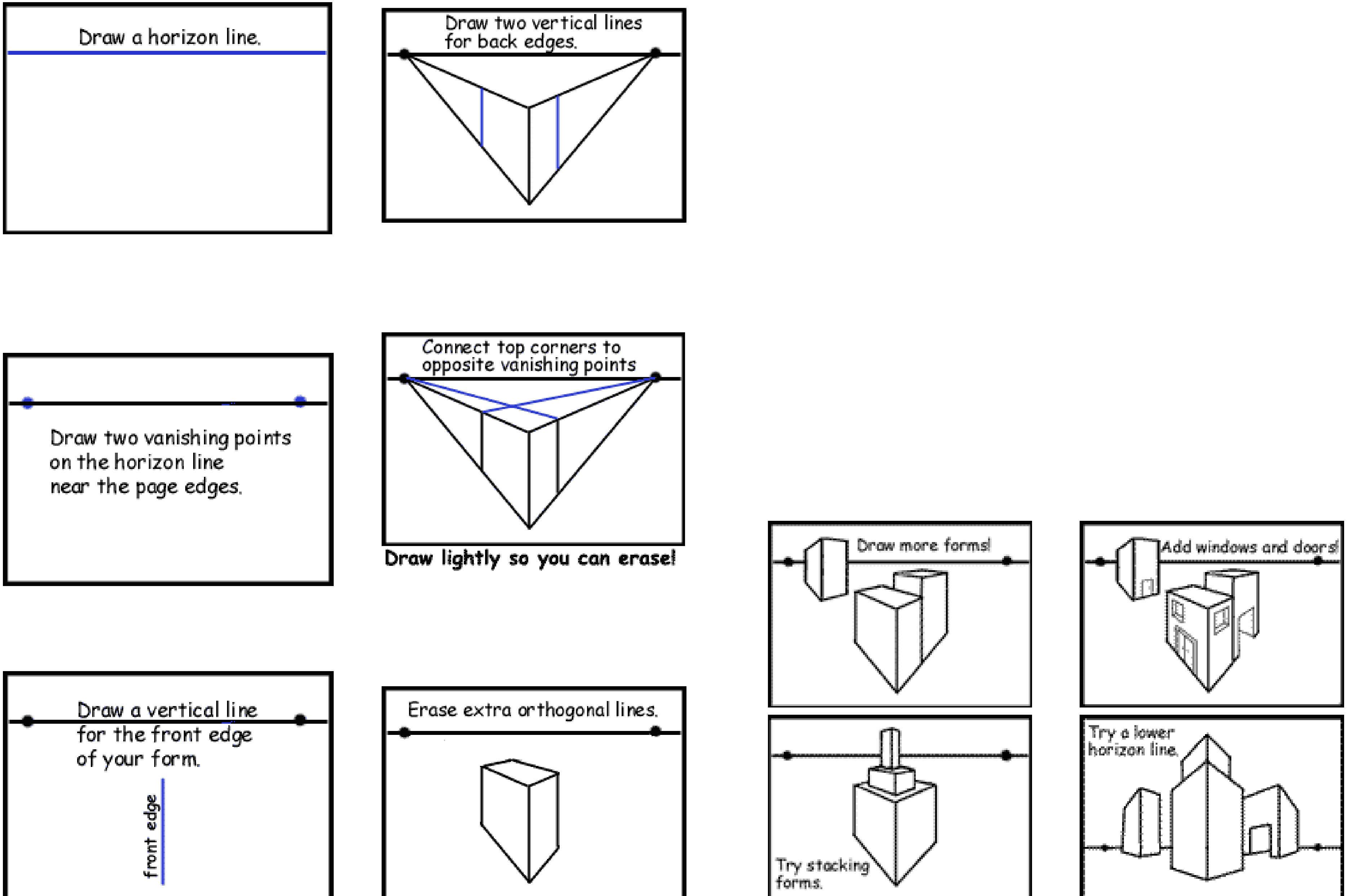




**Now you have a 3-D form in one-point perspective!**

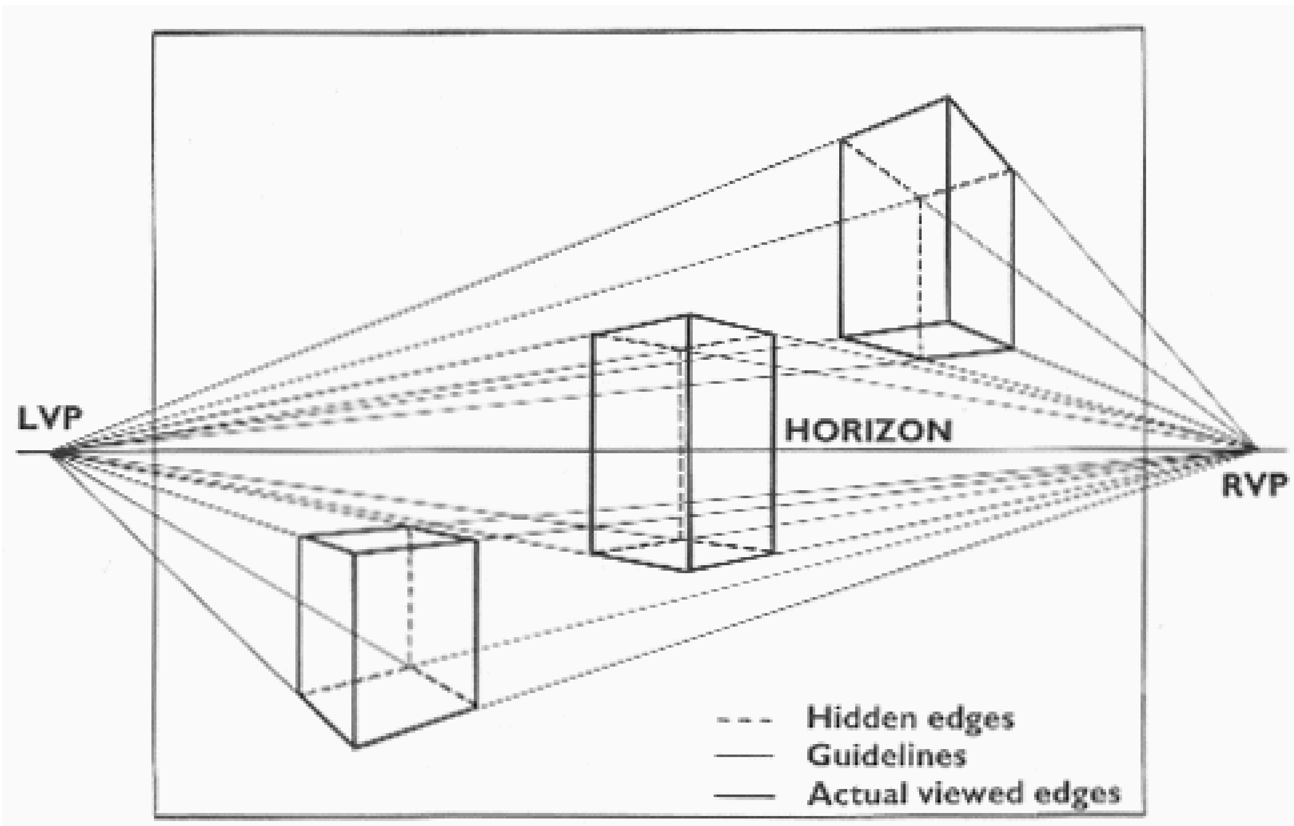
## 10. Add details and experiment!




http://www.sanford-artedventures.com/create/tech_1pt_perspective.html

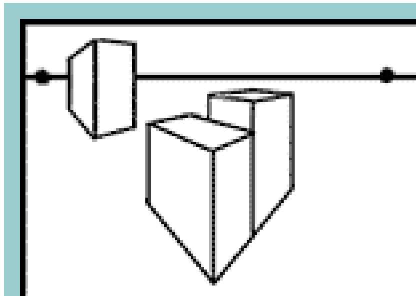



http://www.sanford-artedventures.com/create/tech_2pt_perspective.html




http://www.siggraph.org/education/materials/HyperGraph/viewing/view3d/perspect.htm

# Two-point perspective

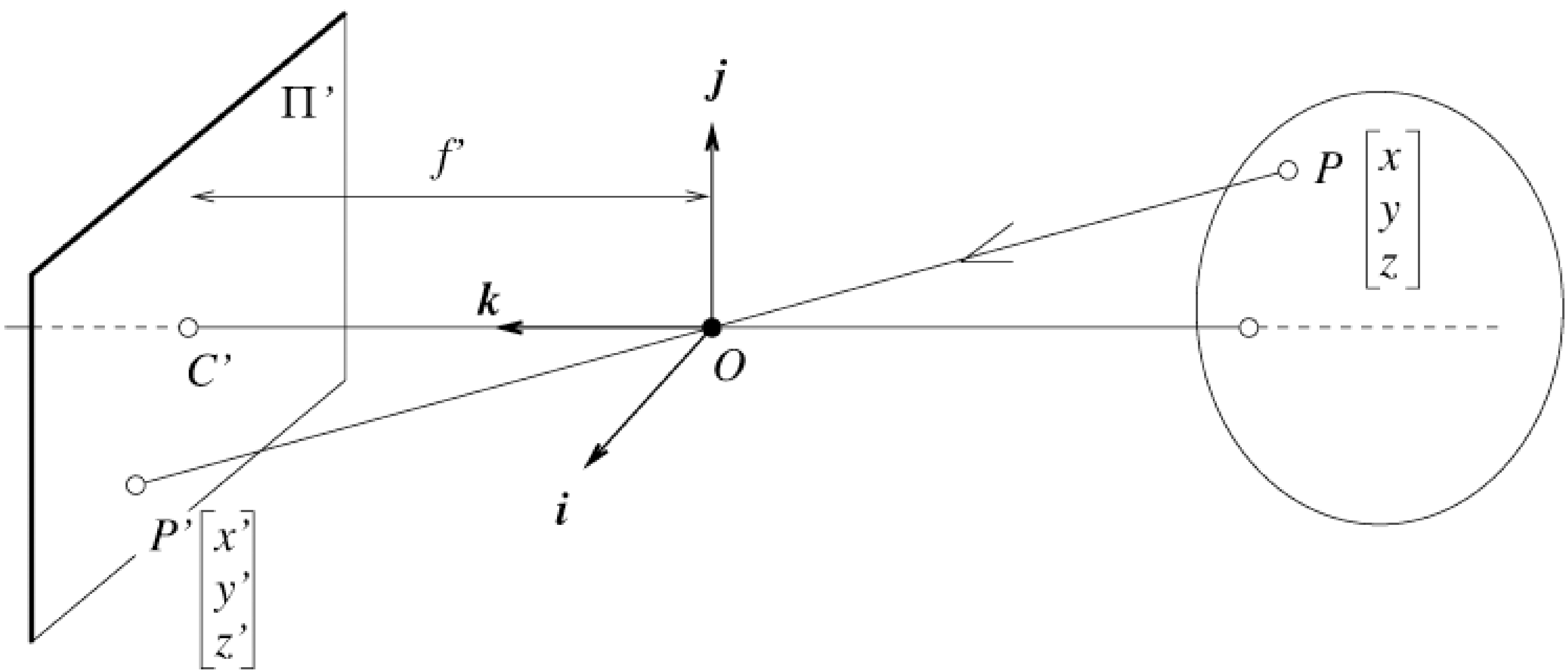


It's easy to draw simple forms in two-point perspective.

Linear perspective allows artists to trick the eye into seeing depth on a flat surface.

http://www.sanford-artedventures.com/create/tech_2pt_perspective.html

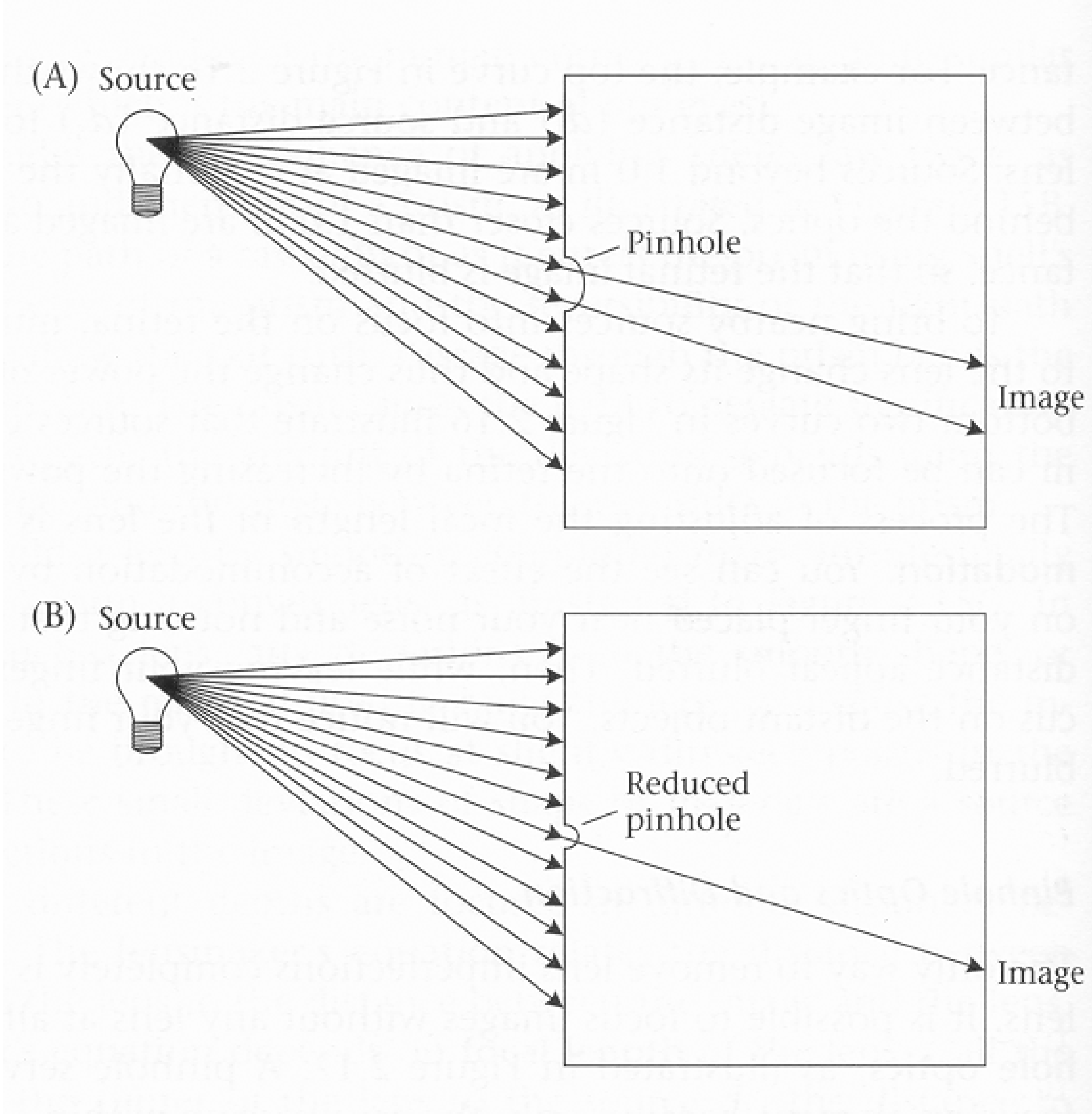
# The equation of projection



# The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, -f)
  - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

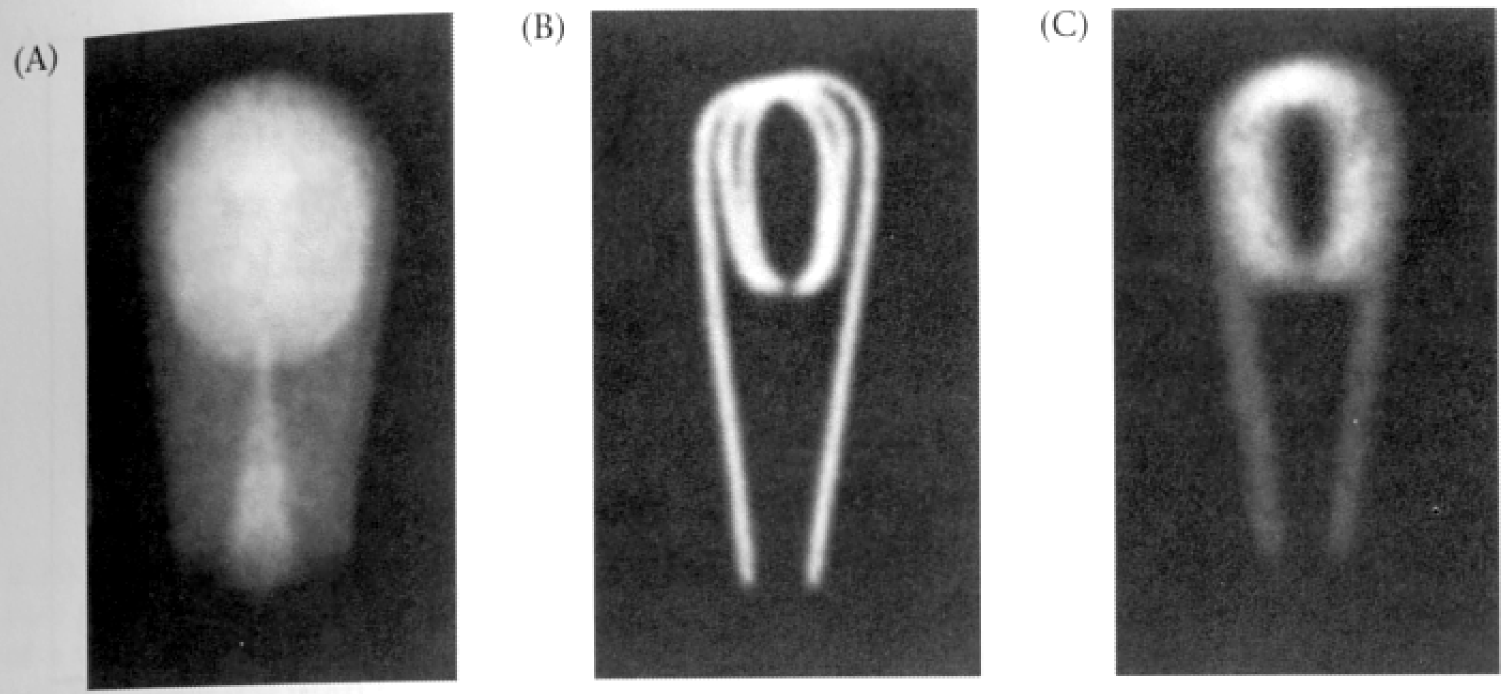



Wandell, Foundations of Vision, Sinauer, 1995

# Pinhole camera demonstrations

- Film camera, box, demo.  Apertures, lens.

- The image is the convolution of the aperture with the scene.

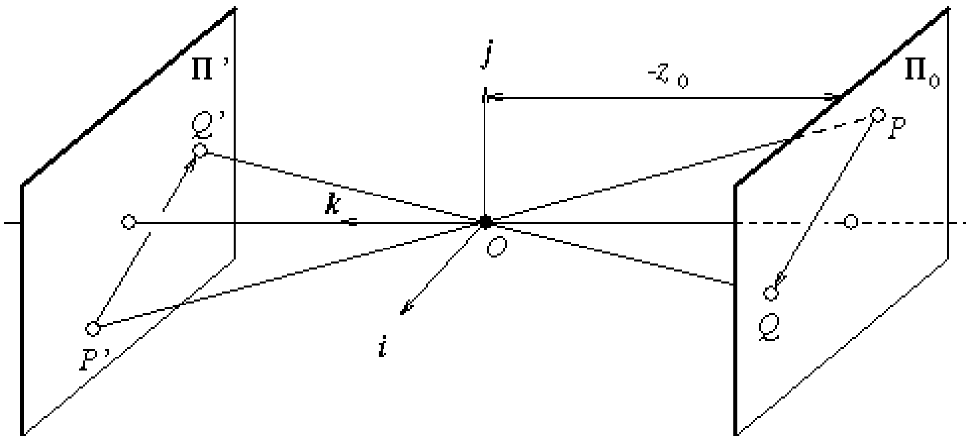



**2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS.** These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

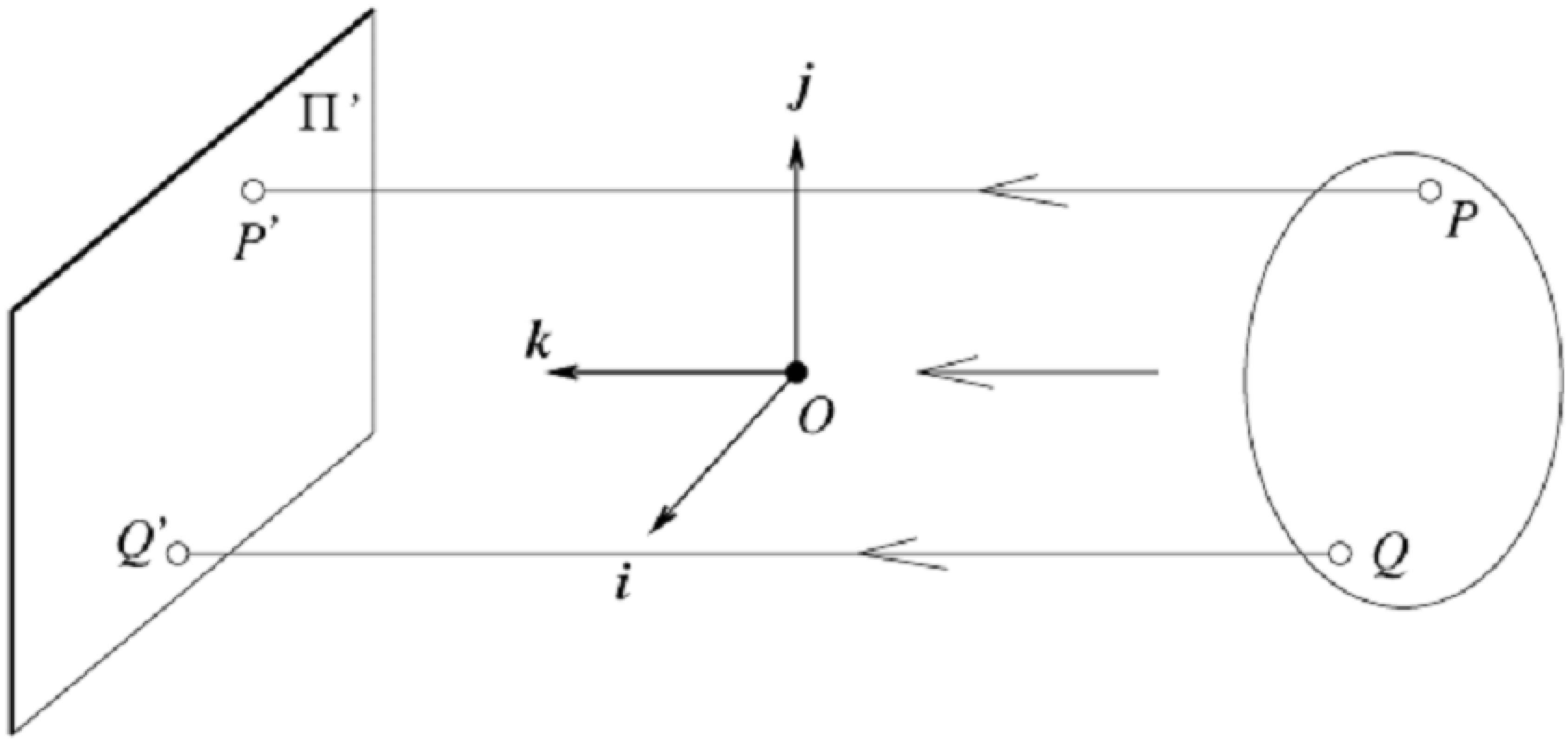
Wandell, Foundations of Vision, Sinauer, 1995

# Weak perspective

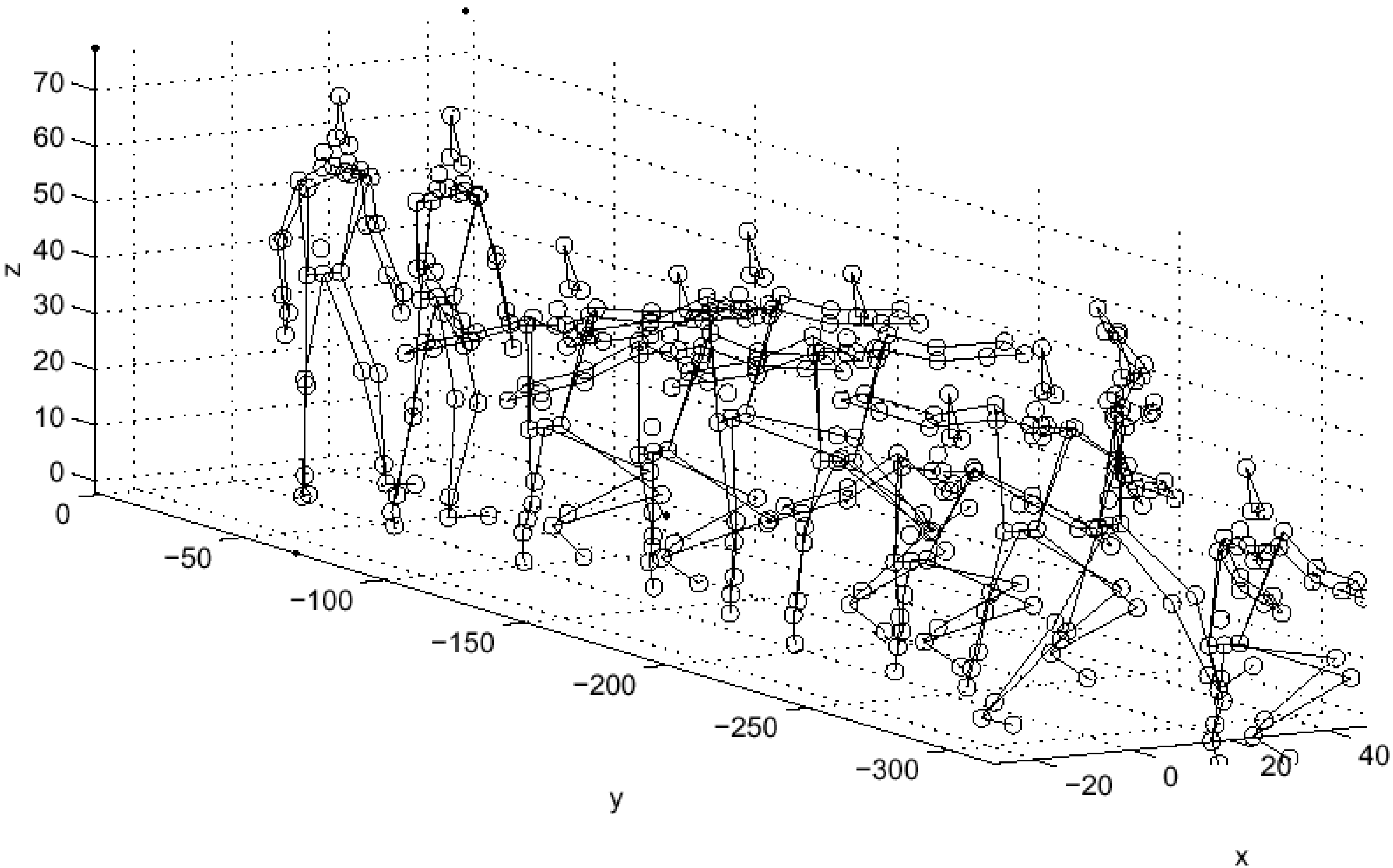
- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



# Orthographic projection



# Example use of orthographic projection: inferring human body motion in 3-d



# Advantage of orthographic projection

Our simplified rendering conditions are as follows: the body is transparent, and each marker is rendered to the image plane orthographically. For figural motion described by human motion basis coefficients $\vec{\alpha}$, the rendered image sequence, $\vec{y}$, is:

$$\vec{y} = PU\vec{\alpha}, \tag{1}$$

where $P$ is the projection operator which collapses the $y$ dimension of the image sequence $U\vec{\alpha}$.

Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

# Orthography can lead to analytic solutions

have our multi-dimensional gaussian,

Prior probability

$$P(\vec{\alpha}) = k_2 e^{-\vec{\alpha}' \Lambda^{-1} - \vec{\alpha}}, \tag{3}$$

where $k_2$ is another normalization constant. If we model the observation noise as i.i.d. gaussian with variance $\sigma$, we have, for the likelihood term of Bayes theorem,

Likelihood function

$$P(\vec{y}|\vec{\alpha}) = k_3 e^{-|\vec{y} - PU\vec{\alpha}|^2/(2\sigma^2)}, \tag{4}$$

with normalization constant $k_3$.

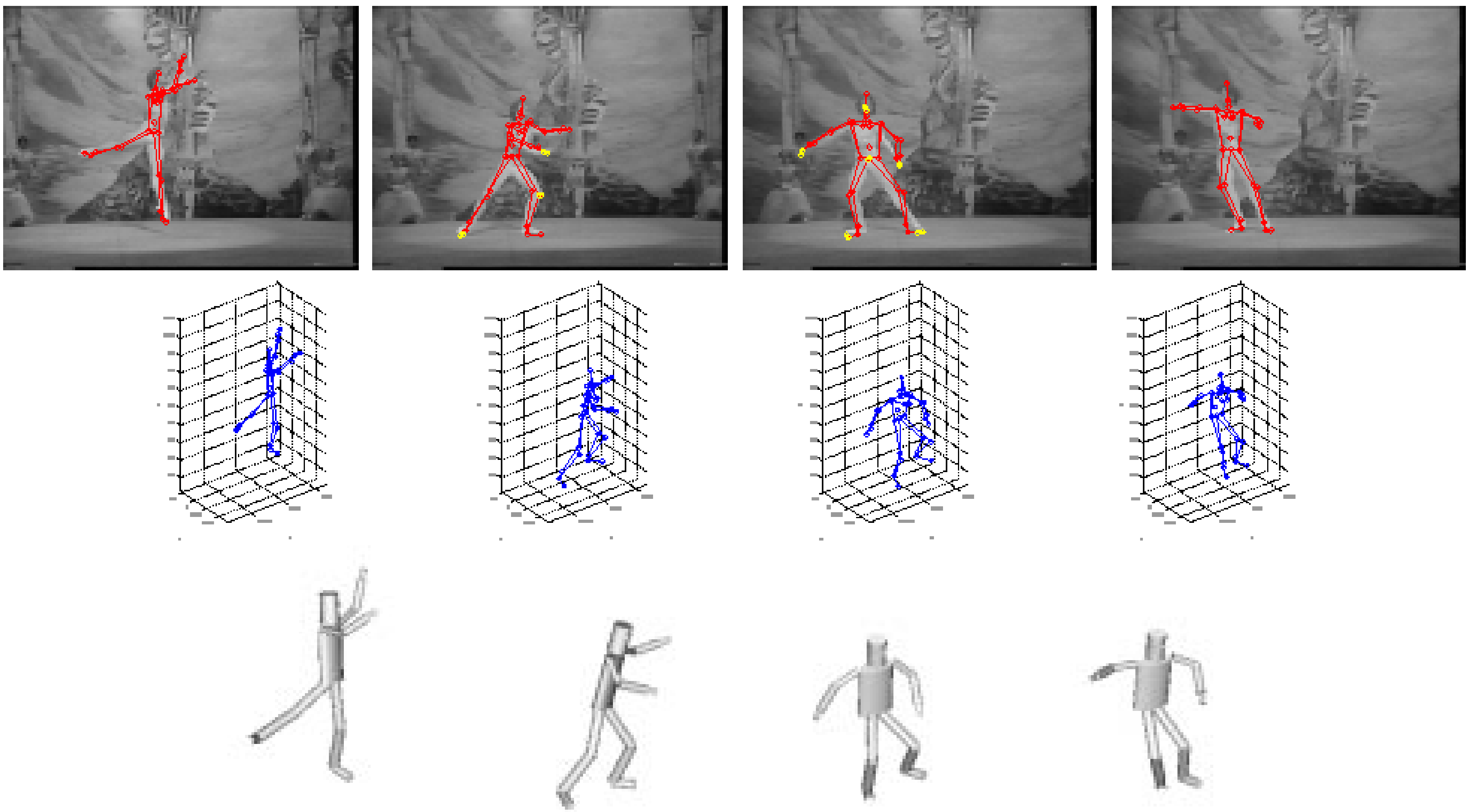
The posterior distribution is the product of these two gaussians. That yields another gaussian, with mean and covariance found by a matrix generalization of "completing the square" [7]. The squared error optimal estimate for $\alpha$ is then

$$\alpha = SU'P'(PUSU'P' + \sigma I)^{-1}(\vec{y} - (P\vec{m})) \tag{5}$$

Analytic solution for inferred 3-d motion

Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

# Results



Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

# But, alas

“The results for the simplified problem appear promising. However serious questions arise because of the simplifying assumptions, which trivialize a number of the hard issues of the problem in the real world. Eg. scaling effects that arise from perpective projection are ignored, by assuming orthographic projection. …”

**Reviewer’s comments**

# Crossed-slit camera model

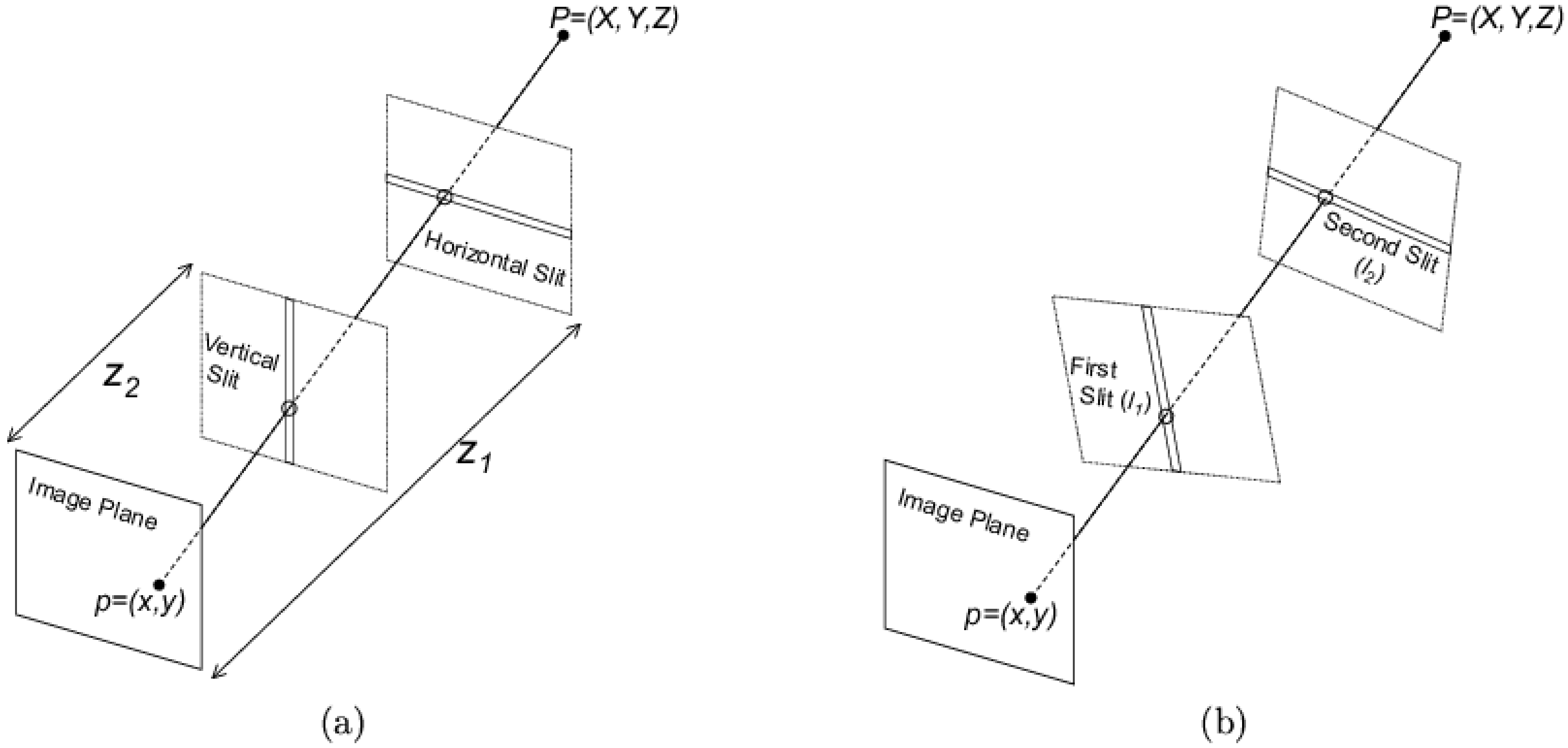
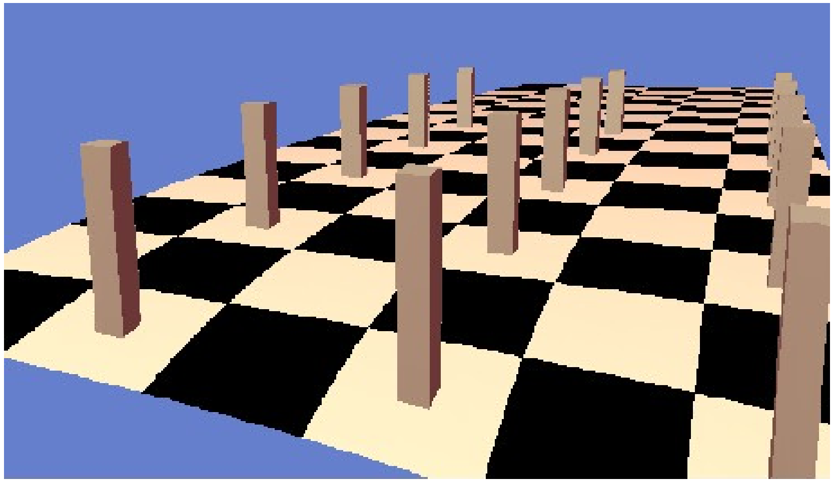


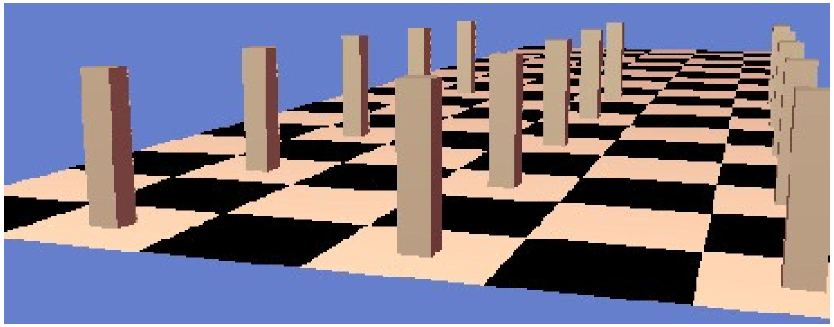

Figure 1: (a) A design of a X-Slits camera where the slits are orthogonal to each other and parallel to the image plane (POX-Slits camera). $Z_1$ denotes the horizontal focal length and $Z_2$ denotes the vertical focal length. The projection ray of a 3-D point $\mathbf{p} = (\mathbf{X}, \mathbf{Y}, \mathbf{Z})$ is shown, with circles showing its intersection points with the 2 slits. (b) A general X-Slits design, with two arbitrary slits $\mathbf{l}_1, \mathbf{l}_2$.

http://www.cs.huji.ac.il/~daphna/papers/xslits.ps.gz

# Crossed-slit camera model



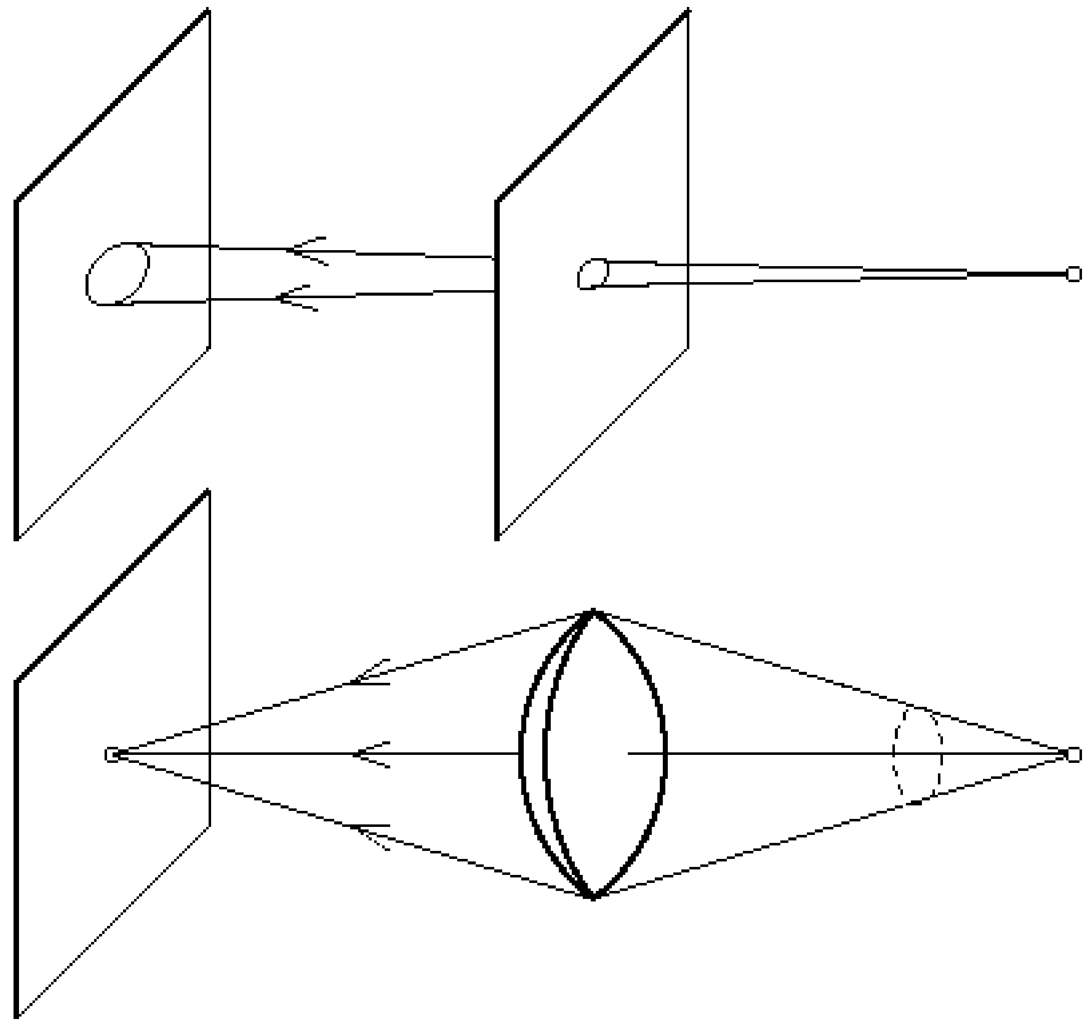
X-slit camera view



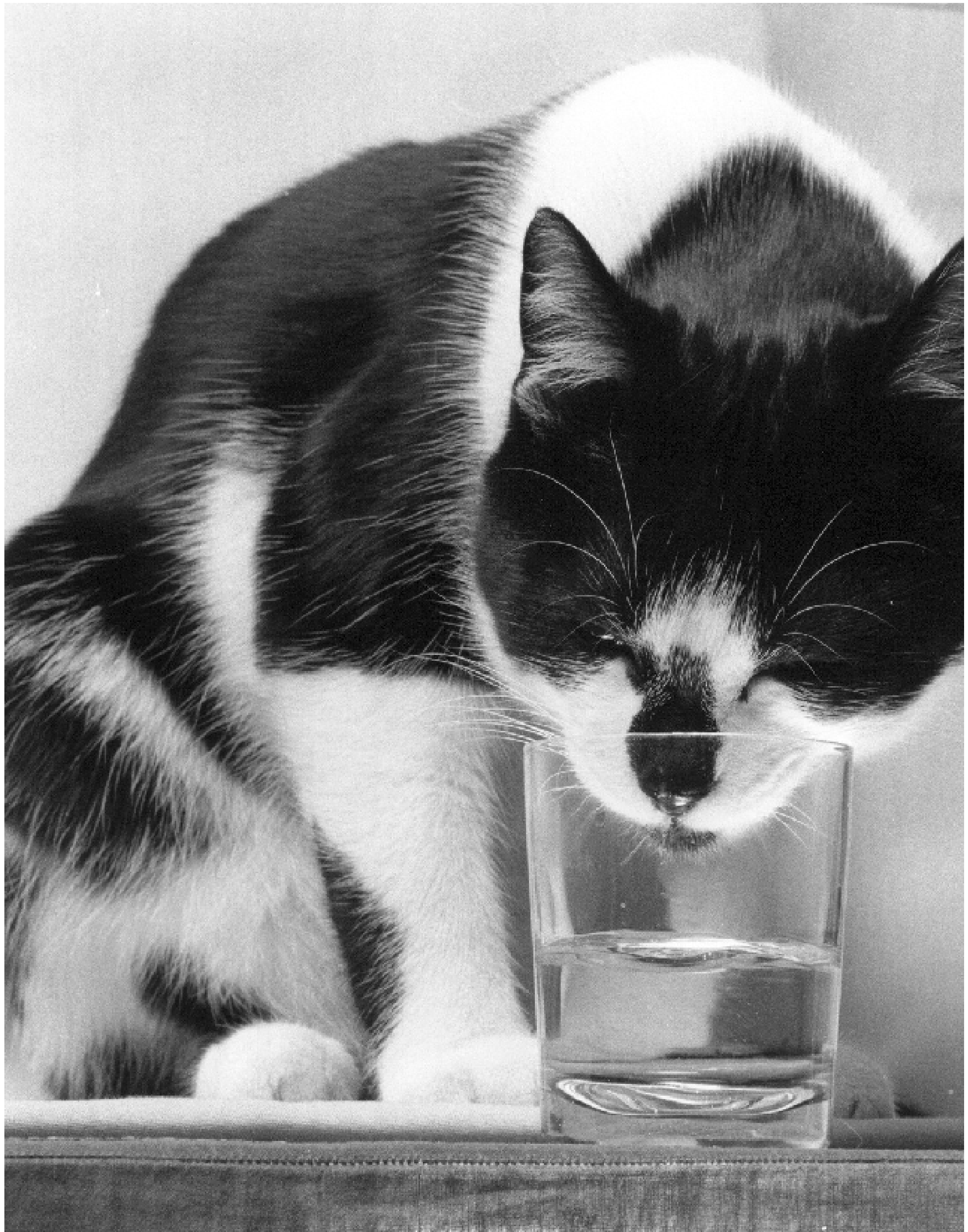
pinhole camera view

http://www.cs.huji.ac.il/~daphna/papers/xslits.ps.gz

# The reason for lenses

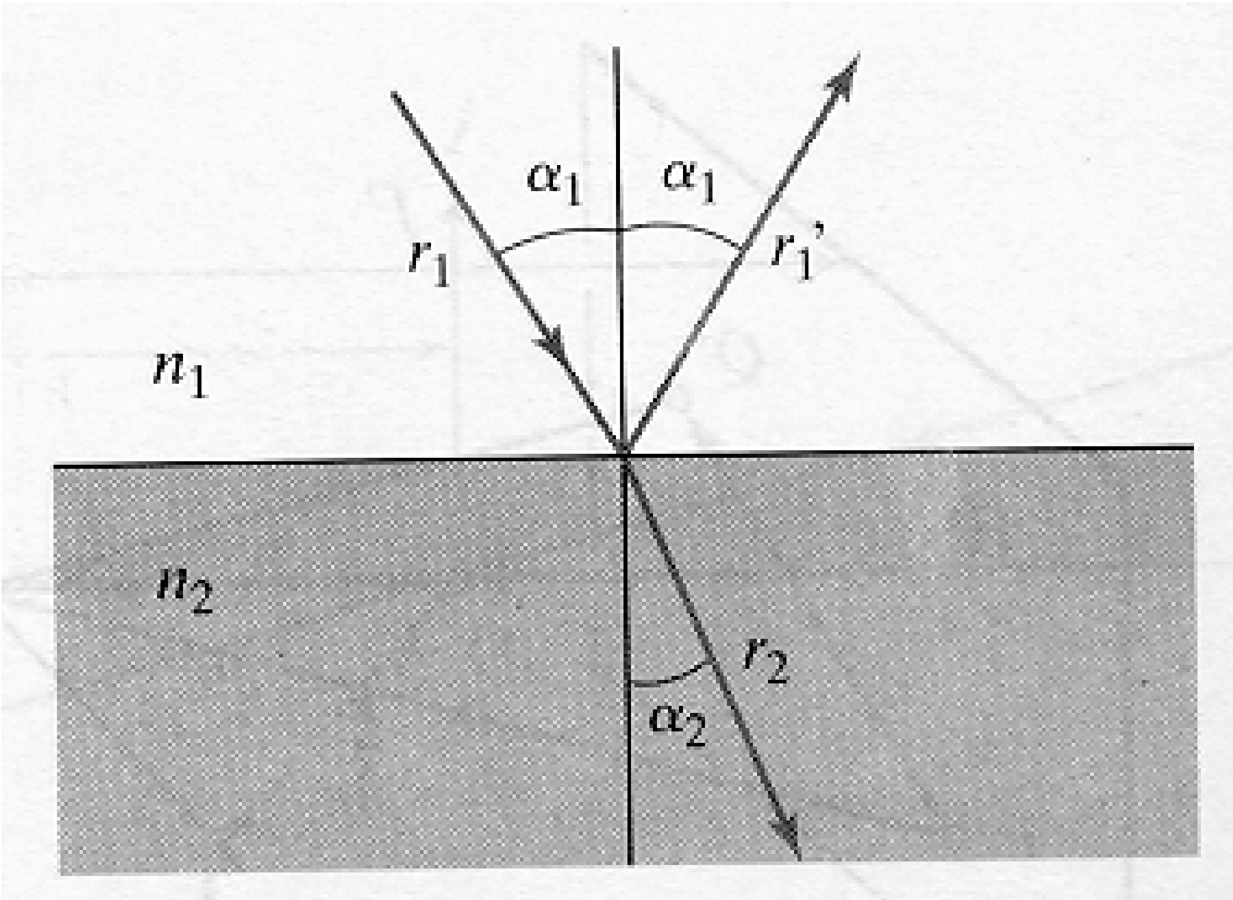


# Water glass refraction



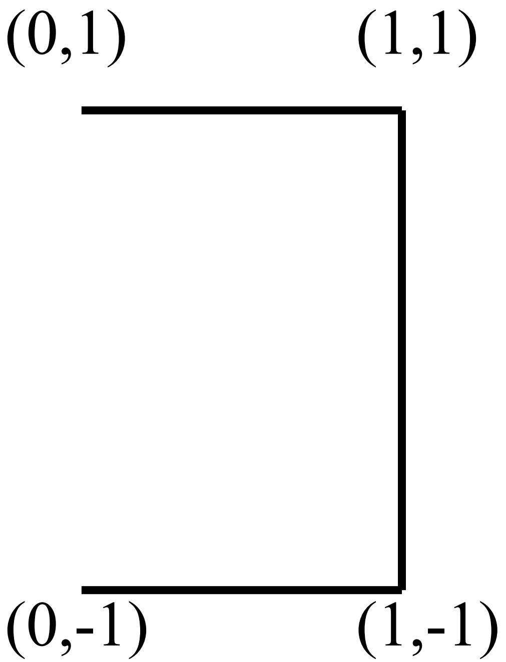
**http://data.pg2k.hd.org/_exhibits/natural-science/cat-black-and-white-domestic-short-hair-DSH-with-nose-in-glass-of-water-on-bedside-table-tweaked-mono-1-AJHD.jpg**

# Snell's law



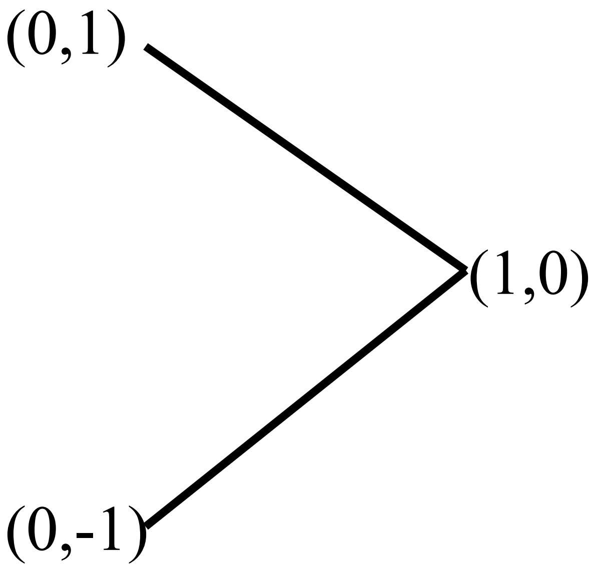

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

# Lens shape



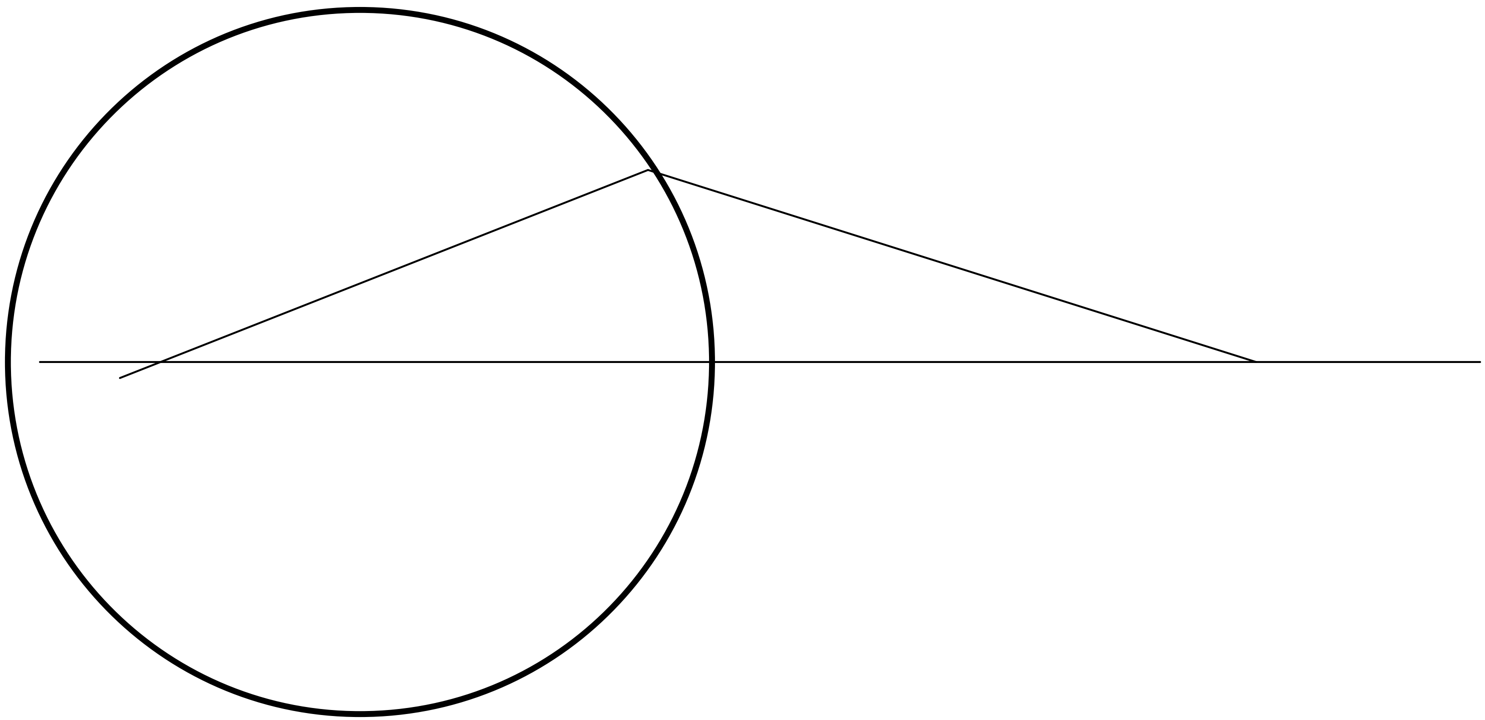

The simplest shape that comes to mind for a computer scientist

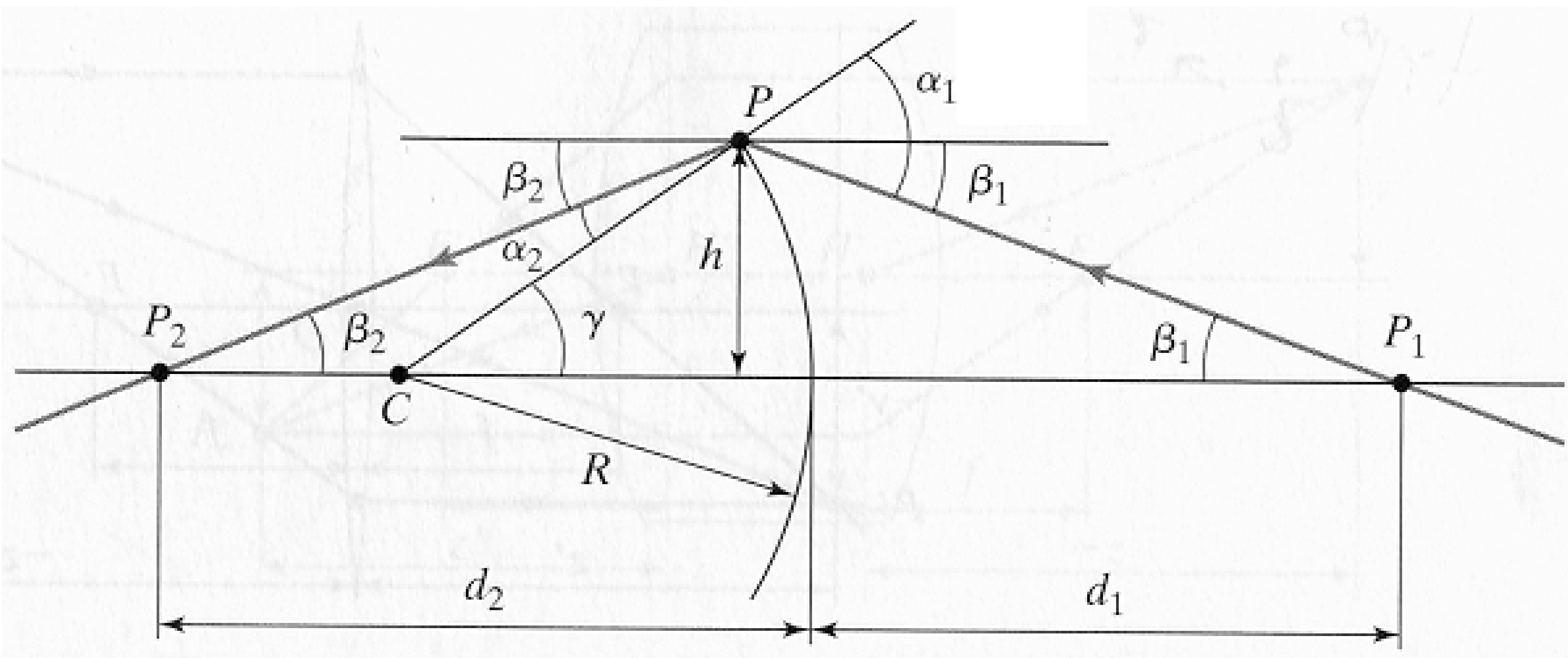
# Lens shape




The next simplest shape

# Spherical lens

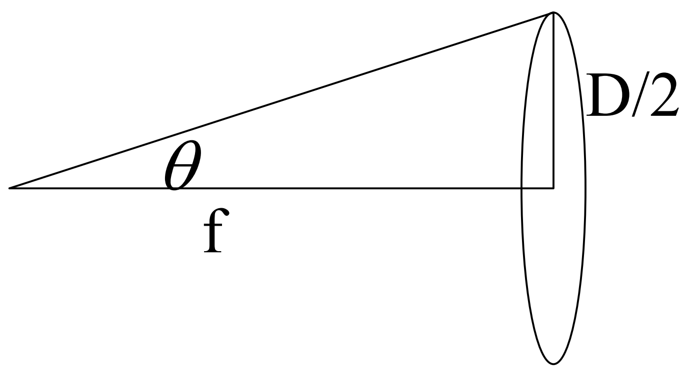





Forsyth and Ponce

# First order optics

$$\sin(\theta) \approx \theta$$




$$\theta \approx \frac{D/2}{f}$$

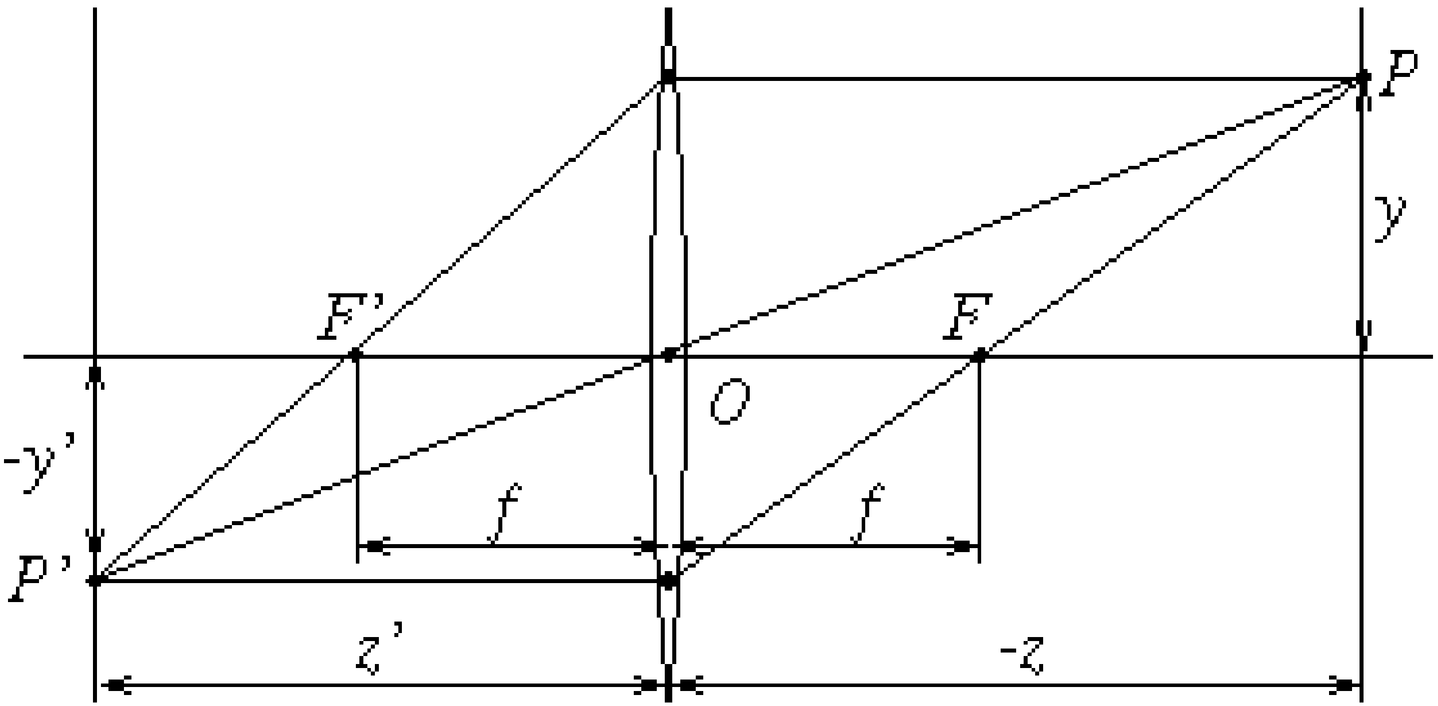
# Paraxial refraction equation

$$\alpha_1 = \gamma + \beta_1 \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$

$$\alpha_2 = \gamma - \beta_2 \approx h\left(\frac{1}{R} - \frac{1}{d_2}\right)$$

$$n_1\alpha_1 \approx n_2\alpha_2 \Leftrightarrow \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

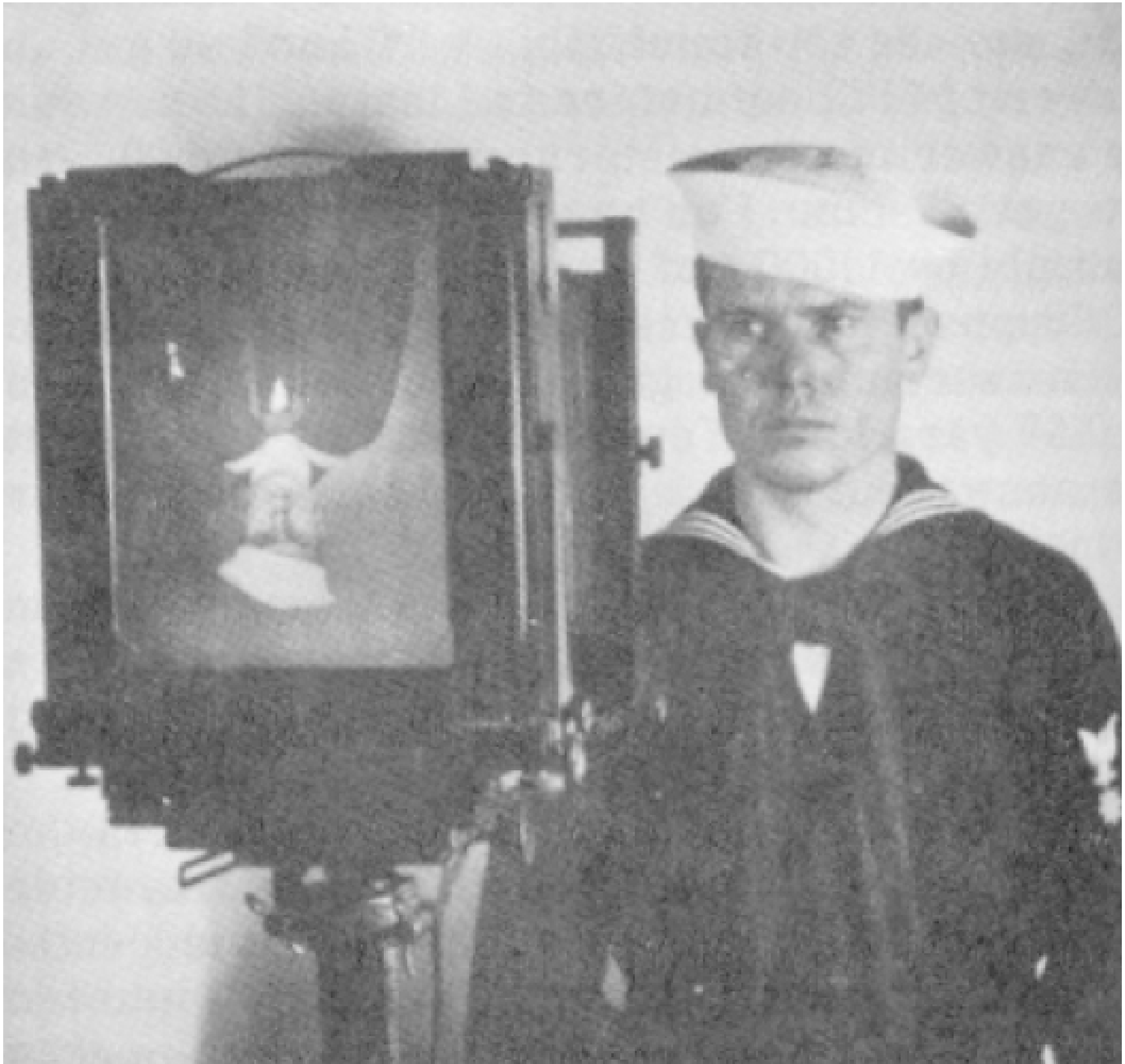
# The thin lens, first order optics




$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$f = \frac{R}{2(n-1)}$$

Forsyth&Ponce

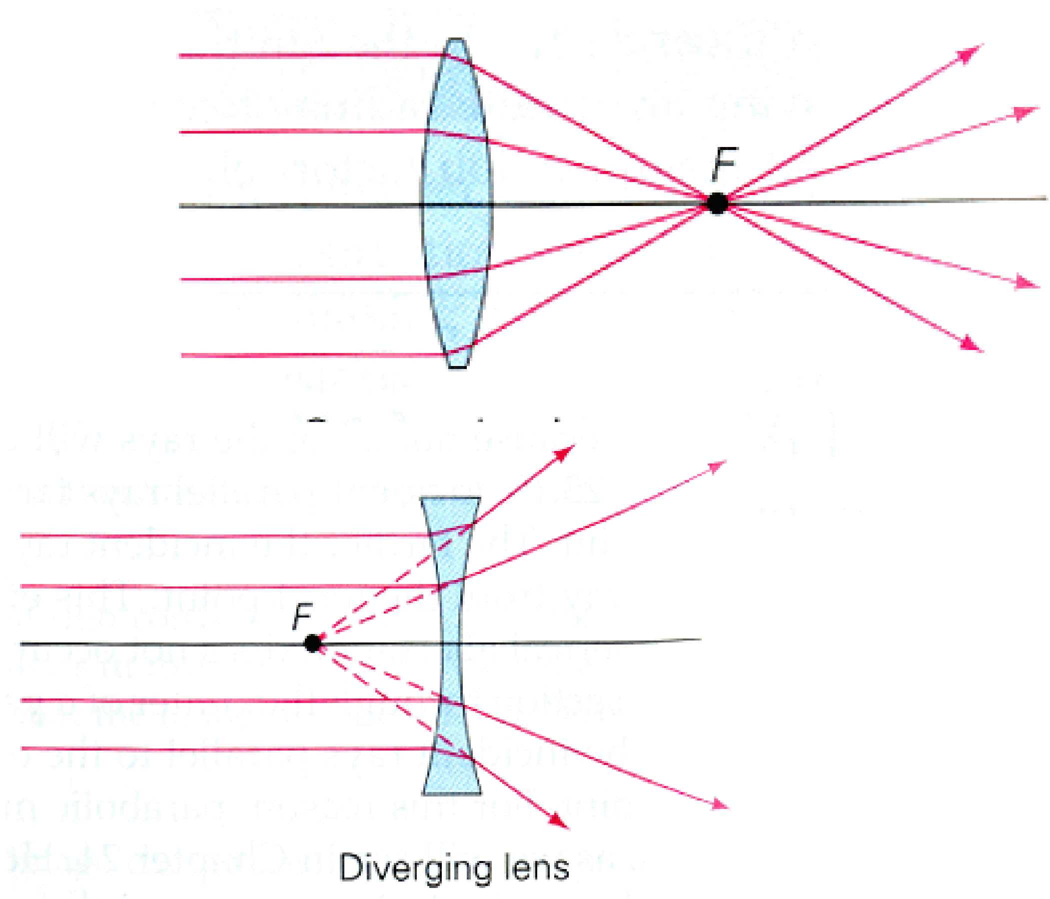


US Navy Manual

# What projection model applies?

# Candle and laser pointer demo

# Convex and concave lenses

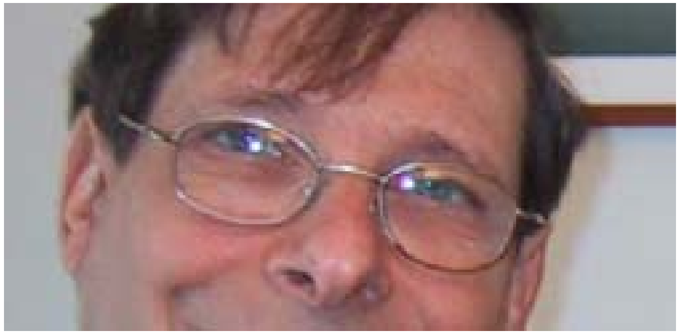



http://www.physics.uiowa.edu/~umallik/adventure/light/lenses.gif

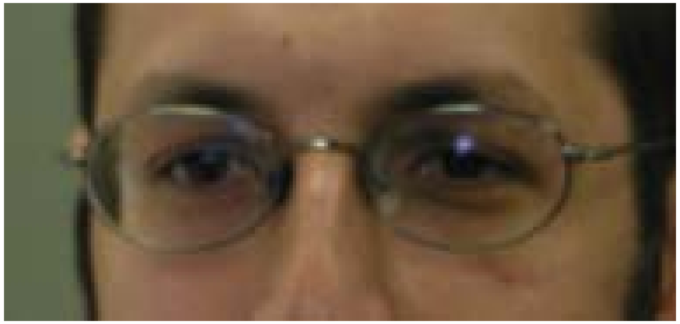
A far-sighted person wearing eyeglasses.

A near-sighted person wearing eyeglasses.

Why do glasses on a far-sighted person make their eyes look larger, while those on a near-sighted person make their eyes look smaller?



Far-sighted



Near-sighted

# More accurate models of real lenses

- Finite lens thickness
- Higher order approximation to $\sin(\theta)$
- Chromatic aberration
- Vignetting

# Thick lens

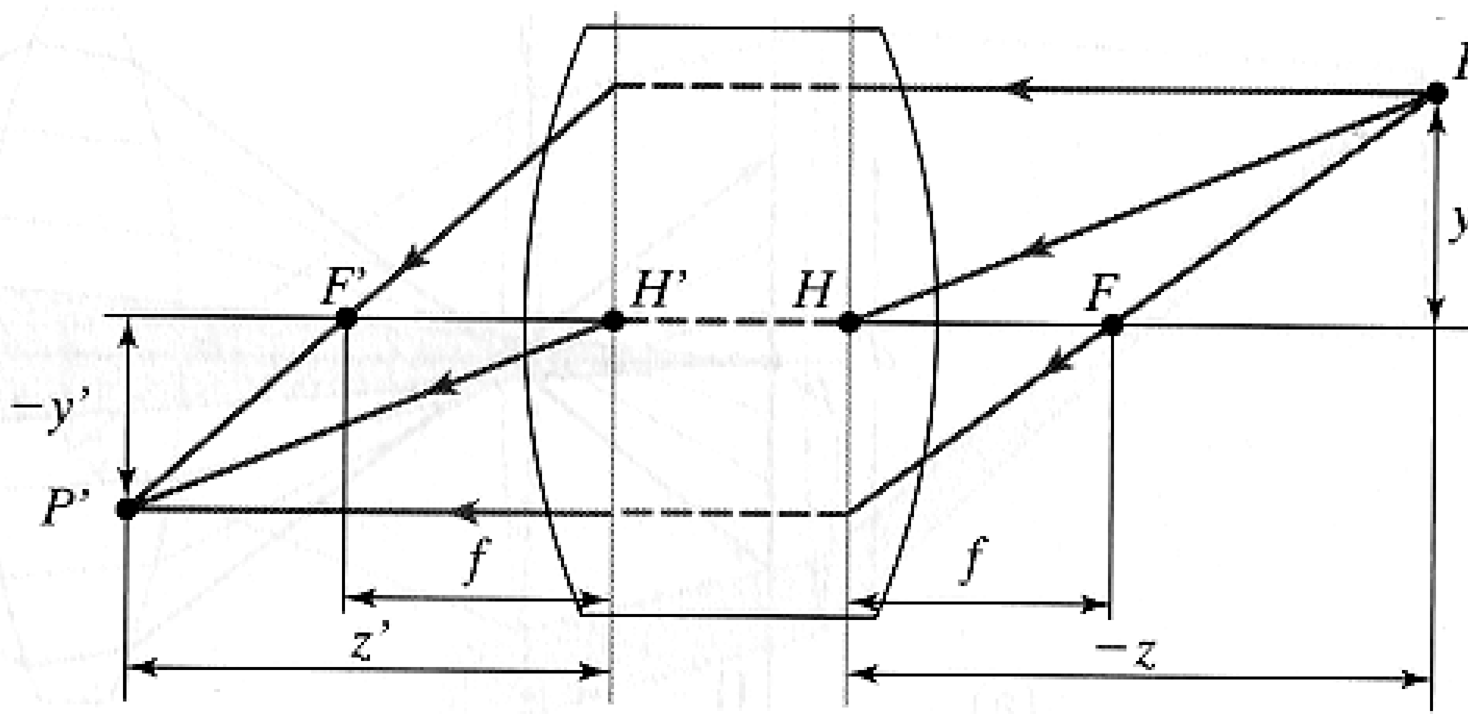
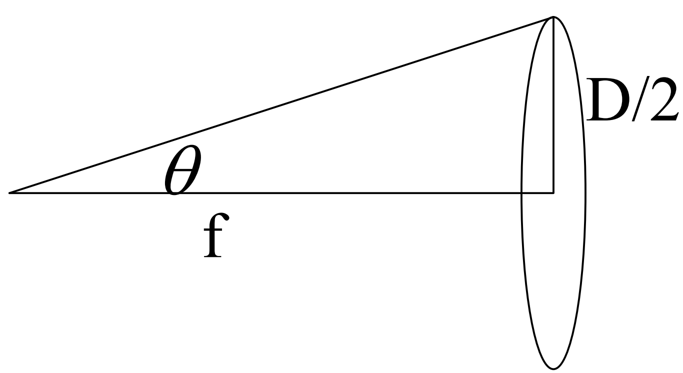



**Figure 1.11** A simple thick lens with two spherical surfaces.

Forsyth&Ponce

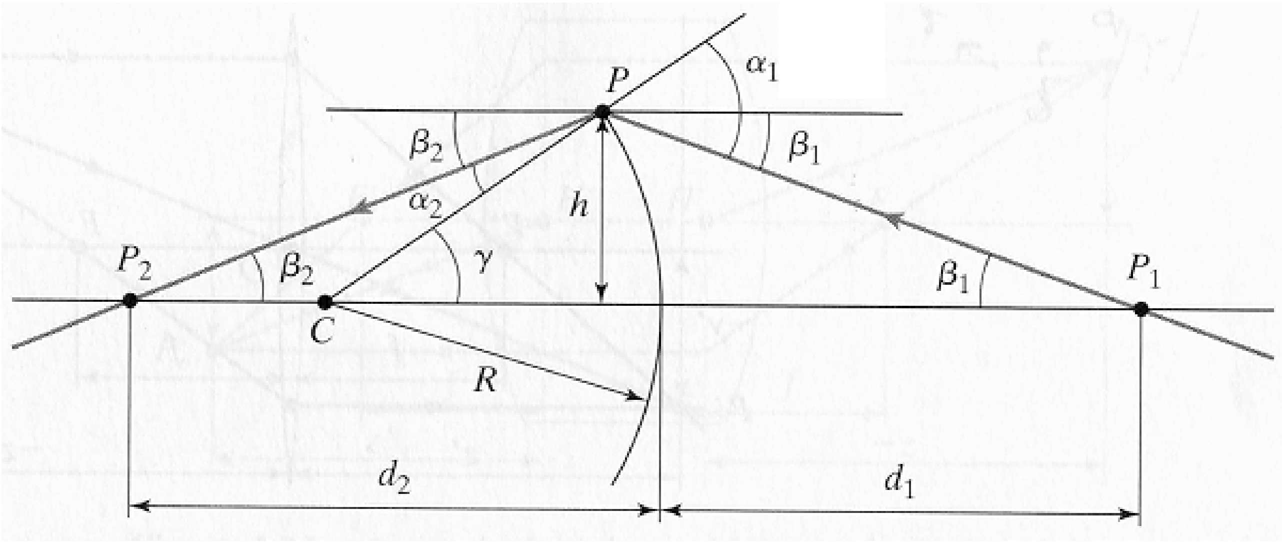
# Third order optics

$$\sin(\theta) \approx \theta - \frac{\theta^3}{6}$$




$$\theta \approx \frac{D/2}{f} - \frac{\left(\frac{D/2}{f}\right)^3}{6}$$

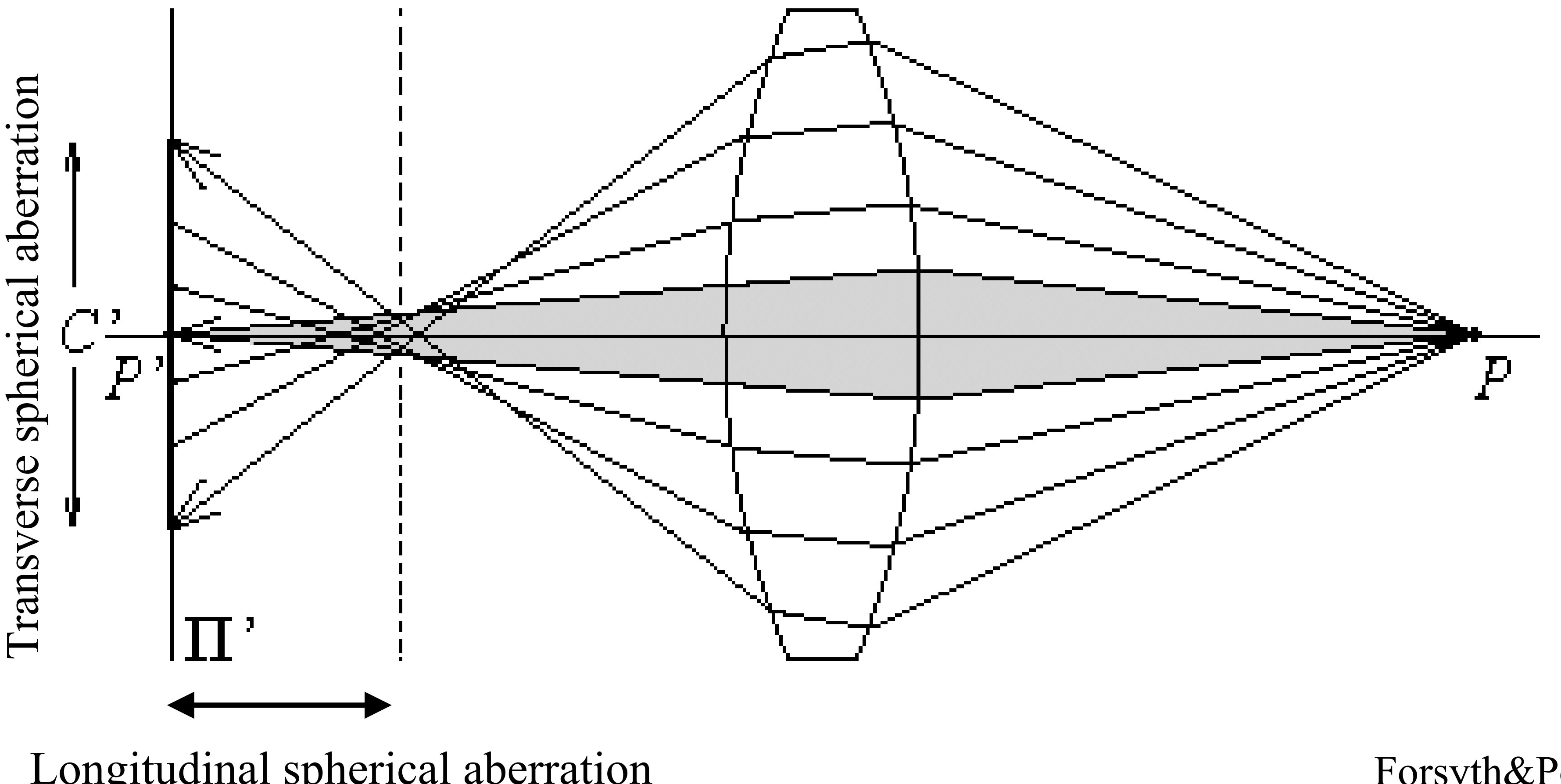
# Paraxial refraction equation, 3rd order optics




$$\frac{n_1}{d_1}+\frac{n_2}{d_2}=\frac{n_2-n_1}{R}+h^2\left[\frac{n_1}{2d_1}\left(\frac{1}{R}+\frac{1}{d_1}\right)^2+\frac{n_2}{2d_2}\left(\frac{1}{R}-\frac{1}{d_2}\right)^2\right]$$

Forsyth&Ponce

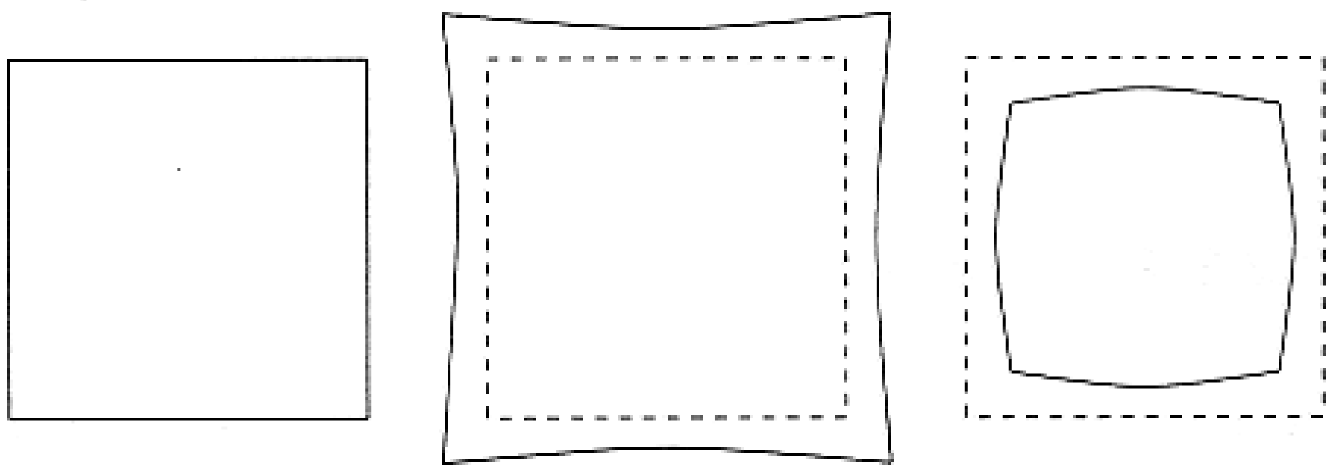
# Spherical aberration (from 3rd order optics




Forsyth&Ponce

# Other 3$^{rd}$ order effects

- Coma, astigmatism, field curvature, distortion.



no distortion

pincushion distortion

barrel distortion

Forsyth&Ponce

# Astigmatic distortion

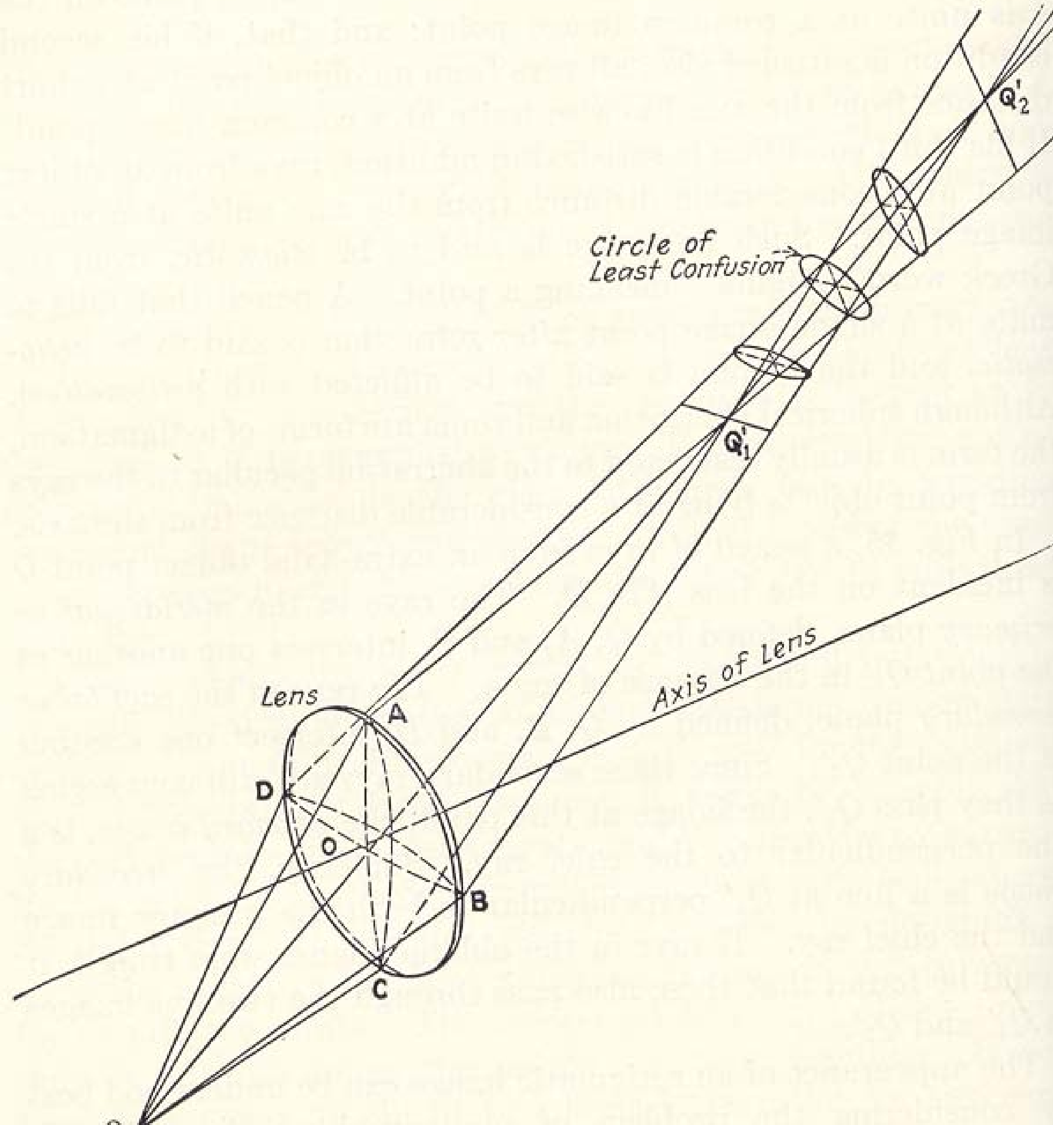
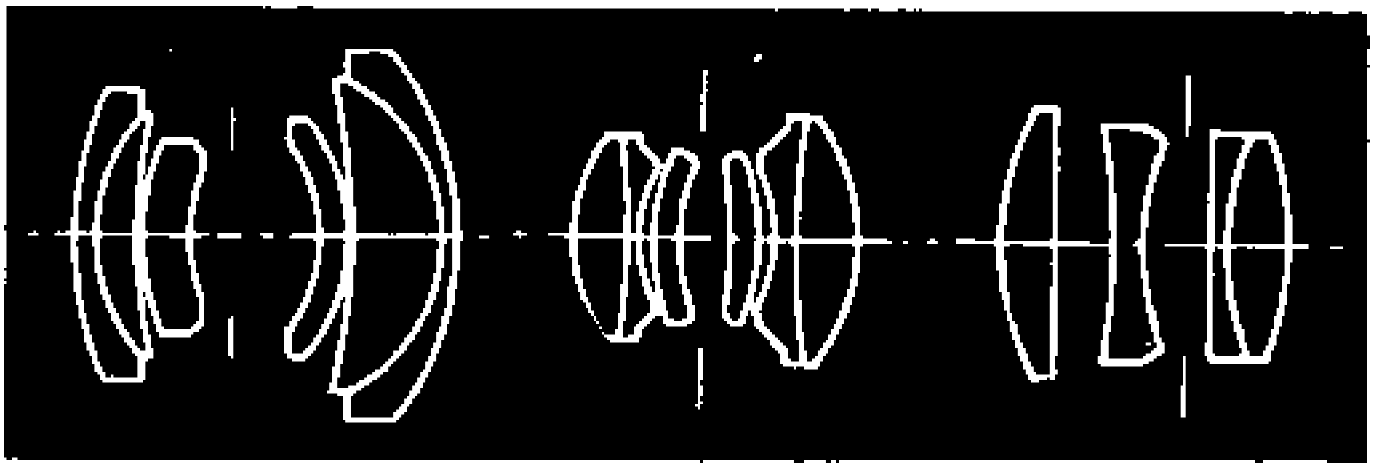



FIG. 45.—An illustration of the character of astigmatic images.

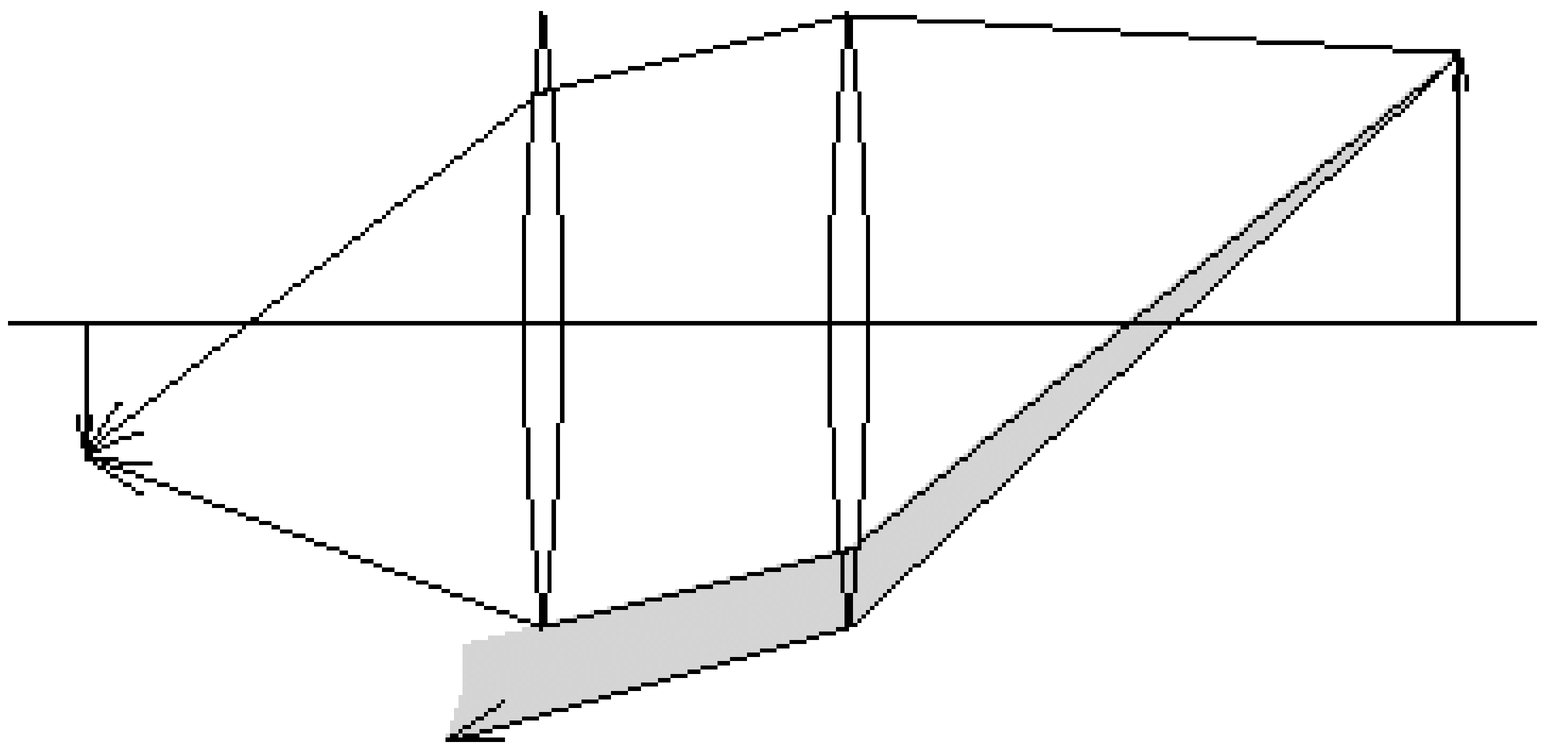
Hardy & Perrin,
The Principles of Optics, 1932

# Lens systems



Lens systems can be designed to correct for aberrations described by $3^{rd}$ order optics

# Vignetting



Forsyth&Ponce

# Chromatic aberration

(great for prisms, bad for lenses)

# Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it
- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)

# Summary

- Want to make images
- Pinhole camera models the geometry of perspective projection
- Lenses make it work in practice
- Models for lenses
  - Thin lens, spherical surfaces, first order optics
  - Thick lens, higher-order optics, vignetting.