Color

- Reading:
  – Chapter 6, Forsyth & Ponce
- Optional reading:
  – Chapter 4 of Wandell, Foundations of Vision, Sinauer, 1995 has a good treatment of this.

Sept. 26, 2002
MIT 6.801/6.866
Profs. Freeman and Darrell

Next class: shape-from-shading

- Reading:
  - Chapter 11 Horn

Oct. 1, 2002
MIT 6.801/6.866
Profs. Freeman and Darrell

Why does a visual system need color?

http://www.hobbylinc.com/gr/pll/pll5019.jpg

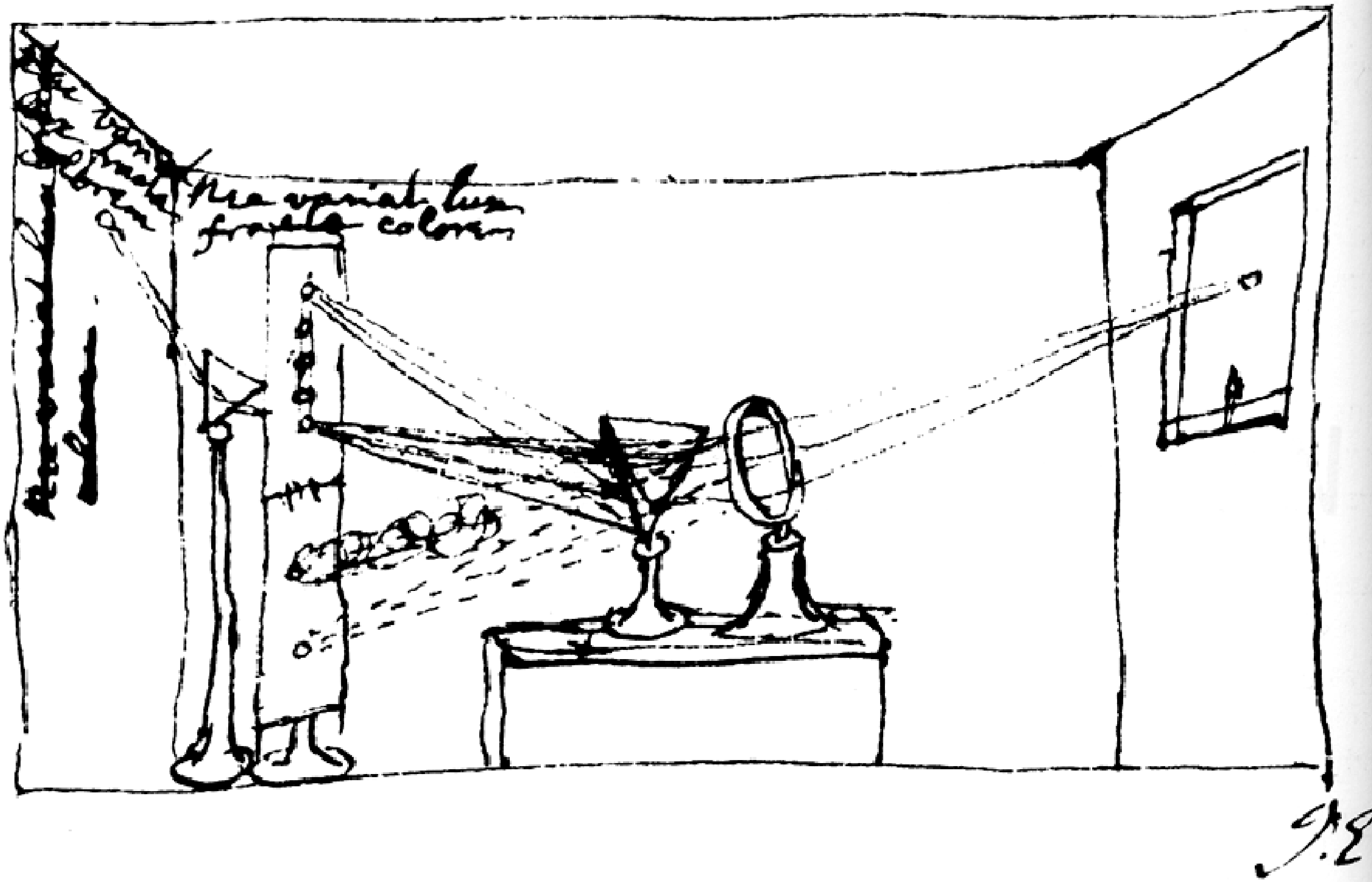
Why does a visual system need color? (an incomplete list…)

- To tell what food is edible.
- To distinguish material changes from shading changes.
- To group parts of one object together in a scene.
- To find people’s skin.
- Check whether someone’s appearance looks normal/healthy.
- To compress images

Lecture outline

- Color physics.
- Color perception and color matching.
- Inference about the world from color observations.

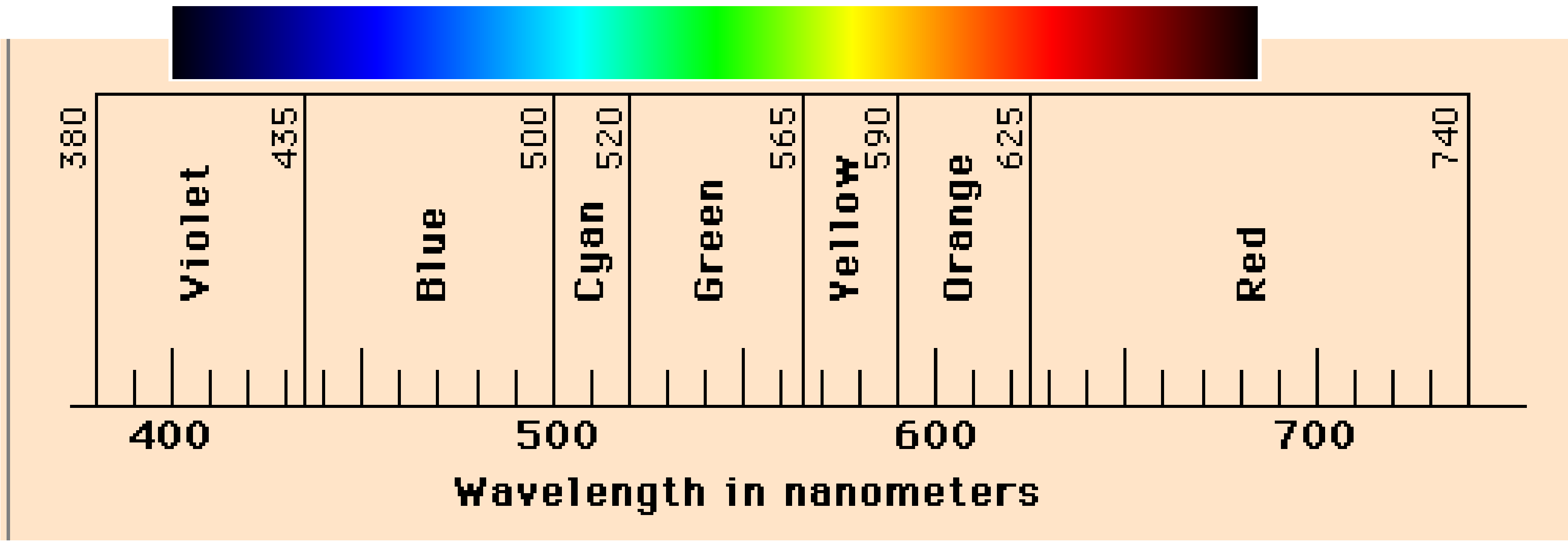
Color



4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

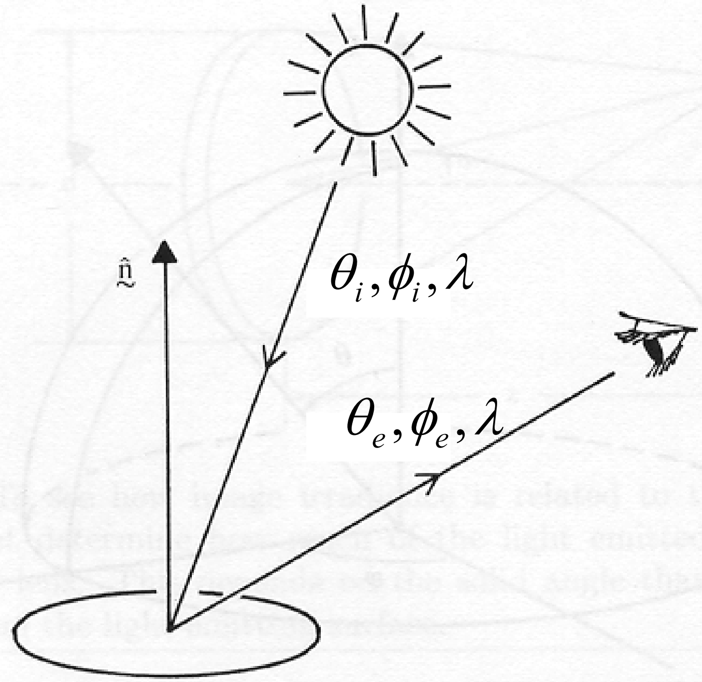
Spectral colors



http://hyperphysics.phy-astr.gsu.edu/hbase/vision/specol.html#c2

Radiometry for colour

- All definitions are now “per unit wavelength”
- All units are now “per unit wavelength”
- All terms are now “spectral”
- Radiance becomes spectral radiance
  - watts per square meter per steradian per unit wavelength
- Irradiance becomes spectral irradiance
  - watts per square meter per unit wavelength



Horn, 1986

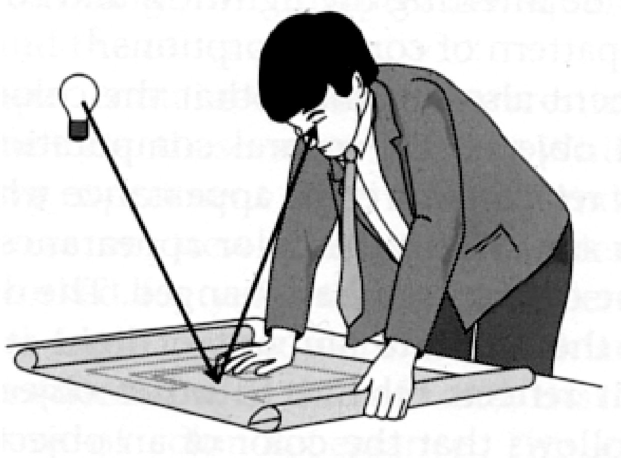
Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction (θ_e, ϕ_e) to the irradiance resulting from illumination from the direction (θ_i, ϕ_i).

Spectral radiance

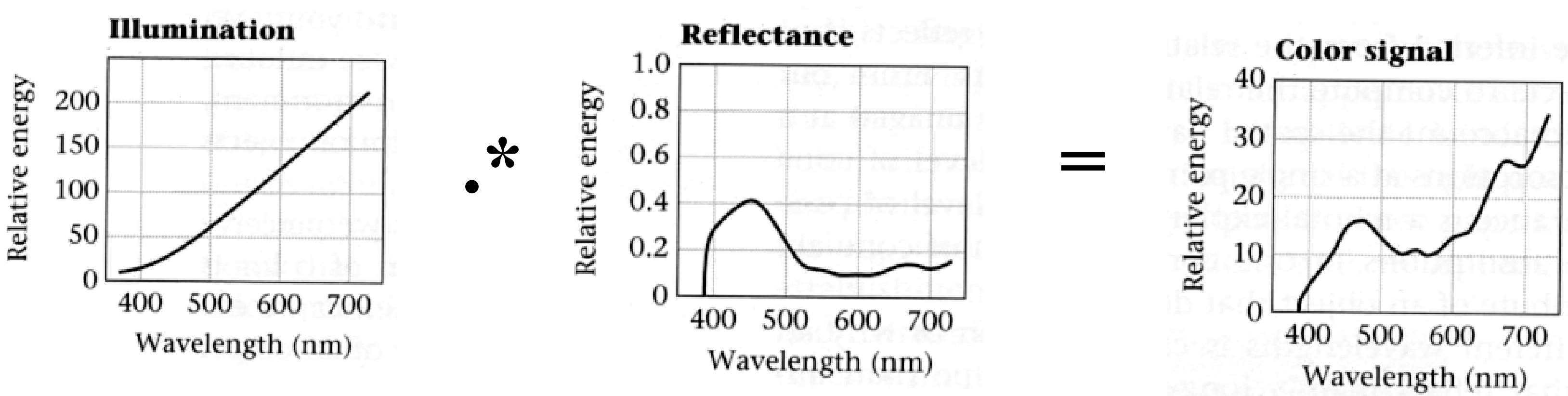
$$BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e, \lambda) = \frac{L(\theta_e, \phi_e, \lambda)}{E(\theta_i, \phi_i, \lambda)}$$

Spectral irradiance

Simplified rendering models: reflectance

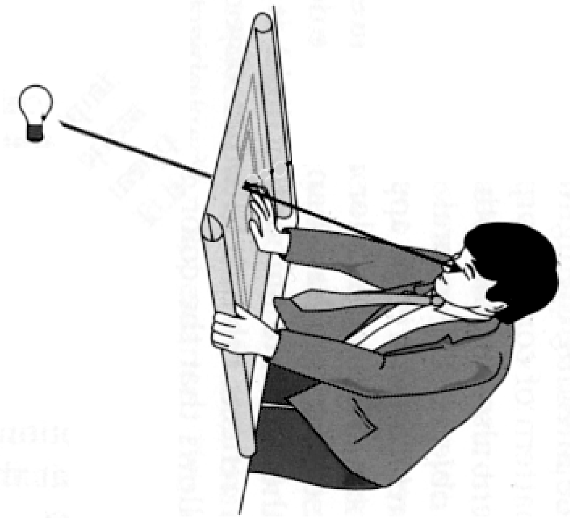


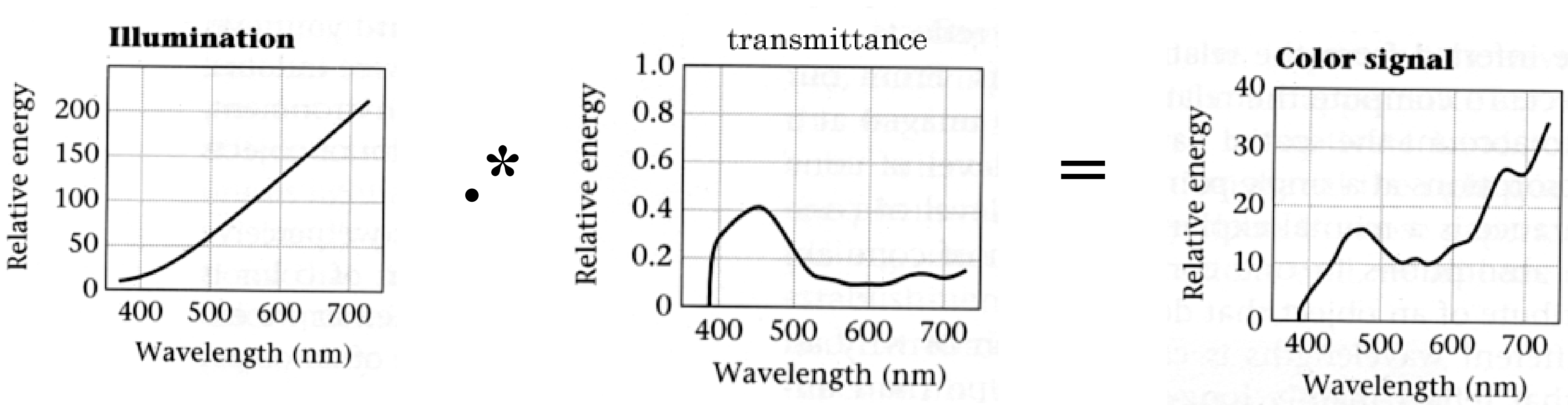
Often are more interested in relative spectral composition than in overall intensity, so the spectral BRDF computation simplifies a wavelength-by-wavelength multiplication of relative energies.



Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Simplified rendering models: transmittance

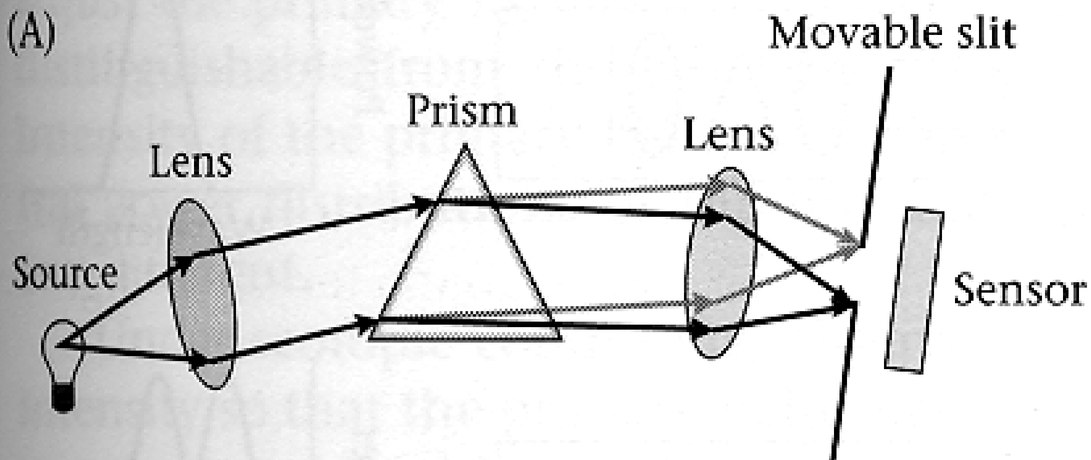




Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Spectrophotometer

(A)



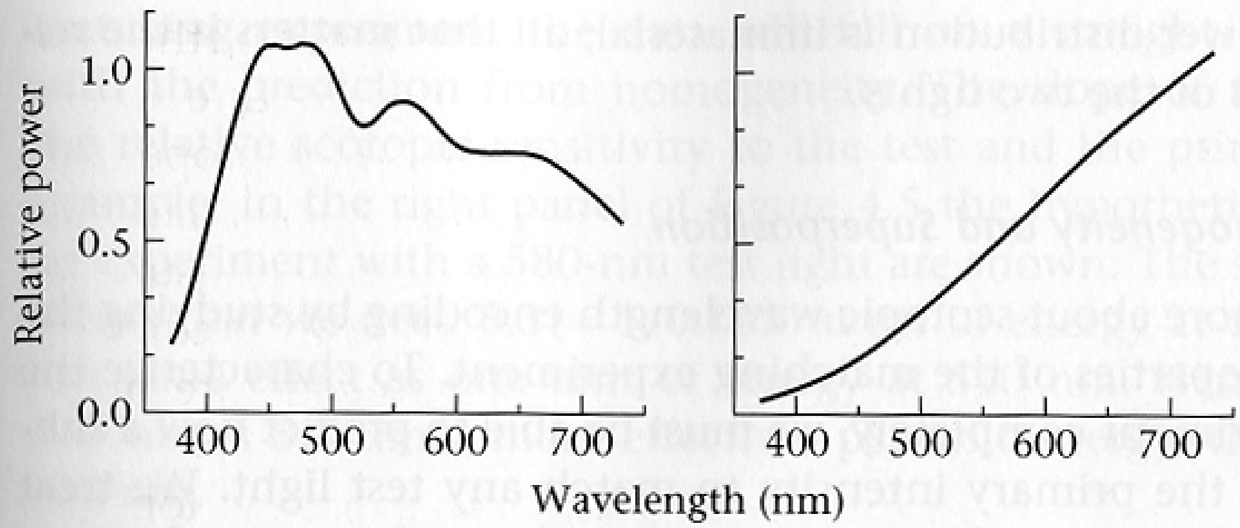
(B)

Wavelength (nm)	Color (viewed in isolation)
700	Red
610	Orange
580	Yellow
540	Green
480	Blue
400	Violet

4.2 A SPECTRORADIOMETER is used to measure the spectral power distribution of light. (A) A schematic design of a spectroradiometer includes a means for separating the input light into its different wavelengths and a detector for measuring the energy at each of the separate wavelengths. (B) The color names associated with the appearance of lights at a variety of wavelengths are shown. After Wyszecki and Stiles, 1982.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

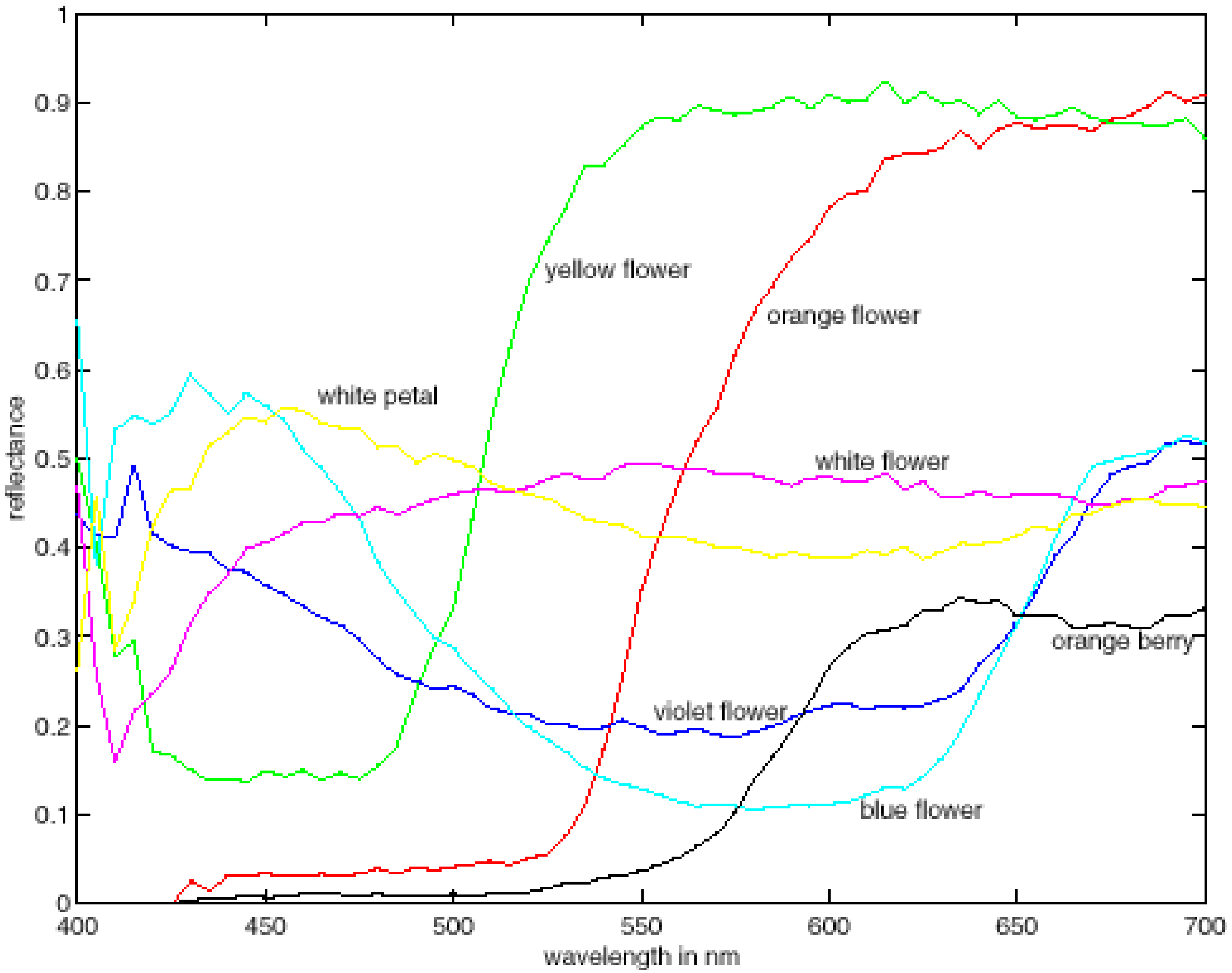
Two illumination spectra



4.4 THE SPECTRAL POWER DISTRIBUTION of two important light sources are shown: (left) blue skylight and (right) a tungsten bulb.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

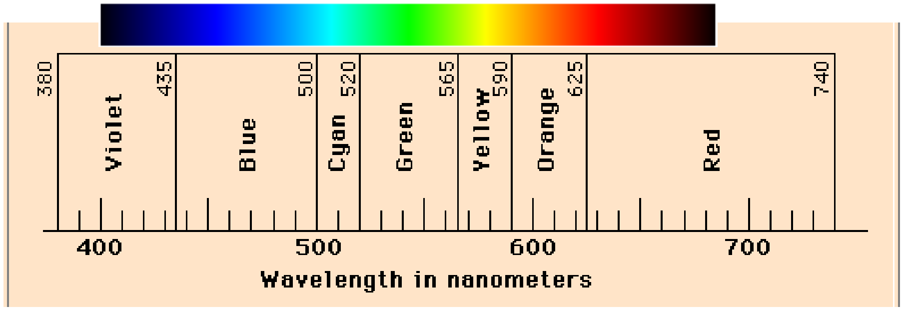
Some reflectance spectra

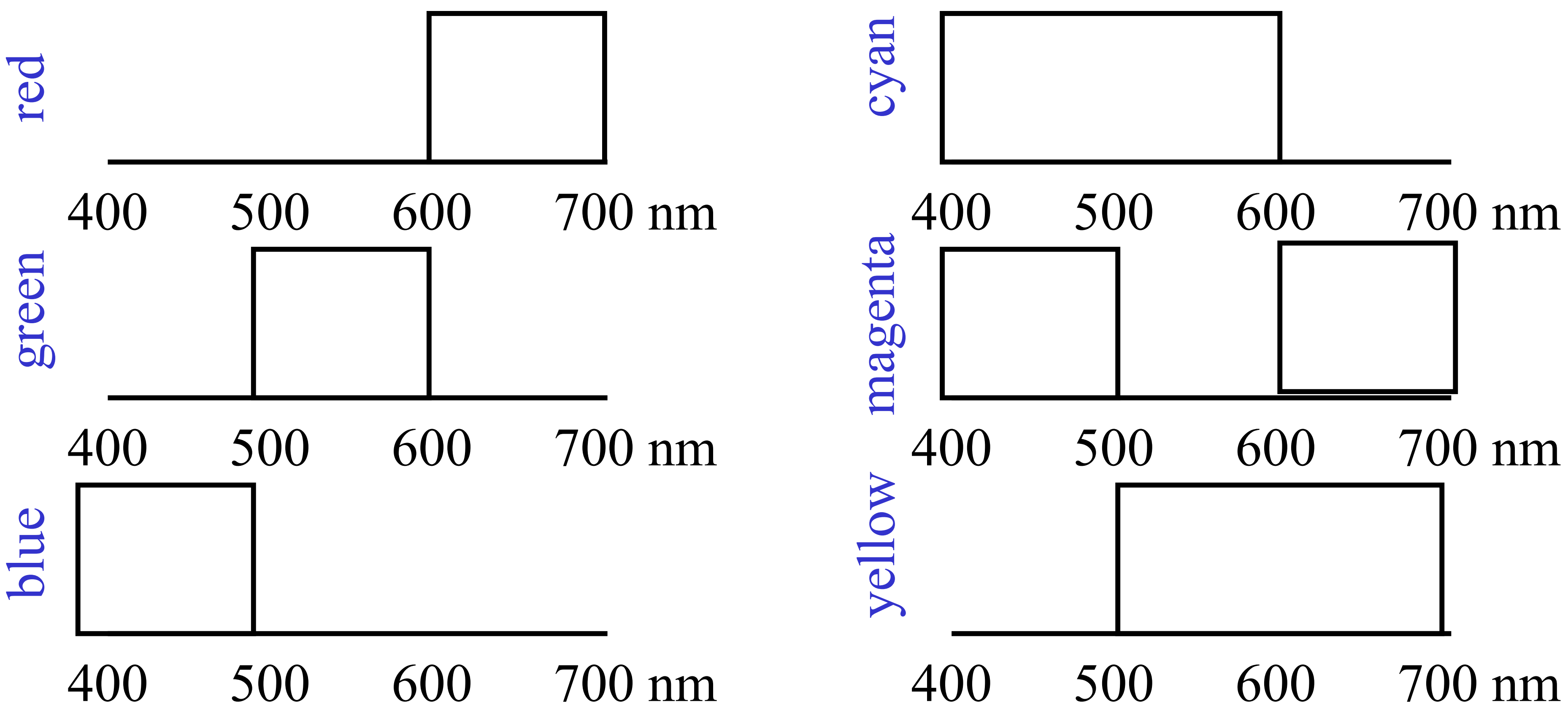


Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.

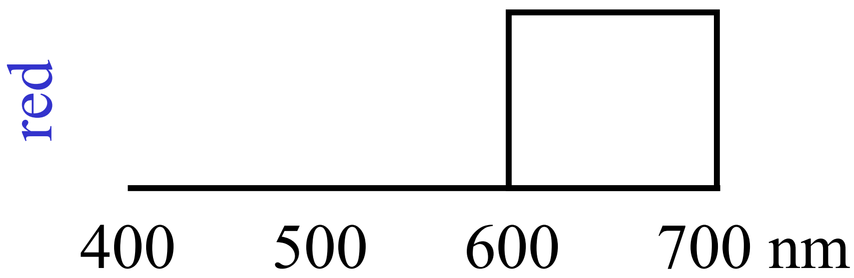
Forsyth, 2002

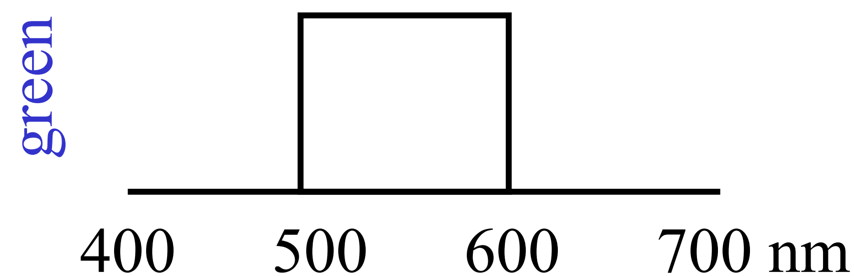
Color names for cartoon spectra

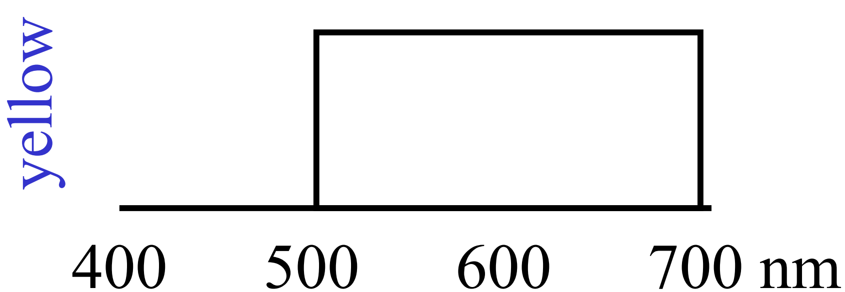




Additive color mixing





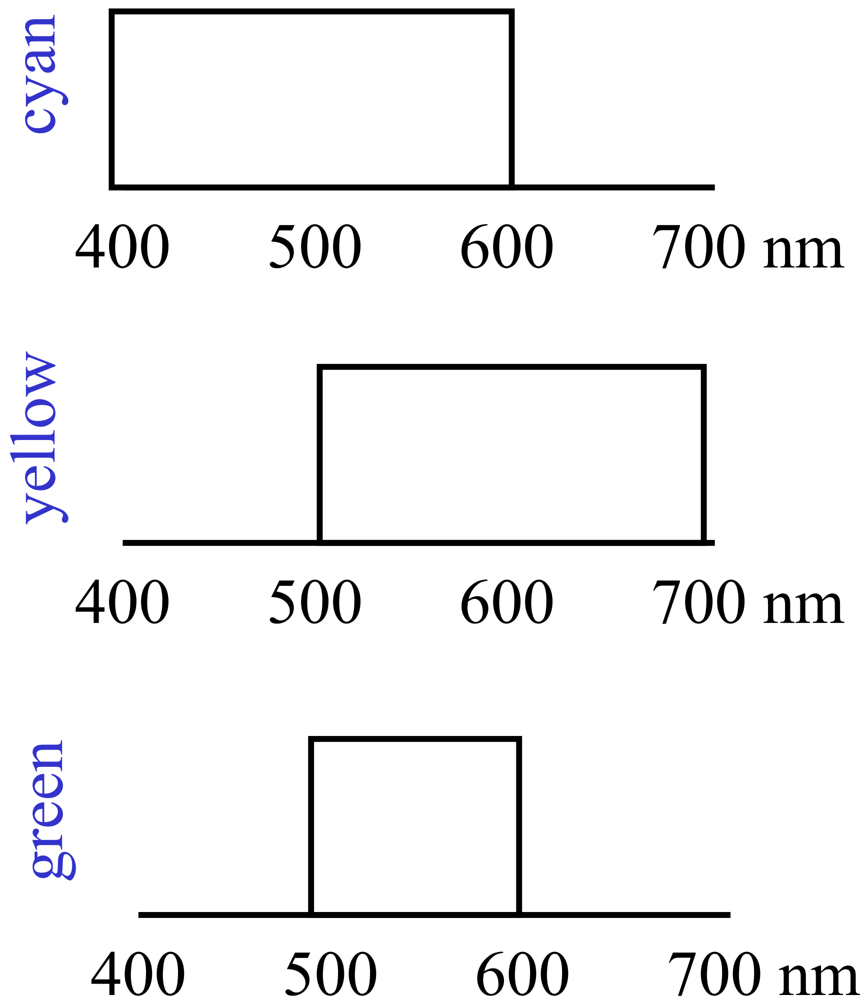


When colors combine by *adding* the color spectra. Examples that follow this mixing rule: CRT phosphors, multiple projectors aimed at a screen, Polachrome slide film.

Red and green make…

Yellow!

Subtractive color mixing



When colors combine by *multiplying* the color spectra. Examples that follow this mixing rule: most photographic films, paint, cascaded optical filters, crayons.

Cyan and yellow (in crayons, called "blue" and yellow) make…

Green!

demos

- Additive color
- Subtractive color

Low-dimensional models for color spectra

$$\begin{pmatrix} \vdots \\ e(\lambda) \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots & \vdots & \vdots \\ E_1(\lambda) & E_2(\lambda) & E_3(\lambda) \\ \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

How to find a linear model for color spectra:

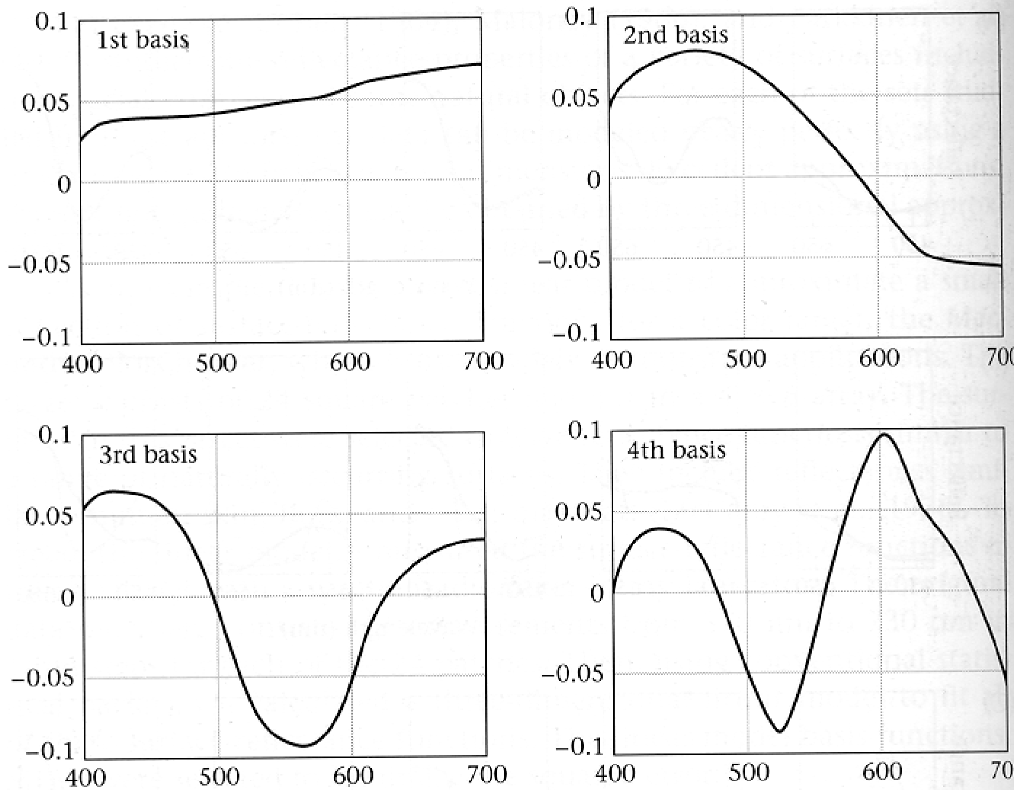
--form a matrix, D, of measured spectra, 1 spectrum per column.

--[u, s, v] = svd(D) satisfies D = u*s*v‘

--the first n columns of u give the best (least-squares optimal) n-dimensional linear bases for the data, D:

$$D \approx u(:,1:n) * s(1:n,1:n) * v(1:n,:)'$$

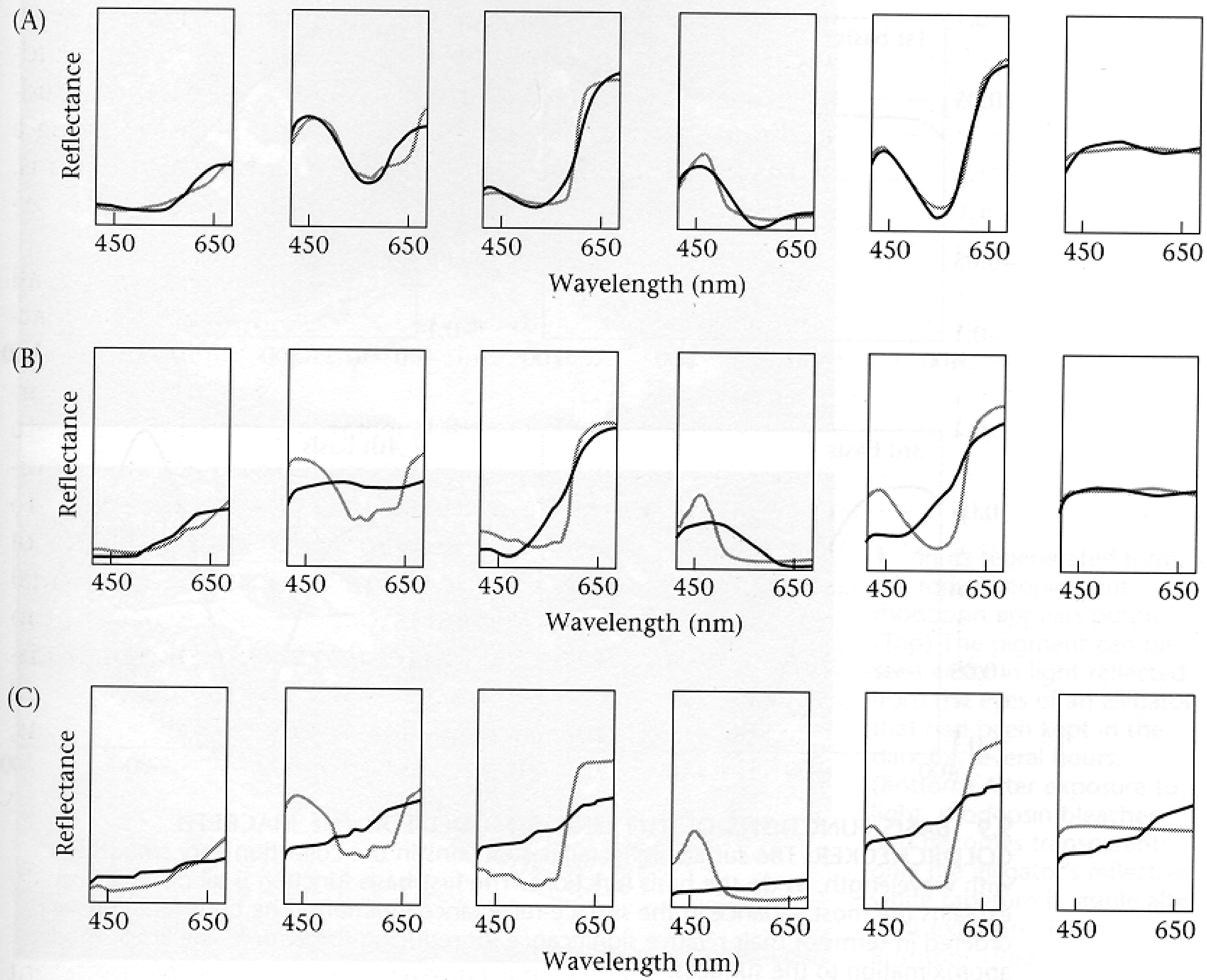
Basis functions for Macbeth color checker



9.9 BASIS FUNCTIONS OF THE LINEAR MODEL FOR THE MACBETH COLORCHECKER. The surface-reflectance functions in the collection vary smoothly with wavelength, as do the basis functions. The first basis function is all positive and explains the most variance in the surface-reflectance functions. The basis functions are ordered in terms of their relative significance for reducing the error in the linear-model approximation to the surfaces.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Low-dimensional models for color spectra



9.8 A LINEAR MODEL TO APPROXIMATE THE SURFACE REFLECTANCES IN THE MACBETH COLORCHECKER. The panels in each row of this figure show the surface-reflectance functions of six colored surfaces (shaded lines) and the approximation to these functions using a linear model (solid lines). The approximations using linear models with (A) three, (B) two, and (C) one dimension are shown.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Outline

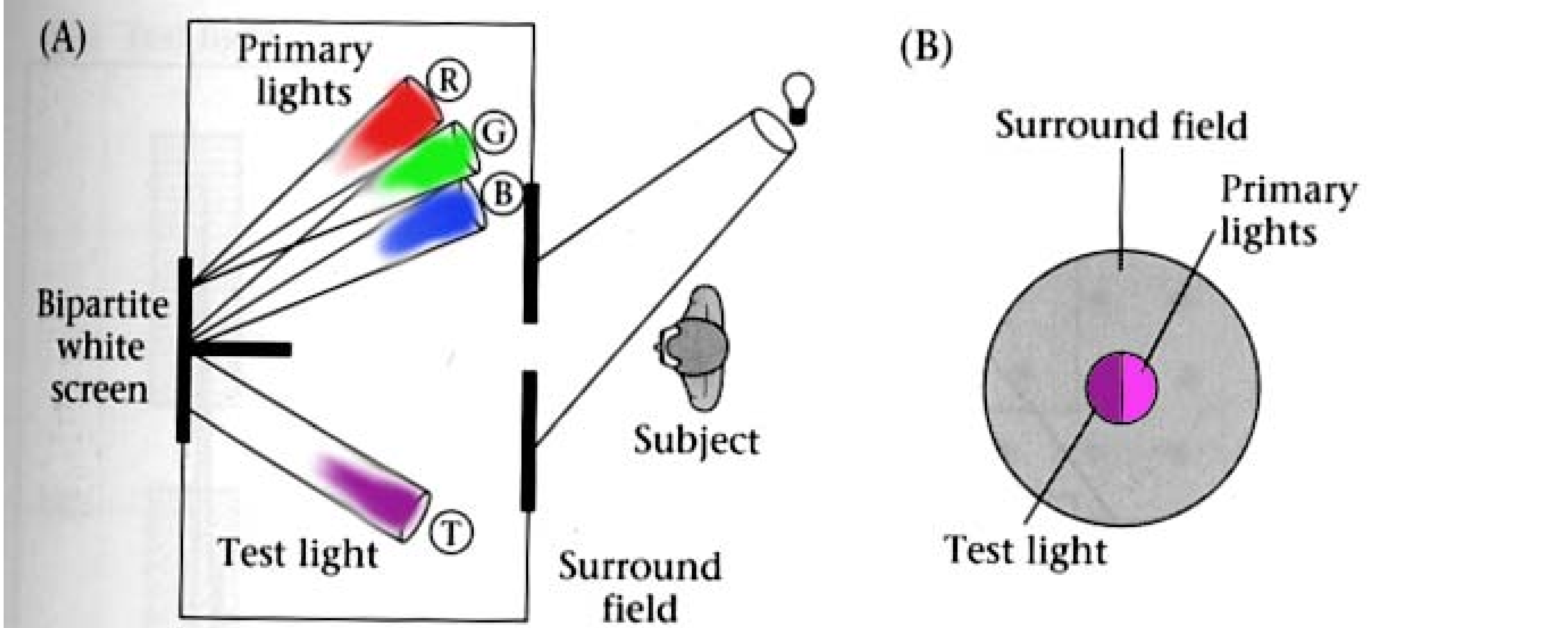
- Color physics.
- Color perception and color matching.
- Inference about the world from color observations.

Why specify color numerically?

- Accurate color reproduction is commercially valuable
  - Many products are identified by color (“golden” arches;
- Few color names are widely recognized by English speakers -
  - About 10; other languages have fewer/more, but not many more.
  - It’s common to disagree on appropriate color names.
- Color reproduction problems increased by prevalence of digital imaging - eg. digital libraries of art.
  - How do we ensure that everyone sees the same color?

Color standards are important in industry

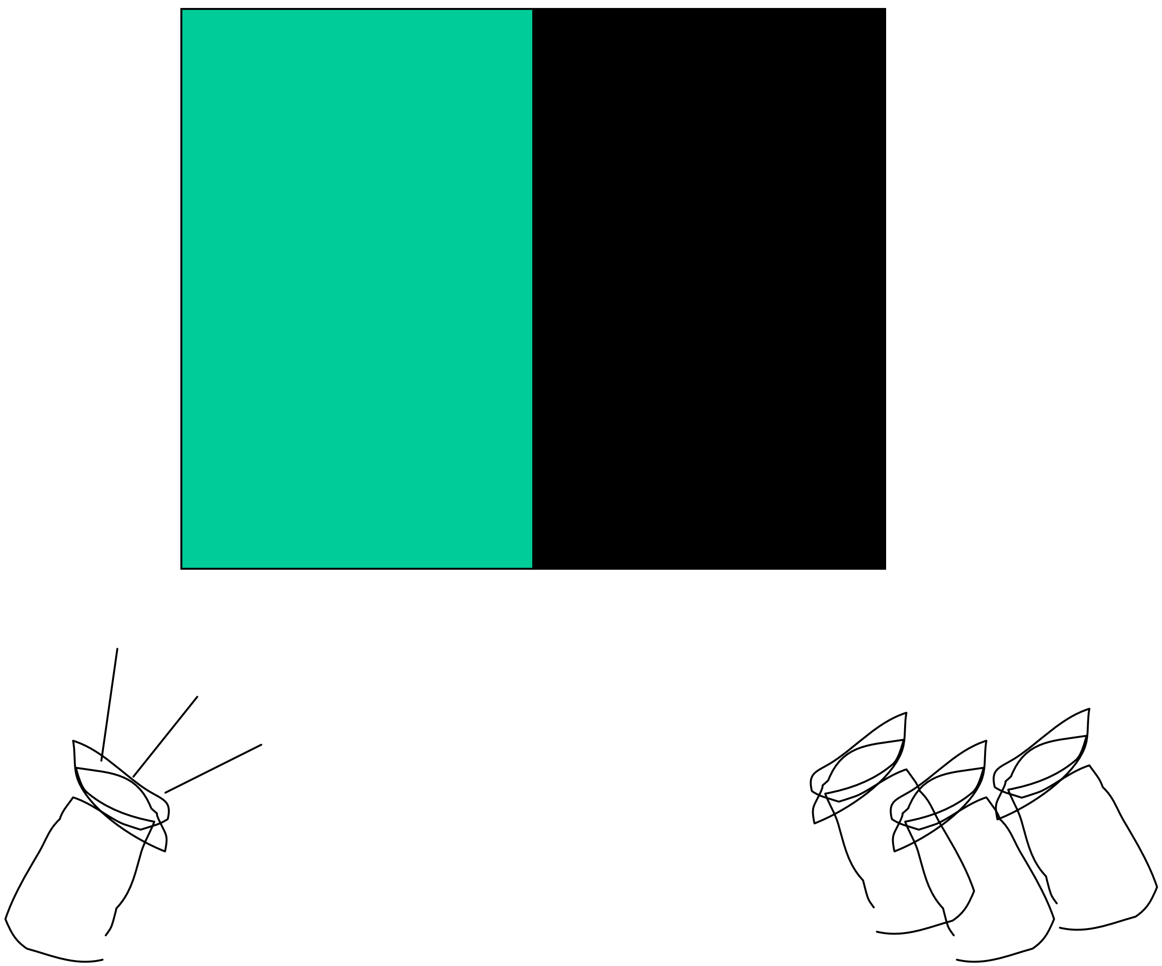
Color matching experiment



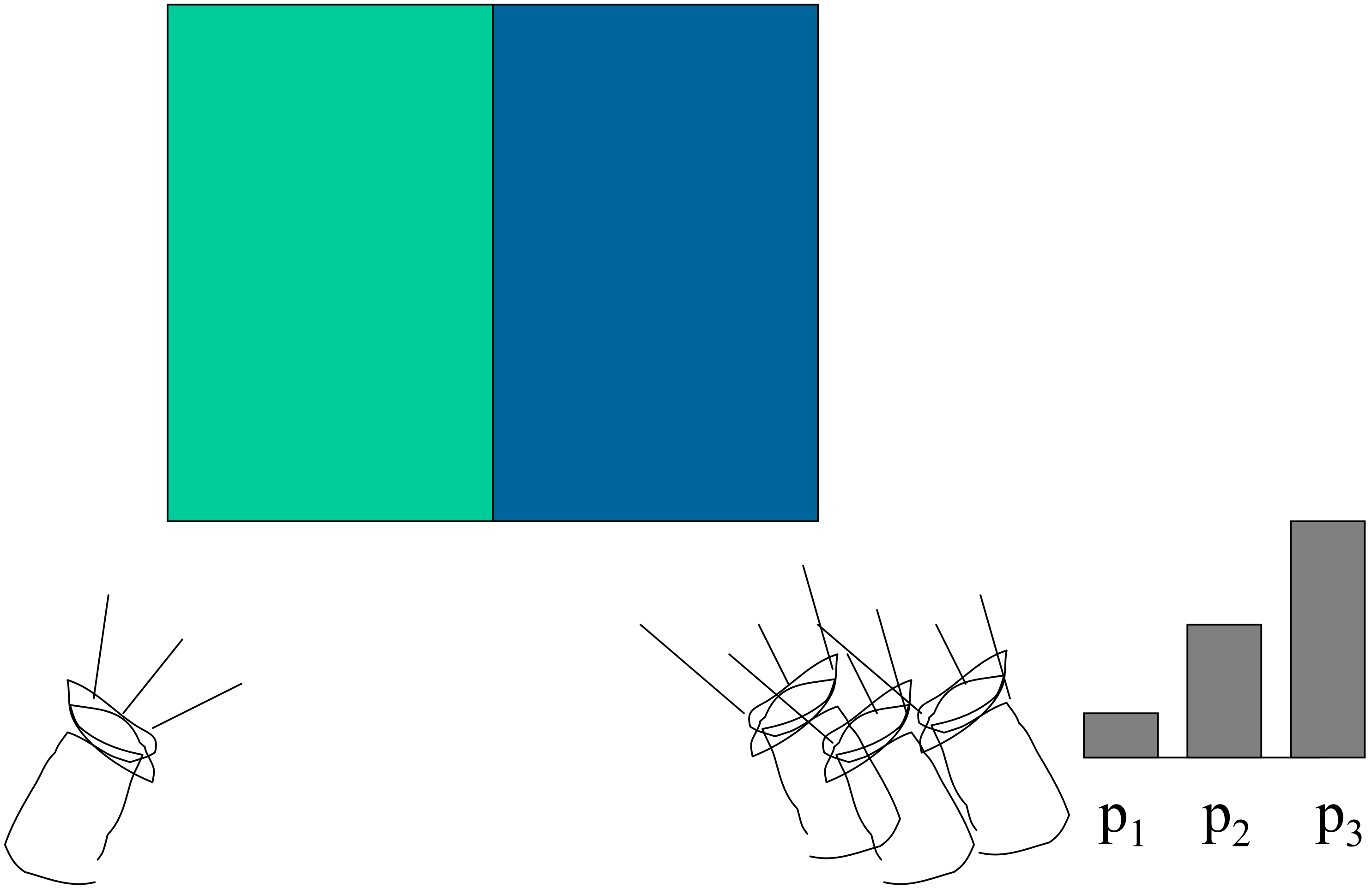
4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

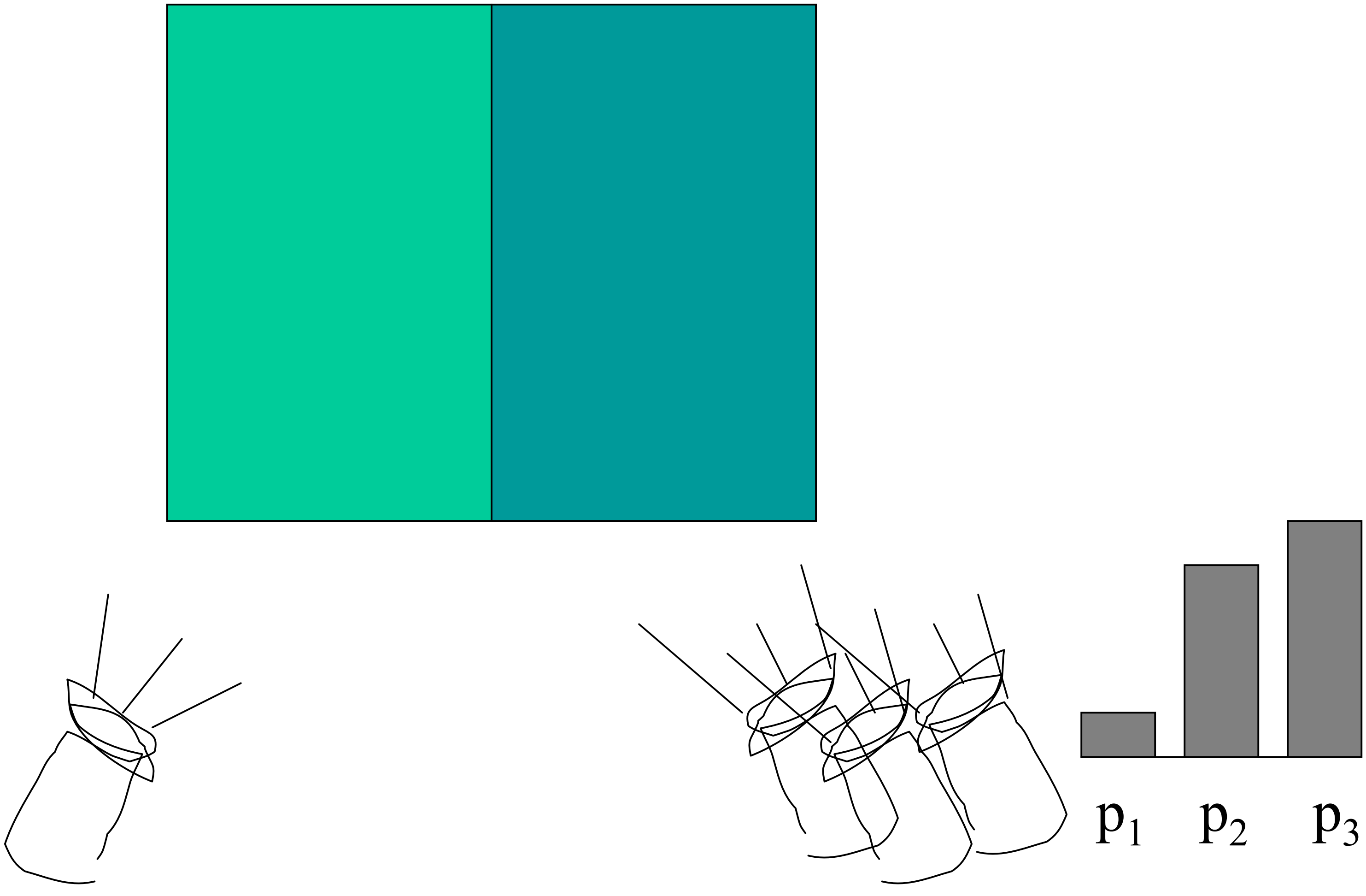
Color matching experiment 1



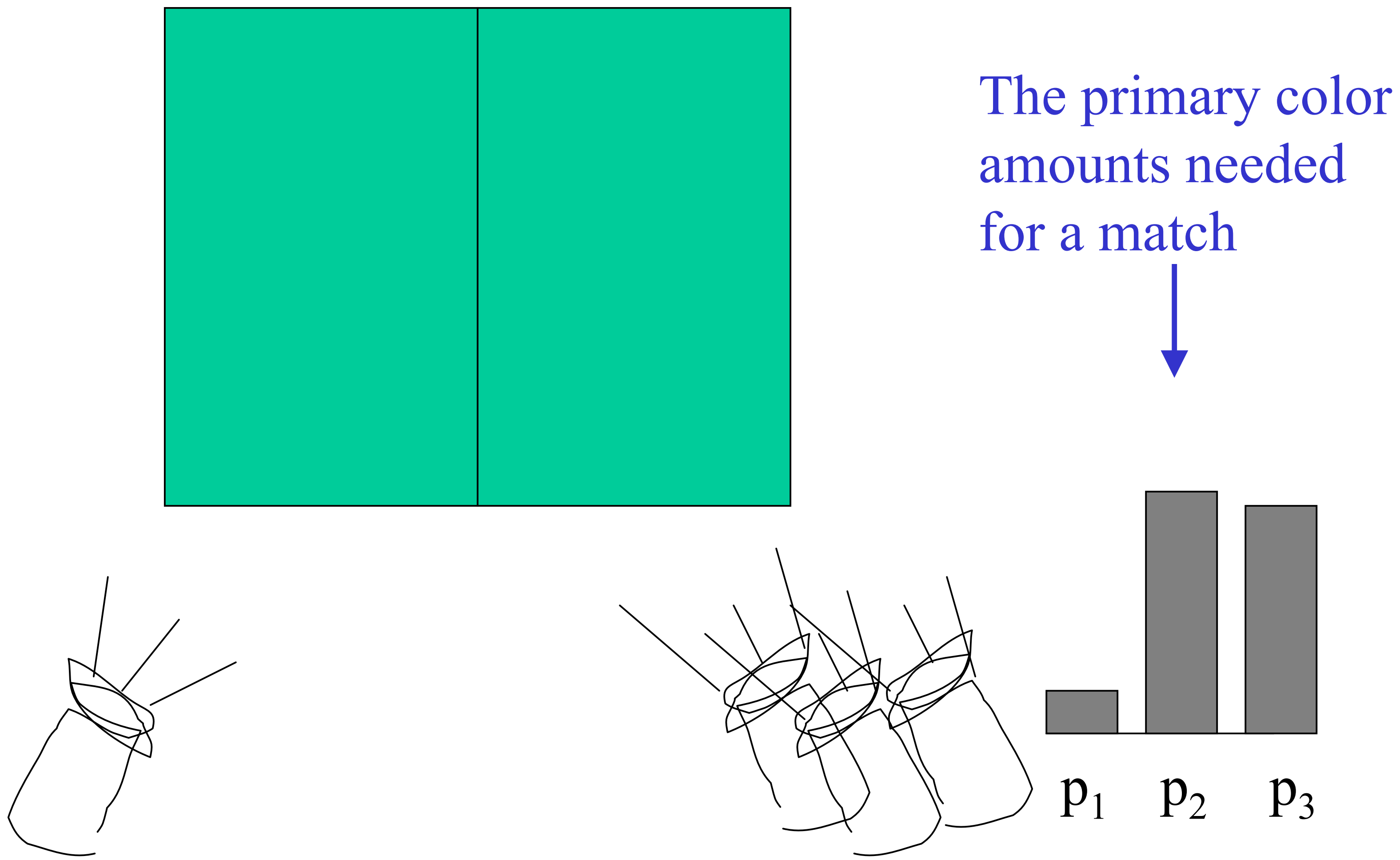
Color matching experiment 1



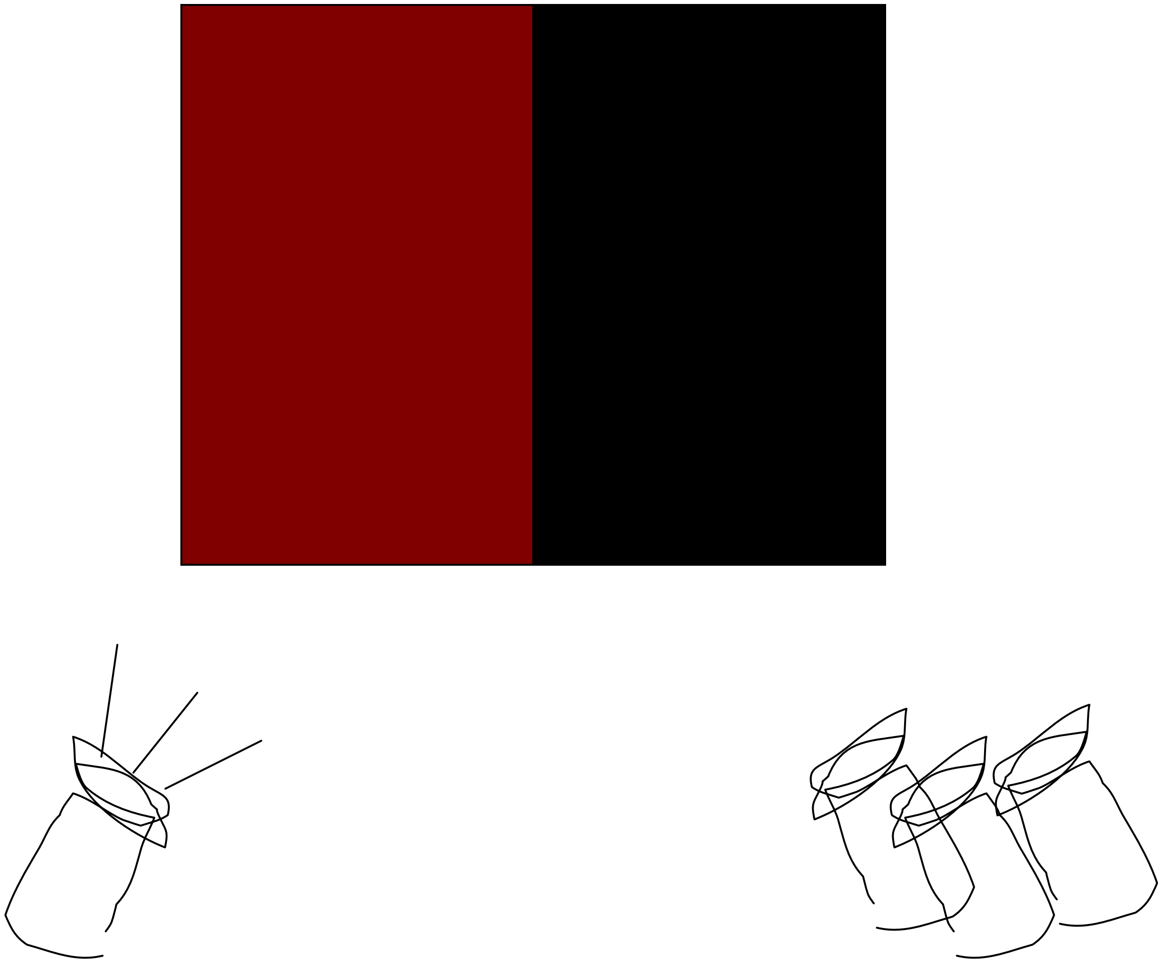
Color matching experiment 1



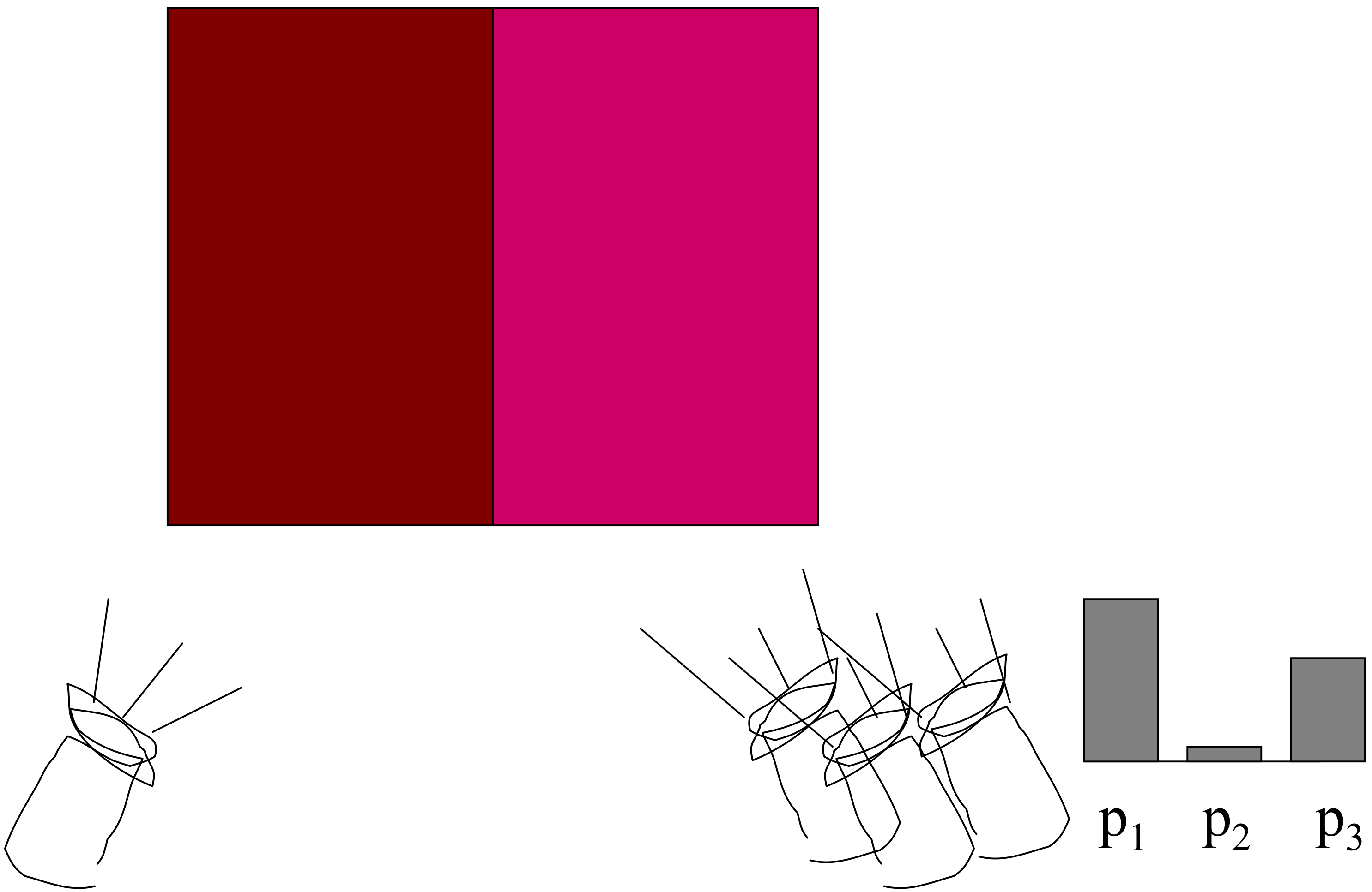
Color matching experiment 1



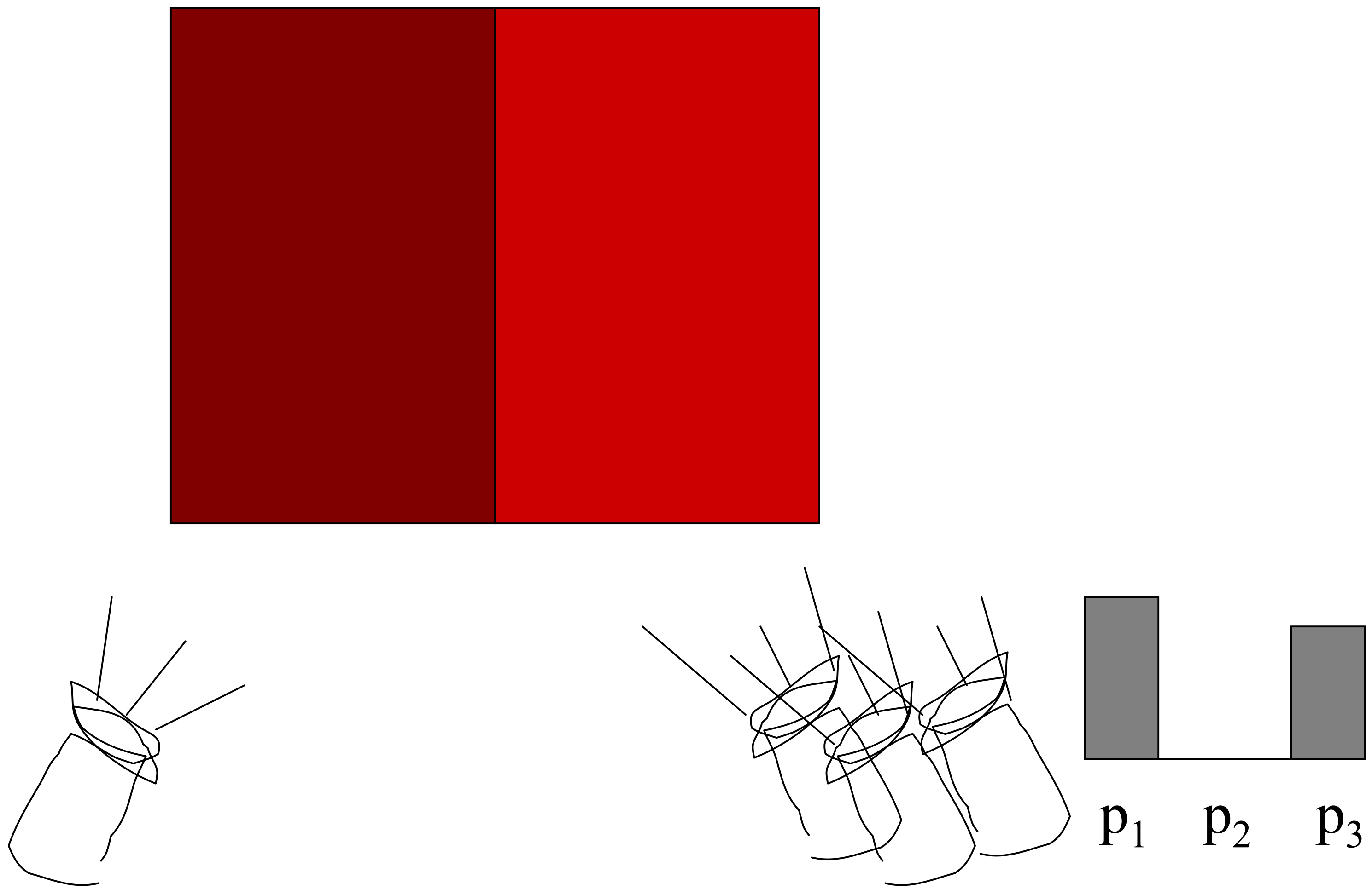
Color matching experiment 2



Color matching experiment 2

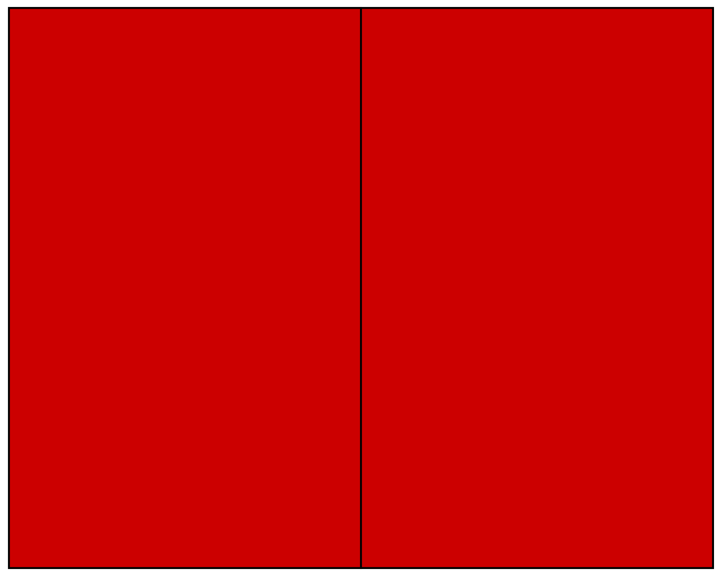


Color matching experiment 2

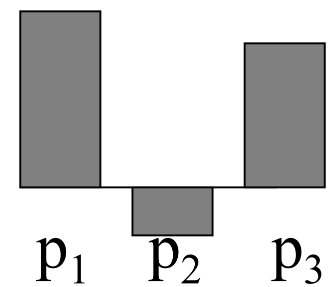


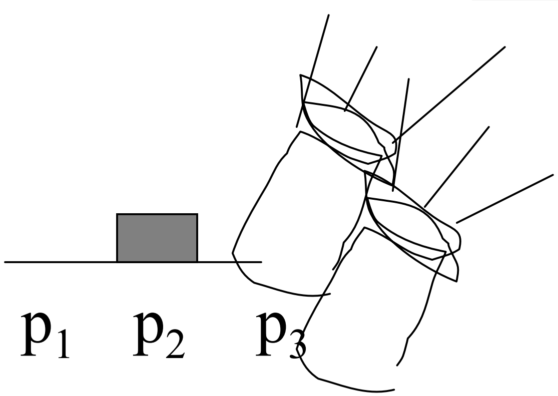
Color matching experiment 2

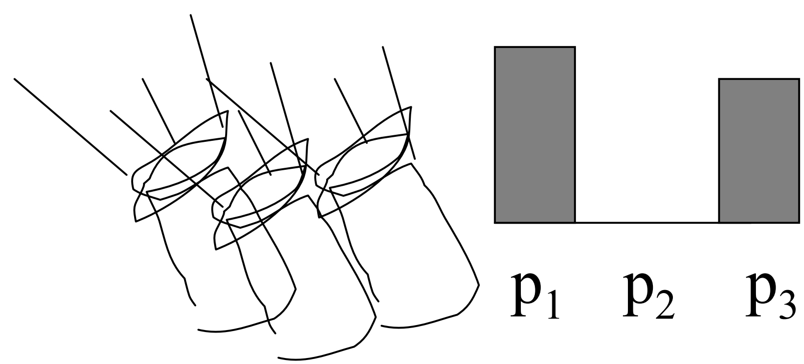
We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.

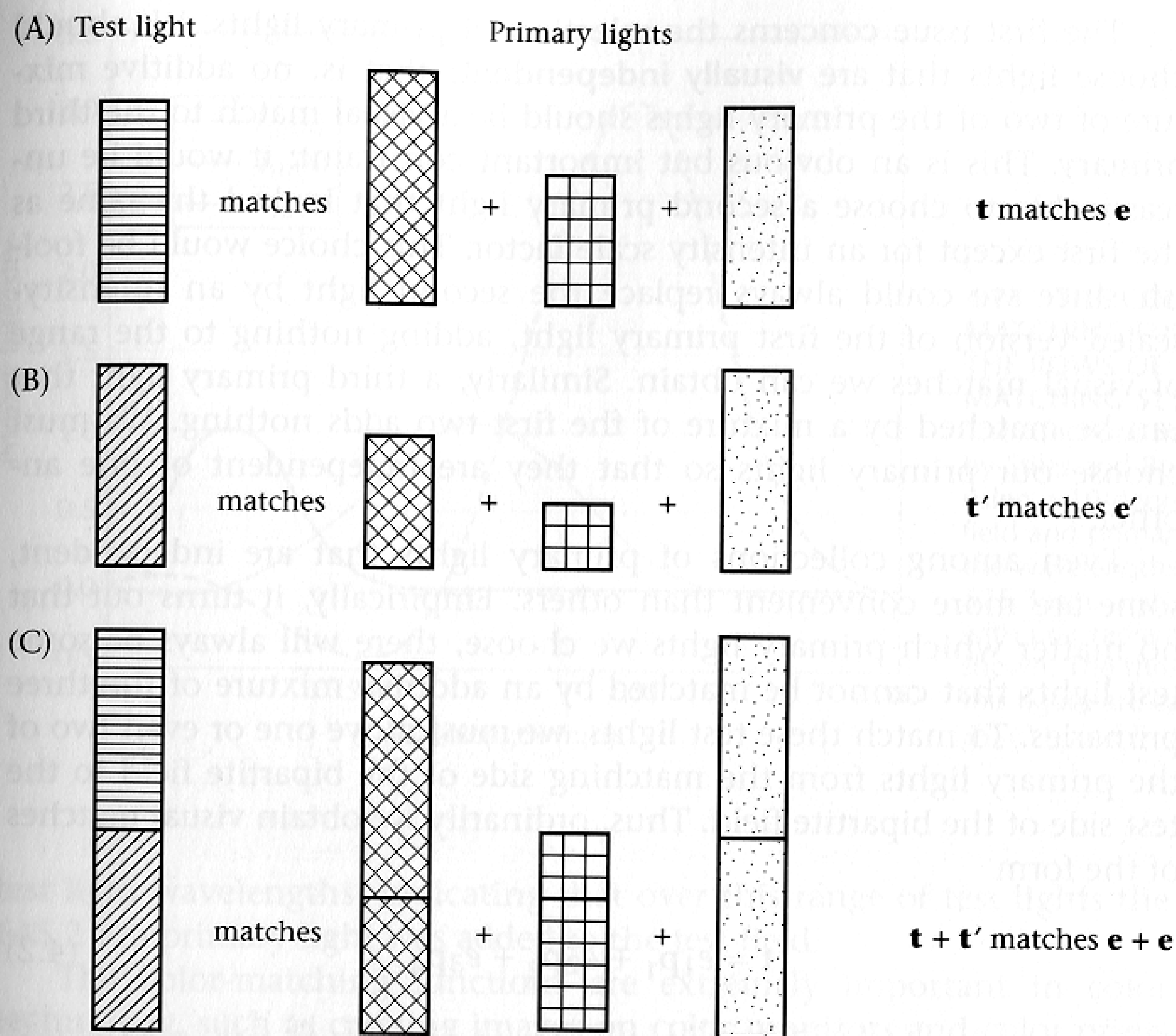


The primary color amounts needed for a match:









4.12 THE COLOR-MATCHING EXPERIMENT SATISFIES THE PRINCIPLE OF SUPERPOSITION. In parts (A) and (B), test lights are matched by a mixture of three primary lights. In part (C) the sum of the test lights is matched by the additive mixture of the primaries, demonstrating superposition.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Grassman's Laws

- For color matches:
  - symmetry: U=V <=>V=U
  - transitivity: U=V and V=W => U=W
  - proportionality: U=V <=> tU=tV
  - additivity: if any two (or more) of the statements

    U=V,

    W=X,

    (U+W)=(V+X) are true, then so is the third
- These statements are as true as any biological law. They mean that color matching in film color mode is linear.

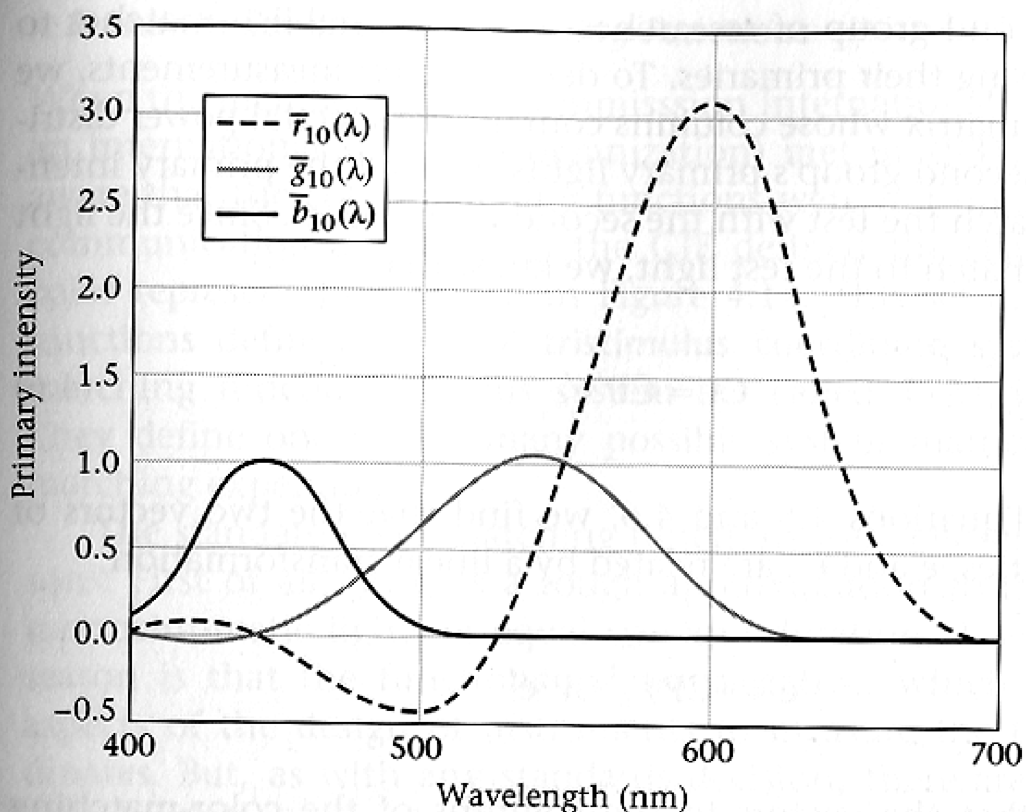
Measure color by color-matching paradigm

- Pick a set of 3 primary color lights.
- Find the amounts of each primary, e_1, e_2, e_3, needed to match some spectral signal, t.
- Those amounts, e_1, e_2, $e_{3,}$ describe the color of t. If you have some other spectral signal, s, and s matches t perceptually, then e_1, e_2, e_3 will also match s.

How to do this, mathematically

- Pick a set of primaries.
- Measure the amount of each primary needed to match monochromatic light at each spectral wavelength (pick some spectral step size).

Color matching functions for a particular set of monochromatic primaries



p_1 = 645.2 nm
p_2 = 525.3 nm
p_3 = 444.4 nm

4.13 THE COLOR-MATCHING FUNCTIONS ARE THE ROWS OF THE COLOR-MATCHING SYSTEM MATRIX. The functions measured by Stiles and Burch (1959) using a 10-degree bipartite field and primary lights at the wavelengths 645.2 nm, 525.3 nm, and 444.4 nm with unit radiant power are shown. The three functions in this figure are called $\bar{r}_{10}(\lambda)$, $\bar{g}_{10}(\lambda)$, and $\bar{b}_{10}(\lambda)$.

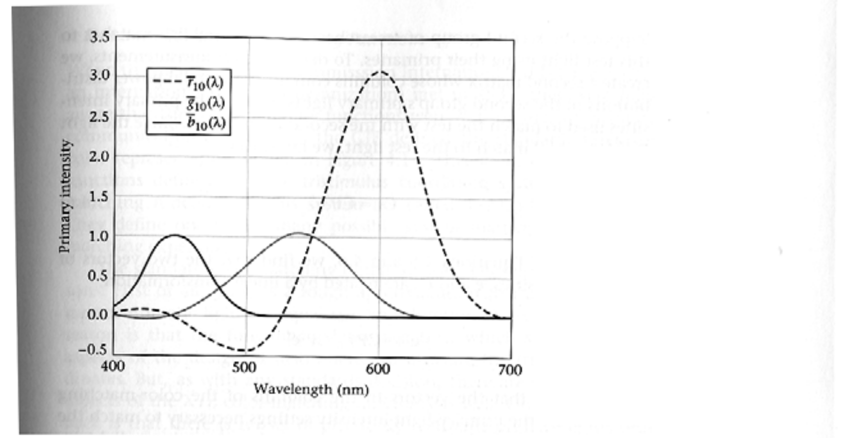
test light wavelengths

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Using the color matching functions to predict the primary match for a new spectral signal

Store the color matching functions in the rows of the matrix, C

$$C = \begin{pmatrix} c_1(\lambda_1) & \cdots & c_1(\lambda_N) \\ c_2(\lambda_1) & \cdots & c_2(\lambda_N) \\ c_3(\lambda_1) & \cdots & c_3(\lambda_N) \end{pmatrix}$$



Let the new spectral signal to be characterized be the vector t.

$$\vec{t} = \begin{pmatrix} t(\lambda_1) \\ \vdots \\ t(\lambda_N) \end{pmatrix}$$

Then the amounts of each primary needed to match t are:

$$C\vec{t}$$

How do you translate colors between different systems of primaries?

$p_1 = (0\ 0\ 0\ 0\ 0 \ldots\ 0\ 1\ 0)^T$

$p_2 = (0\ 0\ \ldots\ 0\ 1\ 0\ \ldots 0\ 0)^T$

$p_3 = (0\ 1\ 0\ 0\ \ldots\ 0\ 0\ 0\ 0)^T$

Primary spectra, P

Color matching functions, C

$p'_1 = (0\ 0.2\ 0.3\ 4.5\ 7\ \ldots .\ 2.1)^T$

$p'_2 = (0.1\ 0.44\ 2.1\ \ldots\ 0.3\ 0)^T$

$p'_3 = (1.2\ 1.7\ 1.6\ \ldots .\ 0\ 0)^T$

Primary spectra, P'

Color matching functions, C'

Any input spectrum, t

The color of t, as described by the primaries, P.

$$C\vec{t} = CP'C'\vec{t}$$

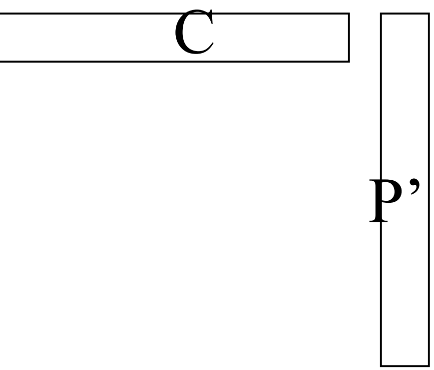
The color of that match to t, described by the primaries, P.

A perceptual match to t, made using the primaries P'

So color matching functions translate like this:

From previous slide $C\vec{t} = CP'C'\vec{t}$ But this holds for any input spectrum, t, so…

$$C = \underbrace{CP'}_{\text{a 3x3 matrix}}C'$$

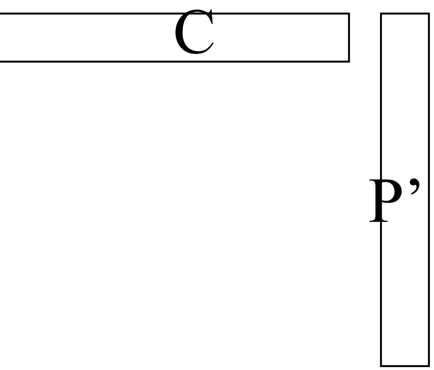


P’ are the old primaries

C are the new primaries’ color matching functions

How do you translate from the color in one set of primaries to that in another?

$$e = \underbrace{CP'}_{\text{the same 3x3 matrix}} e'$$

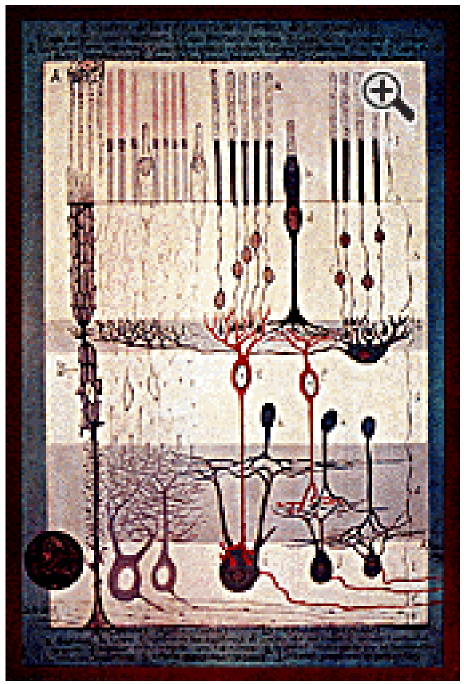


P’ are the old primaries

C are the new primaries’ color matching functions

What’s the machinery in the eye?

Eye Photoreceptor responses

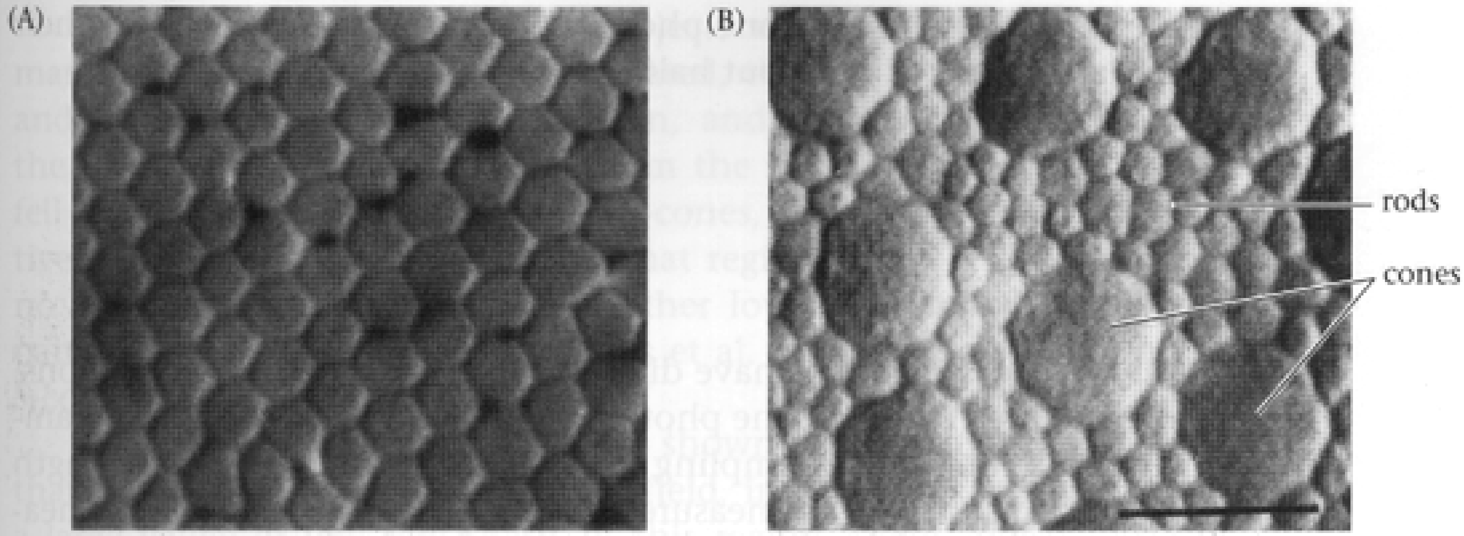


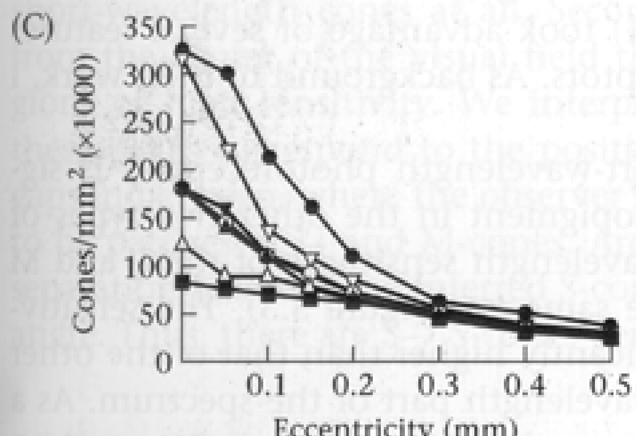
The intricate layers and connections of nerve cells in the retina were drawn by the famed Spanish anatomist Santiago Ramón y Cajal around 1900. Rod and cone cells are at the top. Optic nerve fibers leading to the brain may be seen at bottom right.

Instituto Cajal. CSIC. Madrid.

(Where do you think the light comes in?)

Human Photoreceptors



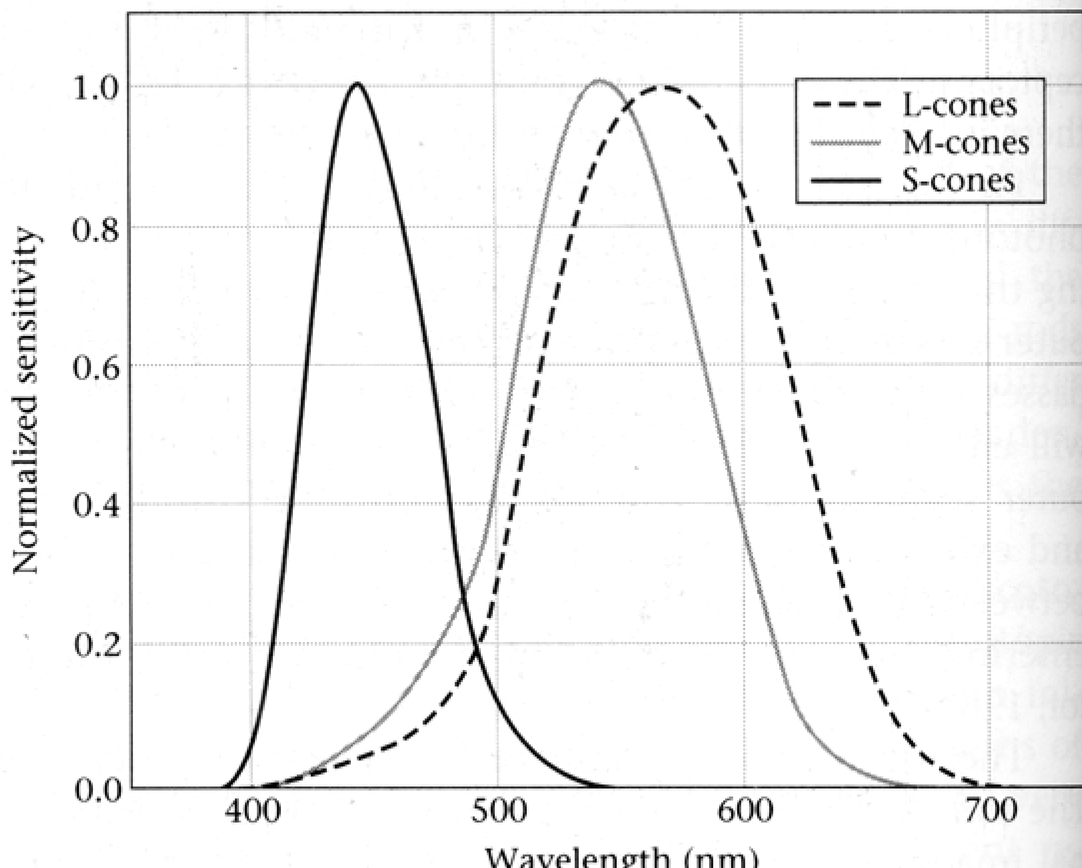


3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Human eye photoreceptor spectral sensitivities

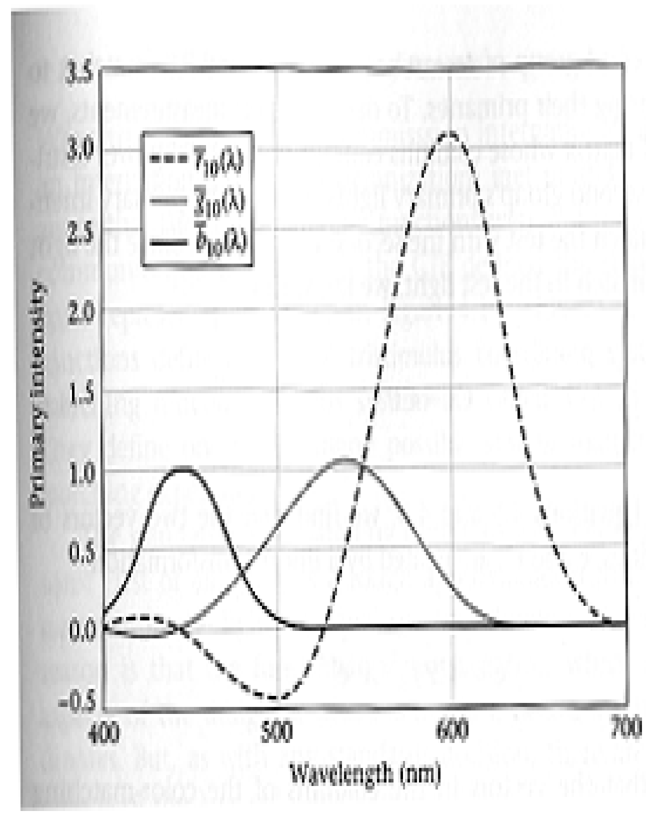
3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.



Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

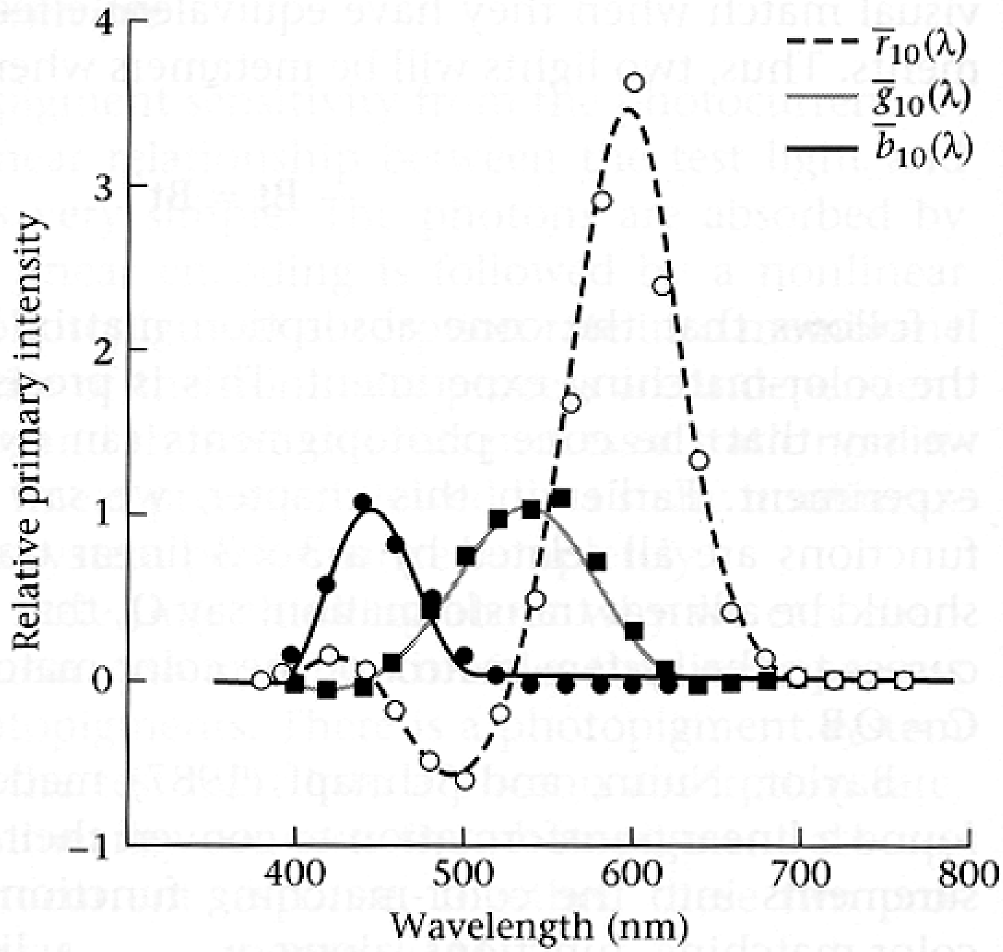
Are the color matching functions we observe obtainable from some 3x3 matrix transformation of the human photopigment response curves?

Color matching functions (for a particular set of spectral primaries



Comparison of color matching functions with best 3x3 transformation of cone responses

4.20 COMPARISON OF CONE PHOTOCURRENT RESPONSES AND THE COLOR-MATCHING FUNCTIONS. The cone photocurrent spectral responsivities are within a linear transformation of the color-matching functions, after a correction has been made for the optics and inert pigments in the eye. The smooth curves show the Stiles and Burch (1959) color-matching functions. The symbols show the matches predicted from the photocurrents of the three types of macaque cones. The predictions included a correction for absorption by the lens and other inert pigments in the eye. Source: Baylor, 1987.



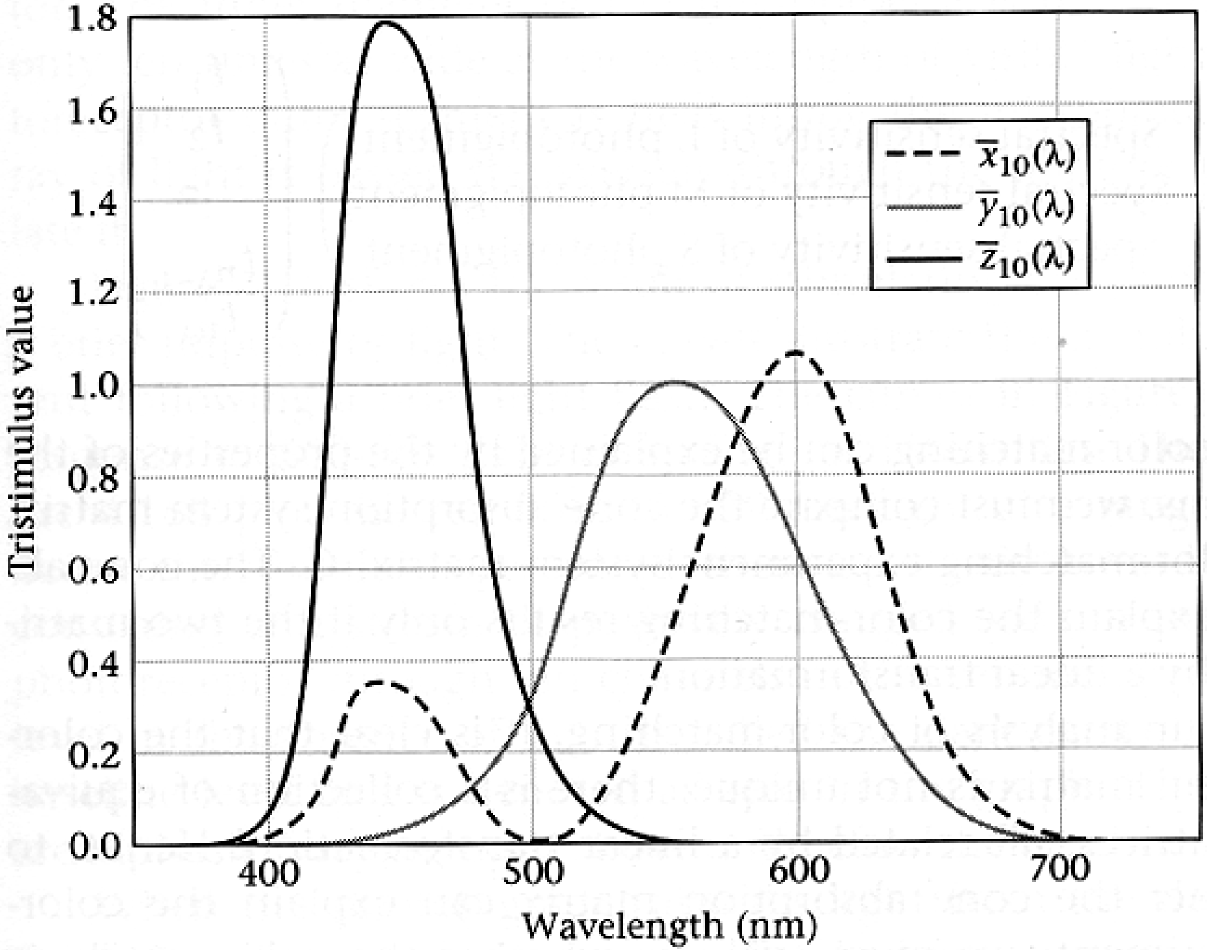
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Since we can define colors using almost any set of primary colors, let's agree on a set of primaries and color matching functions for the world to use…

CIE XYZ color space

- Commission Internationale d'Eclairage, 1931
- "…as with any standards decision, there are some irratating aspects of the XYZ color-matching functions as well…no set of physically realizable primary lights that by direct measurement will yield the color matching functions."
- "Although they have served quite well as a technical standard, and are understood by the mandarins of vision science, they have served quite poorly as tools for explaining the discipline to new students and colleagues outside the field."

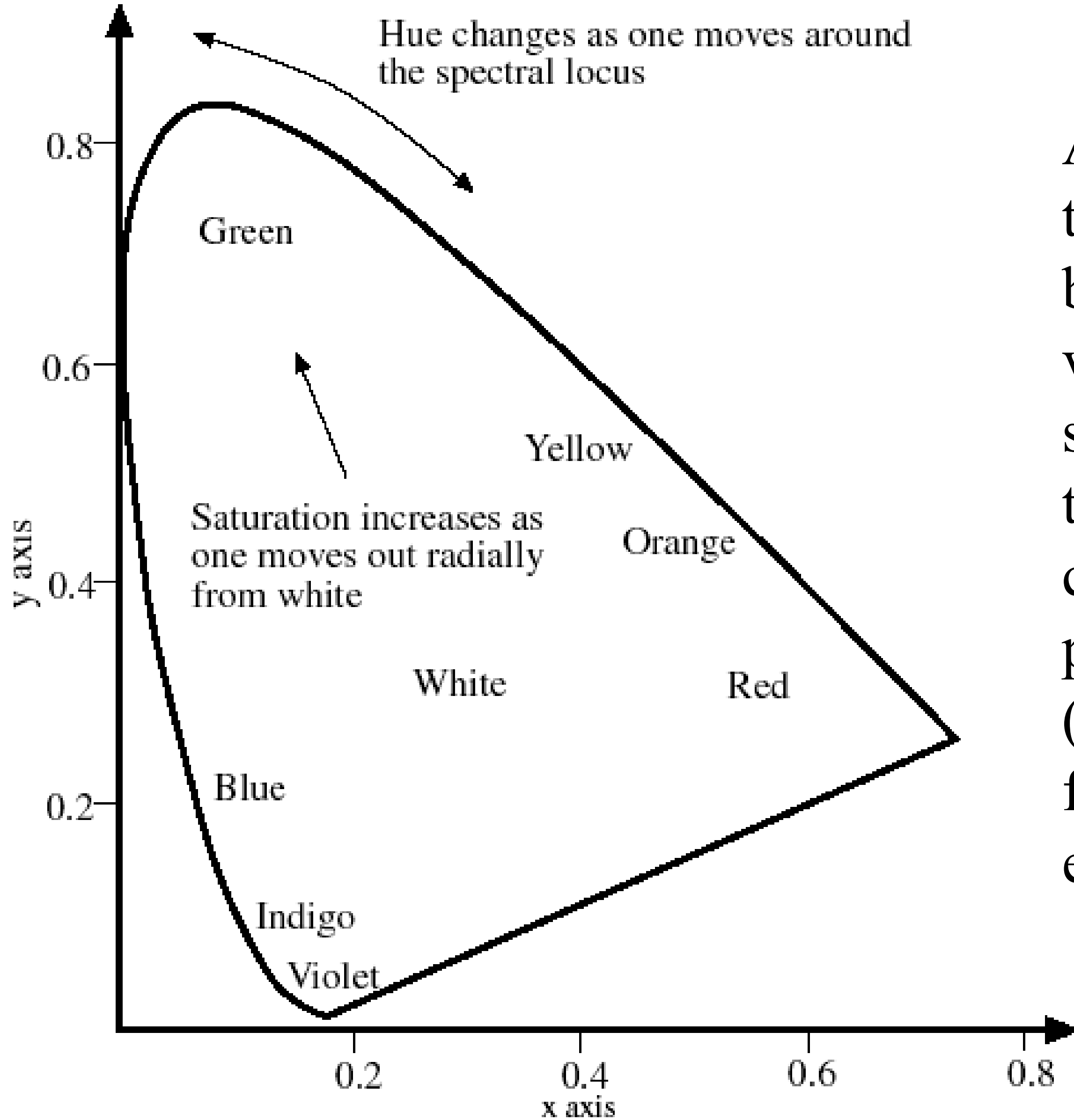
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995



4.14 THE XYZ STANDARD COLOR-MATCHING FUNCTIONS. In 1931 the CIE standardized a set of color-matching functions for image interchange. These color-matching functions are called $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$. Industrial applications commonly describe the color properties of a light source using the three primary intensities needed to match the light source that can be computed from the XYZ color-matching functions.

CIE XYZ: Color matching functions are positive everywhere, but primaries are imaginary. Usually draw x, y, where x=X/(X+Y+Z)
y=Y/(X+Y+Z)

Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995



A qualitative rendering of the CIE (x,y) space. The blobby region represents visible colors. There are sets of (x, y) coordinates that don’t represent real colors, because the primaries are not real lights (so that the color matching functions could be positive everywhere).

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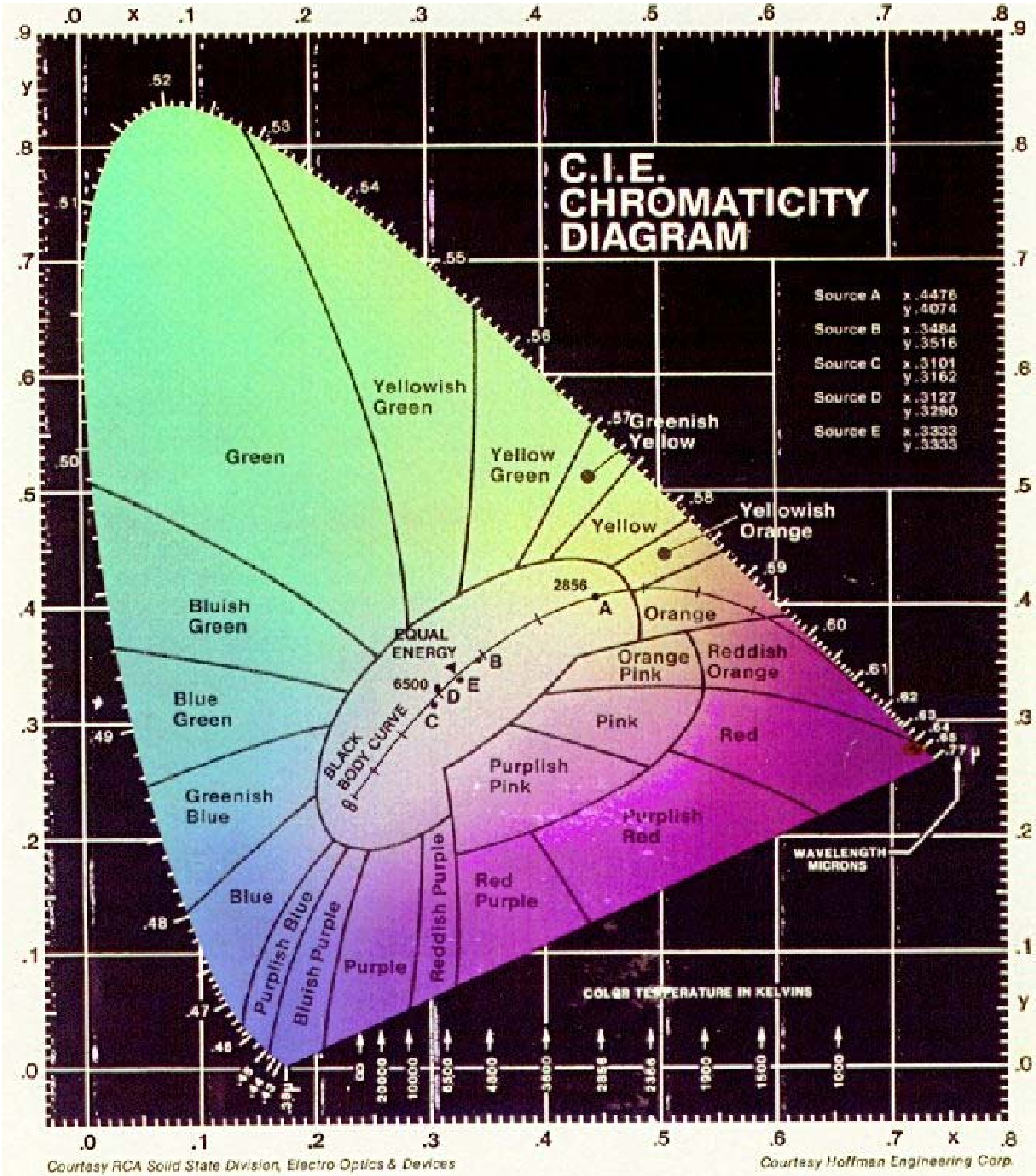
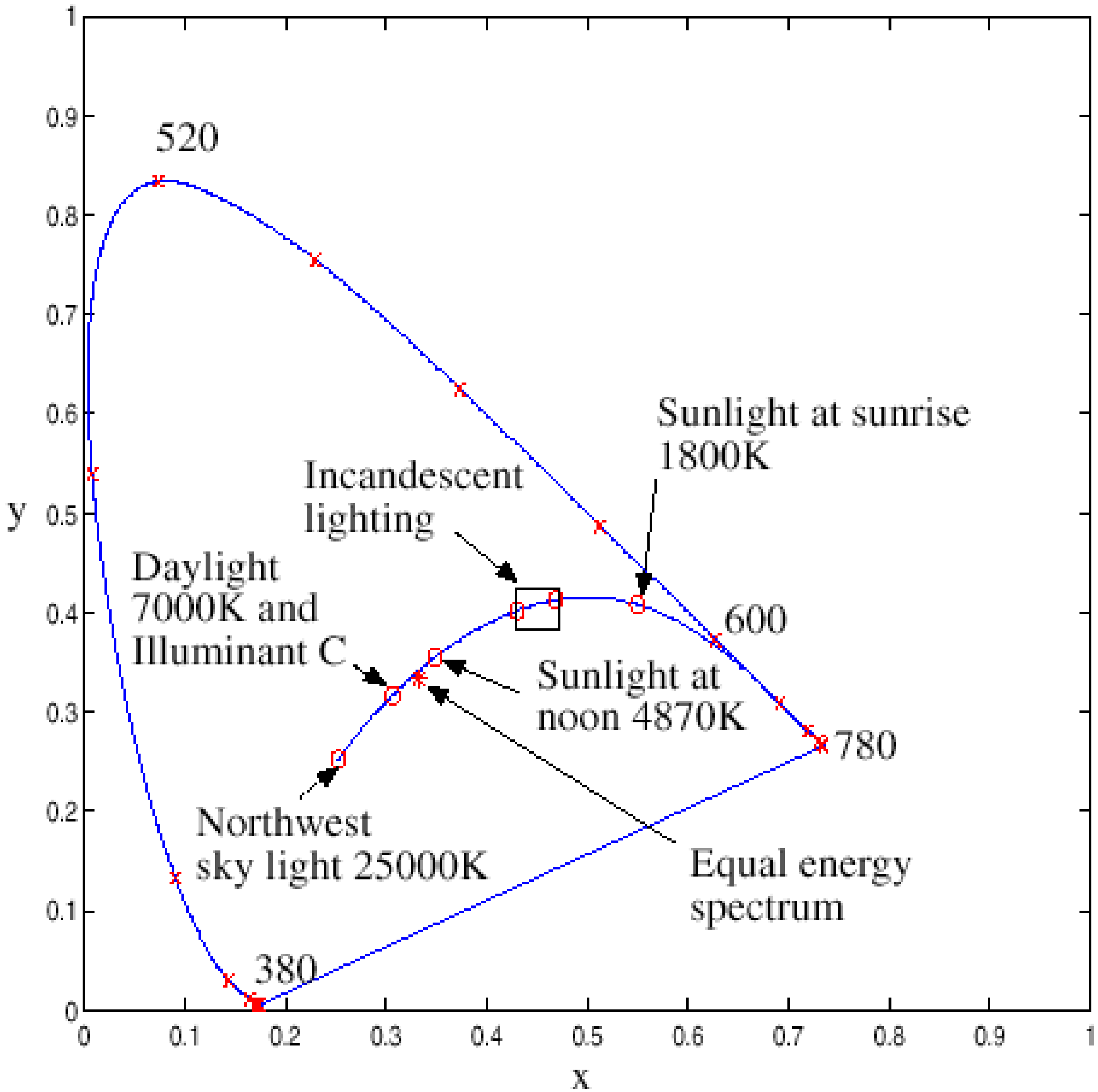

C.I.E. CHROMATICITY DIAGRAM
Source A x .4476 y .4074
Source B x .3484 y .3516
Source C x .3101 y .3162
Source D x .3127 y .3290
Source E x .3333 y .3333
Green
Yellowish Green
Yellow Green
Greenish Yellow
Yellow
Yellowish Orange
Orange
Reddish Orange
Orange Pink
Pink
Red
Purplish Pink
Purplish Red
Red Purple
Reddish Purple
Purple
Bluish Purple
Purplish Blue
Blue
Greenish Blue
Blue Green
Bluish Green
EQUAL ENERGY
BLACK BODY CURVE
WAVELENGTH MICRONS
COLOR TEMPERATURE IN KELVINS
Courtesy RCA Solid State Division, Electro Optics & Devices
Courtesy Hoffman Engineering Corp.



A plot of the CIE (x,y) space. We show the spectral locus (the colors of monochromatic lights) and the black-body locus (the colors of heated black-bodies). I have also plotted the range of typical incandescent lighting.

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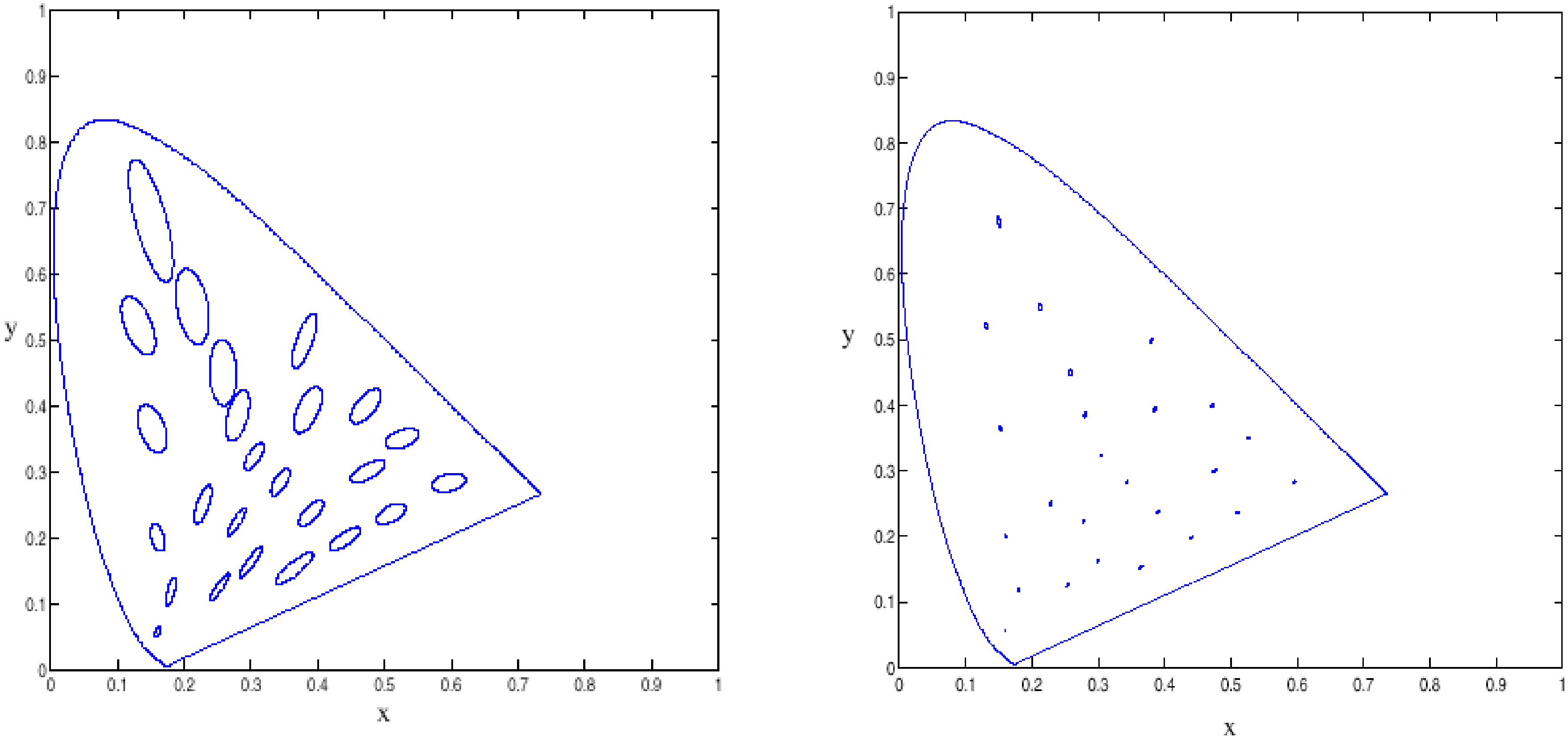
Some other color spaces…

Uniform color spaces

- McAdam ellipses (next slide) demonstrate that differences in x,y are a poor guide to differences in color
- Construct color spaces so that differences in coordinates are a good guide to differences in color.



Variations in color matches on a CIE x, y space. At the center of the ellipse is the color of a test light; the size of the ellipse represents the scatter of lights that the human observers tested would match to the test color; the boundary shows where the just noticeable difference is. The ellipses on the left have been magnified 10x for clarity; on the right they are plotted to scale. The ellipses are known as MacAdam ellipses after their inventor. The ellipses at the top are larger than those at the bottom of the figure, and that they rotate as they move up. This means that the magnitude of the difference in x, y coordinates is a poor guide to the difference in color.

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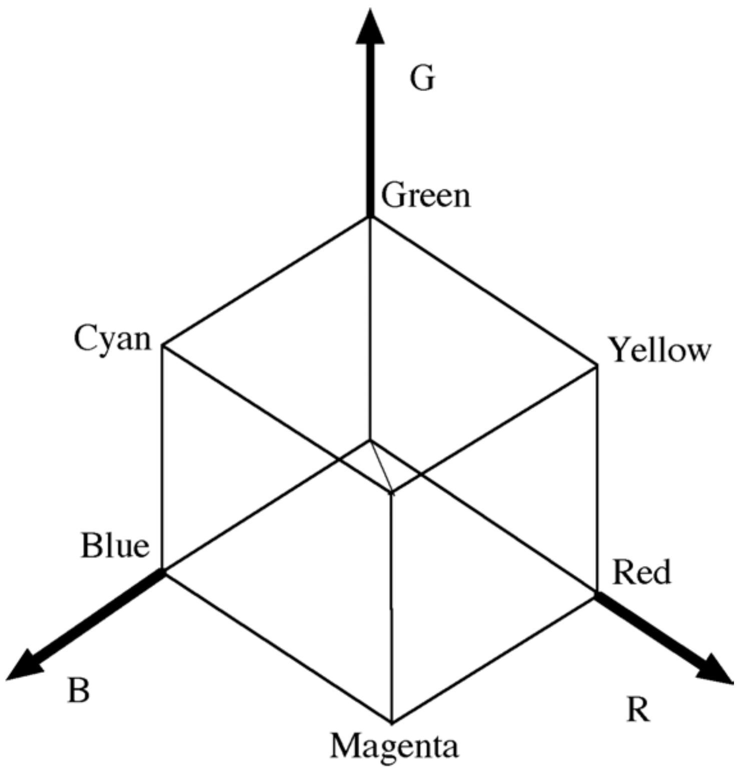
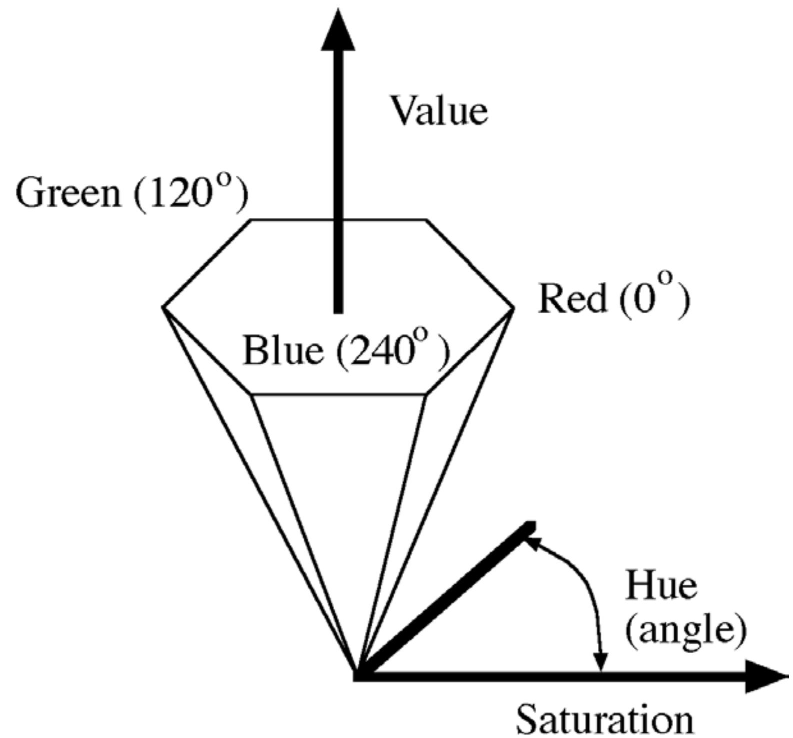
HSV hexcone





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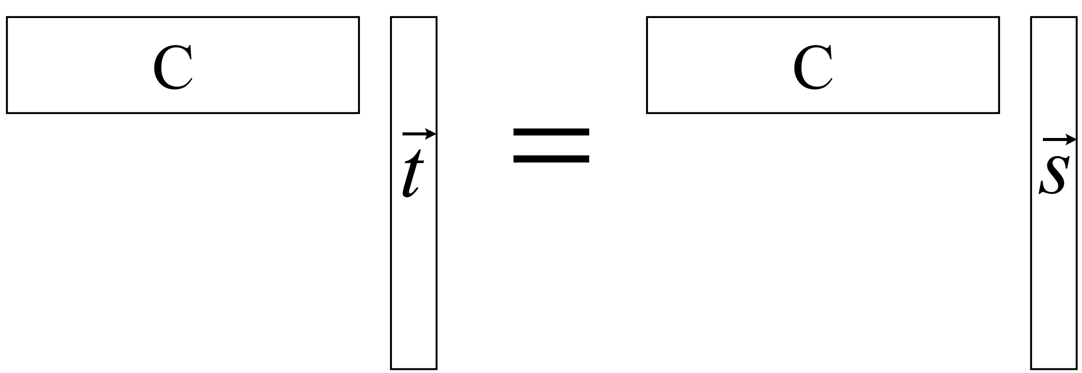
Color metamerism

Two spectra, t and s, perceptually match when

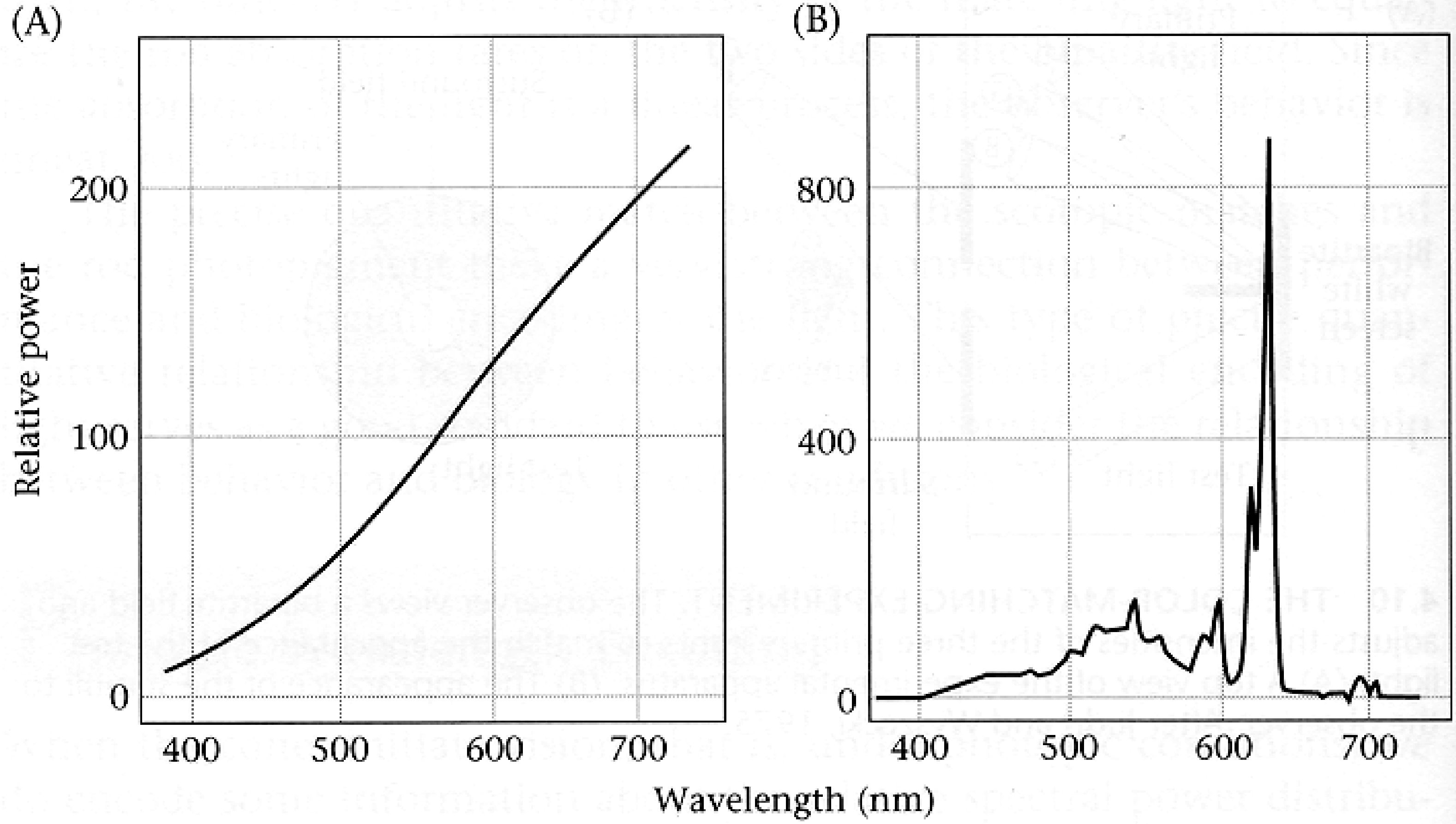
$$C\vec{t} = C\vec{s}$$

where C are the color matching functions for some set of primaries.

Graphically,



Metameric lights



4.11 METAMERIC LIGHTS. Two lights with these spectral power distributions appear identical to most observers and are called metamers. (A) An approximation to the spectral power distribution of a tungsten bulb. (B) The spectral power distribution of light emitted from a conventional television monitor whose three phosphor intensities were set to match the light in panel A in appearance.

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