

Lecture 17: Recitation Problem and Solutions

December 12, 2001

1. **How many possible network structures are there over three nodes?**

There are 25: 1 with no arcs; 6 with one arc; 12 with two arcs; and 6 with three arcs.

2. **How many parameters are there in each of the network structures given in the figure?**

S1: 3; S2: 7; S3: 5; S4: 6; S5: 5; S6: 5

3. **What parameter estimates would you get for the CPTs in each of the network structures given in the figure, using the data in Table 1, without the Bayesian correction?**

- $\Pr(A) = 1/3$
- $\Pr(B) = 2/3$
- $\Pr(C) = 2/3$
- $\Pr(B|A) = 0$
- $\Pr(B|\neg A) = 1$
- $\Pr(C|A) = 0$
- $\Pr(C|\neg A) = 1$
- $\Pr(A|B) = 0$
- $\Pr(A|\neg B) = 1$
- $\Pr(C|B) = 1$
- $\Pr(C|\neg B) = 0$

A	B	C
0	1	1
0	1	1
1	0	0

Table 1: Data set

- $\Pr(A|C) = 0$
- $\Pr(A|\neg C) = 1$
- $\Pr(B|C) = 1$
- $\Pr(B|\neg C) = 0$
- $\Pr(C|A, B) = \text{undefined}$
- $\Pr(C|A, \neg B) = 0$
- $\Pr(C|\neg A, B) = 1$
- $\Pr(C|\neg A, \neg B) = \text{undefined}$
- $\Pr(A|B, C) = 0$
- $\Pr(A|B, \neg C) = \text{undefined}$
- $\Pr(A|\neg B, C) = \text{undefined}$
- $\Pr(A|\neg B, \neg C) = 1$

4. What is the log likelihood of the data given each of the models and the parameter estimates without the Bayesian correction?

S1:

$$\begin{aligned}
 \Pr(D|M) &= 2 \ln \Pr(\neg A, B, C) + \ln \Pr(A, \neg B, \neg C) \\
 &= 2 \ln(2/3 \cdot 2/3 \cdot 2/3) + \ln(1/3 \cdot 1/3 \cdot 1/3) \\
 &= -5.73
 \end{aligned}$$

S2:

$$\begin{aligned}
 \Pr(D|M) &= 2 \ln(\Pr(\neg A) \Pr(B|\neg A) \Pr(C|\neg A, B)) \\
 &\quad + \ln(\Pr(A) \Pr(\neg B|A) \Pr(\neg C|A, \neg B)) \\
 &= 2 \ln(2/3 \cdot 1 \cdot 1) + \ln(1/3 \cdot 1 \cdot 1) \\
 &= -1.91
 \end{aligned}$$

S3:

$$\begin{aligned}
 \Pr(D|M) &= 2 \ln(\Pr(\neg A) \Pr(B|\neg A) \Pr(C|\neg A)) \\
 &\quad + \ln(\Pr(A) \Pr(\neg B|A) \Pr(\neg C|A)) \\
 &= 2 \ln(2/3 \cdot 1 \cdot 1) + \ln(1/3 \cdot 1 \cdot 1) \\
 &= -1.91
 \end{aligned}$$

S4:

$$\begin{aligned}
 \Pr(D|M) &= 2 \ln(\Pr(B) \Pr(C) \Pr(\neg A|B, C)) \\
 &\quad + \ln(\Pr(\neg B) \Pr(\neg C) \Pr(A|\neg B, \neg C)) \\
 &= 2 \ln(2/3 \cdot 2/3 \cdot 1) + \ln(1/3 \cdot 1/3 \cdot 1) \\
 &= -3.82
 \end{aligned}$$

S5:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(B) \Pr(\neg A|B) \Pr(C|B)) \\ &\quad + \ln(\Pr(\neg B) \Pr(A|\neg B) \Pr(\neg C|\neg B)) \\ &= 2 \ln(2/3 \cdot 1 \cdot 1) + \ln(1/3 \cdot 1 \cdot 1) \\ &= -1.91\end{aligned}$$

S6:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(C) \Pr(B|C) \Pr(\neg A|C)) \\ &\quad + \ln(\Pr(\neg C) \Pr(\neg B|\neg C) \Pr(A|\neg C)) \\ &= 2 \ln(2/3 \cdot 1 \cdot 1) + \ln(1/3 \cdot 1 \cdot 1) \\ &= -1.91\end{aligned}$$

5. What parameter estimates do you get with the Bayesian correction?

- $\Pr(A) = 2/5$
- $\Pr(B) = 3/5$
- $\Pr(C) = 3/5$
- $\Pr(B|A) = 1/3$
- $\Pr(B|\neg A) = 3/4$
- $\Pr(C|A) = 1/3$
- $\Pr(C|\neg A) = 3/4$
- $\Pr(A|B) = 1/4$
- $\Pr(A|\neg B) = 2/3$
- $\Pr(C|B) = 3/4$
- $\Pr(C|\neg B) = 1/3$
- $\Pr(A|C) = 1/4$
- $\Pr(A|\neg C) = 2/3$
- $\Pr(B|C) = 3/4$
- $\Pr(B|\neg C) = 1/3$
- $\Pr(C|A, B) = 1/2$
- $\Pr(C|A, \neg B) = 1/3$
- $\Pr(C|\neg A, B) = 3/4$
- $\Pr(C|\neg A, \neg B) = 1/2$
- $\Pr(A|B, C) = 1/4$
- $\Pr(A|B, \neg C) = 1/2$

- $\Pr(A|\neg B, C) = 1/2$
- $\Pr(A|\neg B, \neg C) = 2/3$

6. What log likelihoods?

S1:

$$\begin{aligned}\Pr(D|M) &= 2 \ln \Pr(\neg A, B, C) + \ln \Pr(A, \neg B, \neg C) \\ &= 2 \ln(3/5 \cdot 3/5 \cdot 3/5) + \ln(2/5 \cdot 2/5 \cdot 2/5) \\ &= -4.28\end{aligned}$$

S2:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(\neg A) \Pr(B|\neg A) \Pr(C|\neg A, B)) \\ &\quad + \ln(\Pr(A) \Pr(\neg B|A) \Pr(\neg C|A, \neg B)) \\ &= 2 \ln(3/5 \cdot 3/4 \cdot 3/4) + \ln(2/5 \cdot 2/3 \cdot 2/3) \\ &= -3.90\end{aligned}$$

S3:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(\neg A) \Pr(B|\neg A) \Pr(C|\neg A)) \\ &\quad + \ln(\Pr(A) \Pr(\neg B|A) \Pr(\neg C|A)) \\ &= 2 \ln(3/5 \cdot 3/4 \cdot 3/4) + \ln(2/5 \cdot 2/3 \cdot 2/3) \\ &= -3.90\end{aligned}$$

S4:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(B) \Pr(C) \Pr(\neg A|B, C)) \\ &\quad + \ln(\Pr(\neg B) \Pr(\neg C) \Pr(A|\neg B, \neg C)) \\ &= 2 \ln(3/5 \cdot 3/5 \cdot 3/4) + \ln(2/5 \cdot 2/5 \cdot 2/3) \\ &= -4.86\end{aligned}$$

S5:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(B) \Pr(\neg A|B) \Pr(C|B)) \\ &\quad + \ln(\Pr(\neg B) \Pr(A|\neg B) \Pr(\neg C|\neg B)) \\ &= 2 \ln(3/5 \cdot 3/4 \cdot 3/4) + \ln(2/5 \cdot 2/3 \cdot 2/3) \\ &= -3.90\end{aligned}$$

S6:

$$\begin{aligned}\Pr(D|M) &= 2 \ln(\Pr(C) \Pr(B|C) \Pr(\neg A|C)) \\ &\quad + \ln(\Pr(\neg C) \Pr(\neg B|\neg C) \Pr(A|\neg C)) \\ &= 2 \ln(3/5 \cdot 3/4 \cdot 3/4) + \ln(2/5 \cdot 2/3 \cdot 2/3) \\ &= -3.90\end{aligned}$$

7. How many parameters are in each network? 3, 7, 5, 5, 5, 5