6.825 Recitation Problems: Lec. 7-8

Solutions

October 18, 2001

1 Lecture 7: Resolution

1.1 Resolution refutation

1. Using resolution refutation, prove the last sentence in each group from the rest of the sentences in the group.

   (a) $P \rightarrow Q$

   $\neg P \rightarrow R$

   $\neg Q \rightarrow R$

   First, we negate the last sentece, and then we put everything in CNF.

   The last sentence becomes

   $\neg(Q \vee R)$

   $\neg Q \wedge \neg R$

   Now we do resolution:

      i. $\neg P \vee Q$

     ii. $P \vee R$

    iii. $\neg Q$

    iv. $\neg R$

      v. $\neg P$ (i, iii)

    vi. P (ii, iv)

   vii. $false$ (v, vi)

   (b) $(P \rightarrow Q) \vee (R \rightarrow S)$

   $(P \rightarrow S) \vee (R \rightarrow Q)$

      i. $\neg P \vee Q \vee \neg R \vee S$

     ii. P

    iii. $\neg S$

    iv. R

      v. $\neg Q$

    vi. $Q \vee \neg R \vee S$ (i, ii)

   vii. $\neg R \vee S$ (v, vi)

   viii. S (iv, vii)

    ix. $false$ (iii, viii)

   (c) $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$

   $\neg(T \vee Q)$

   $U \rightarrow (\neg T \rightarrow (\neg S \wedge P))$

   $\neg U$

    i. $\neg P \lor Q \lor S \lor T$

   ii. $\neg T$

  iii. $\neg Q$

  iv. $T \lor \neg S \lor \neg U$

   v. $T \lor P \lor \neg U$

  vi. U

 vii. $\neg P \lor Q \lor S$ (i, ii)

viii. $\neg P \lor S$ (iii, vii)

  ix. $T \lor \neg S$ (iv, vi)

   x. $\neg S$ (ii, ix)

  xi. $\neg P$ (viii, x)

 xii. $T \lor \neg U$ (v, xi)

xiii. $\neg U$ (ii, xii)

xiv. $false$ (vi, xiii)

2. Use resolution refutation to do problem 6.5 from R&N.

   Let's write the logical sentences in CNF.

   (a) $Mythical(U) \rightarrow \neg Mortal(U)$
      $\neg Mythical(U) \lor \neg Mortal(U)$

   (b) $\neg Mythical(U) \rightarrow Mortal(U) \land Mammal(U)$

      i. $Mythical(U) \lor Mortal(U)$

     ii. $Mythical(U) \lor Mammal(U)$

   (c) $(\neg Mortal(U) \lor Mammal(U)) \rightarrow Horn(U)$

      i. $Mortal(U) \lor Horn(U)$

     ii. $\neg Mammal(U) \lor Horn(U)$

   (d) $Horn(U) \rightarrow Magical(U)$
      $\neg Horn(U) \lor Magical(U)$

We cannot prove that unicorns are mythical.
Now let's prove that unicorns are magical.

1. $\neg Mythical(U) \lor \neg Mortal(U)$

2. $Mythical(U) \lor Mortal(U)$

3. $Mythical(U) \lor Mammal(U)$

4. $Mortal(U) \lor Horn(U)$

5. $\neg Mammal(U) \lor Horn(U)$

6. $\neg Horn(U) \lor Magical(U)$

7. $\neg Magical(U)$

8. $\neg Horn(U)$ (6, 7)

9. $Mortal(U)$ (4, 8)

10. $\neg Mammal(U)$ (5, 8)

11. $\neg Mythical(U)$ (1, 9)

12. *Mythical(U)* (3, 10)

13. *false* (11, 12)

And we can also prove that unicorns are horned.

1. ¬*Mythical(U)* ∨ ¬*Mortal(U)*

2. *Mythical(U)* ∨ *Mortal(U)*

3. *Mythical(U)* ∨ *Mammal(U)*

4. *Mortal(U)* ∨ *Horn(U)*

5. ¬*Mammal(U)* ∨ *Horn(U)*

6. ¬*Horn(U)* ∨ *Magical(U)*

7. ¬*Horn(U)*

8. *Mortal(U)* (4, 7)

9. ¬*Mammal(U)* (5, 7)

10. ¬*Mythical(U)* (1, 8)

11. *Mythical(U)* (3, 9)

12. *false* (10, 11)

1.2   Unification

For each pair of sentences, give an MGU.

• Color(Tweety, Yellow)	Color(x, y)	{x/Tweety, y/Yellow}
• Color(Tweety, Yellow)	Color(x, x)	Not Unifiable
• Color(Hat(John), Blue)	Color(Hat(y), y)	Not Unifiable
• R(F(x), B)	R(y, z)	{y/F(x), z/B}
• R(F(y), x)	R(x, F(B))	{x/F(y), y/B}
• R(F(y), y, x)	R(x, F(A), F(v))	{x/F(y), y/F(A), v/F(A)}
• Loves(x, y)	Loves(y, x)	{x/y}
• F(G(w), H(w, J(x, y)))	F(G(v), H(u, v))	{w/v, u/v, v/J(x, y)}
• F(G(w), H(w, J(x, u)))	F(G(v), H(u, v))	Not Unifiable
• F(x, F(u, x))	F(F(y, A), F(z, F(B, z)))	{x/F(y, A), u/z, y/B, z/A}

2 Lecture 8

2.1 A silly recitation problem

symbolize the following argument, and then derive the conclusion from the premises using resolution refutation.

- Nobody who really appreciates Beethoven fails to keep silence while the Moonlight sonata is being played.

- Guinea pigs are hopelessly ignorant of music.

- No one who is hopelessly ignorant of music ever keeps silence while the moonlight sonata is being played.

- Therefore, guinea pigs never really appreciate Beethoven.

(Taken from a book by Lewis Carroll, logician and author of *Alice in Wonderland.*)

2.2 Another, sillier problem

You don't have to do this one. It's just for fun. Same type as the previous one. Also from Lewis Carroll

- The only animals in this house are cats

- Every animal that loves to gaze at the moon is suitable for a pet

- When I detest an animal, I avoid it

- No animals are carnivorous unless they prowl at night

- No cat fails to kill mice

- No animals ever like me, except those that are in this house

- Kangaroos are not suitable for pets

- None but carnivorous animals kill mice

- I detest animals that do not like me

- Animals that prowl at night always love to gaze at the moon

- Therefore, I always avoid a kangaroo