Logic Miscellanea

- Completeness and Incompleteness
- Equality
- Paramodulation

Lecture 9 • 1

Completeness and Decidability

Complete: If KB $\models \alpha$ then KB $\vdash \alpha$
- If it's entailed, there is a proof

Semi-decidable:
- If there's a proof, we'll halt with it
- If not, maybe halt, maybe not

Gödel's Completeness Theorem: *There exists a complete proof system for FOL*
Robinson's Completeness Theorem: *Resolution refutation is a complete proof system for FOL*

FOL is semi-decidable: if the desired conclusion follows from the premises then eventually resolution refutation will find a contradiction.

Lecture 9 • 2

Adding Arithmetic

Gödel's Incompleteness Theorem: *There is no consistent, complete proof system for FOL + Arithmetic.*
Either there are sentences that are true, but not provable or there are sentences that are provable, but not true.

Arithmetic gives you the ability to construct code-names for sentences within the logic.
P = "P is not provable."
- If P is true: it's not provable (incomplete)
- If P is false: it's provable (inconsistent)

Lecture 9 • 3

Equality

$\forall x.\ x = x$
$\forall xy.\ x=y \to y = x$
$\forall xyz.\ x=y \quad y = z \to x = z$
$\forall xy.\ x=y \to (P(x) \leftrightarrow P(y))$
etc., etc., etc., …

Lecture 9 • 4

Paramodulation

Need one more rule to deal with resolution and equality.

$$\frac{\alpha \vee (s = t) \qquad \beta \vee \gamma[r]}{(\alpha \vee \beta \vee \gamma[t])\theta}$$

$\gamma[r]$ is a literal containing term r
$\theta = \text{unify}(s,r)$

$$\frac{F(x) = B \qquad Q(y) \vee W(y,F(y))}{Q(y) \vee W(y,B)}$$

$$\frac{P(x) \vee F(x) = B \qquad Q(y) \vee W(y,F(y))}{P(y) \vee Q(y) \vee W(y,B)}$$

where
$s = F(x)$
$t = B$
$\gamma[\ ] = W(y,\ )$
$r = F(y)$
$\theta = \{x/y\}$

Lecture 9 • 5

Recitation Problems

Formalize each group of sentences (using the given function and predicate symbols), then prove the last from the others using resolution and paramodulation.

Jane's lover drives a red car.
Fred is the only person who drives a red car.
Therefore, Fred is Jane's lover.
(L(x) = the lover of x; D(x) = x drives a red car)

Mrs. Abbot only teaches good students.
John and Mary have the same teacher.
Mrs. Abbot is Mary's teacher.
Therefore, John is a good student.
(T(x) = the teacher of x; G(x) = x is a good student)

Lecture 9 • 6

1

More Recitation Problems

- Every part is either made by FooCorp or BarCorp.
- All fragile parts are stored in the warehouse of their manufacturer.
- BarCorp can't manufacture titanium parts.
- The part I need is fragile and made of titanium.
- Therefore, the part I need is the FooCorp's warehouse.

(M(x) = the manufacturer of part x; W(x,y) = part x is stored in the warehouse of company y; T(x) = part x is made of titanium; F(x) part x is fragile; use a constant for "the part I need".)