6.825 Techniques in Artificial Intelligence

Probability

- Logic represents uncertainty by disjunction
- But, cannot tell us how likely the different conditions are
- Probability theory provides a quantitative way of encoding likelihood

Foundations of Probability

Is coin-flipping deterministic?

P(the sun comes up tomorrow) = 0.999

- Frequentist
  - Probability is inherent in the process
  - Probability is estimated from measurements

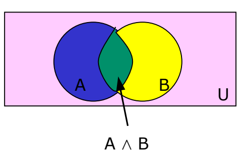
Probs can be wrong!

- Subjectivist (Bayesian)
  - Probability is a model of <u>your</u> degree of belief

Probs can be inconsistent!

Axioms of Probability

- Universe of atomic events (like interpretations in logic).
- Events are sets of atomic events
- P: events $\rightarrow$ [0,1]
  - P(true) = 1 = P(U)
  - P(false) = 0 = P($\emptyset$)
  - P(A v B) = P(A) + P(B) – P(A $\wedge$ B)



Recitation Problem I

Prove that

- P($\neg$ A) = 1 – P(A)
- P(A v B v C) =
  P(A) + P(B) + P(C) –
  P(A $\wedge$ B) – P(A $\wedge$ C) – P(B $\wedge$ C) +
  P(A $\wedge$ B $\wedge$ C)

A Question

Jane is from Berkeley. She was active in anti-war protests in the 60's. She lives in a commune.

- Which is more probable?
  1. Jane is a bank teller
  2. Jane is a feminist bank teller

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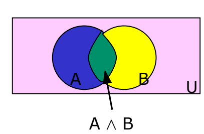
1. A
2. A $\wedge$ B



Dutch Book

- You believe
  - P(A) = 0.3
  - P(A ∧ B) = 0.4 (and also that P(¬ (A ∧ B)) = 0.6)

You		Bet Stakes		A ∧ B	¬ A ∧B	A ∧ ¬B	¬ A ∧ ¬ B
A	0.3	**A**	**3 to 7**	-7	3 + ε	-7	3 + ε
A ∧ B	0.4	**¬(A ∧B)**	**6 to 4**	6 + ε	-4	-4	-4

- No matter what the state of the world, you lose
- This is because your beliefs are inconsistent

If you take a 3 to 7 bet on some condition C, then if C turns out to be true, you lose 7, but if it's false, you win 3.

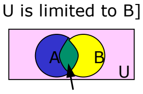
Random Variables

- Random variables
  - Function: discrete domain → [0, 1]
  - Sums to 1 over the domain
    - Raining is a propositional random variable
    - Raining(true) = 0.2
      - P(Raining = true) = 0.2
    - Raining(false) = 0.8
      - P(Raining = false) = 0.8
- Joint distribution
  - Probability assignment to all combinations of values of random variables

Joint Distribution Example

	Toothache	¬Toothache
Cavity	0.04	0.06
¬ Cavity	0.01	0.89

- The sum of the entries in this table has to be 1
- Given this table, one can answer all the probability questions about this domain
- P(cavity) = 0.1 [add elements of cavity row]
- P(toothache) = 0.05 [add elements of toothache column]
- P(A | B) = P(A ∧ B)/P(B) [prob of A when U is limited to B]
- P(cavity | toothache) = 0.04/0.05 = 0.8



A ∧ B

Bayes' Rule

- Bayes' Rule
  - P(A | B) = P(B | A) P(A) / P(B)
  - P(disease | symptom)
    = P(symptom | disease) P(disease)/ P(symptom)
  - Imagine
    - disease = BSE
    - symptom = paralysis
    - P(disease | symptom) is different in England vs US
    - P(symptom | disease) should be the same
    - It is more useful to learn P(symptom | disease)
- Conditioning
  - P(A) = P(A | B) P(B) + P(A | ¬B) P(¬B)
    = P(A ∧ B) + P(A ∧ ¬B)

Independence

- A and B are independent iff
  - P(A ∧ B) = P(A) · P(B)
  - P(A | B) = P(A)
  - P(B | A) = P(B)
- Independence is essential for efficient probabilistic reasoning

- A and B are conditionally independent given C iff
  - P(A | B, C) = P(A | C)
  - P(B | A, C) = P(B | C)
  - P(A ∧ B | C) = P(A | C) · P(B | C)

Examples of Conditional Independence

- Toothache (T)
- Spot in Xray (X)
- Cavity (C)
- None of these propositions are independent of one other
- T and X are conditionally independent given C

- Battery is dead (B)
- Radio plays (R)
- Starter turns over (S)
- None of these propositions are independent of one another
- R and S are conditionally independent given B

Combining evidence

- Bayesian updating given two pieces of information

$$P(C|T,X) = \frac{P(T,X|C)P(C)}{P(T,X)}$$

- Assume that T and X are conditionally independent given C

$$P(C|T,X) = \frac{P(T|C)P(X|C)P(C)}{P(T,X)}$$

- We can do the evidence combination sequentially

Normalizing Factor

$$P(C|T,X) + P(\neg C|T,X) = 1$$

$$\frac{P(T|C)P(X|C)P(C)}{P(T,X)} + \frac{P(T|\neg C)P(X|\neg C)P(\neg C)}{P(T,X)} = 1$$

$$P(T|C)P(X|C)P(C) + P(T|\neg C)P(X|\neg C)P(\neg C) = P(T,X)$$

Recitation Problems II

- Show that P(A) >= P(A,B)
- Show that P(A|B) + P(~A|B) = 1
- Show that the different formulations of conditional independence are equivalent:
  - P(A | B, C) = P(A | C)
  - P(B | A, C) = P(B | C)
  - P(A ∧ B | C) = P(A | C) · P(B | C)
- Conditional Bayes' rule. Write an expression for P(A | B,C) in terms of P(B | A,C).