Decision Making under Uncertainty

- How to make one decision in the face of uncertainty
- In a deterministic problem, making one decision is easy
- Planning is hard because we considered long sequences of actions
- Given uncertainty, even making one decision is difficult

A short survey

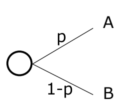
1. Which alternative would you prefer:
   A. A sure gain of $240
   B. A 25% chance of winning $1000 and a 75% chance of winning nothing

- 2. Which alternative would you prefer:
  - C. A sure loss of $750
  - D. A 75% chance of losing $1000 and a 25% chance of losing nothing

- 3. How much would you pay to play the following game: We flip a coin. If it comes up heads, I'll pay you $2. If it comes up tails, we'll flip again, and if it comes up heads, I'll pay you $4. If it comes up tails, we'll flip again, and if it comes up heads, I'll pay you $8. And so on, out to infinity.

Decision Theory

- A calculus for decision-making under uncertainty
- Set of primitive outcomes
- Preferences on primitive outcomes: $A \succ B$
- Subjective degrees of belief (probabilities)
- Lotteries: uncertain outcomes

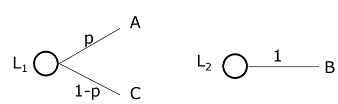


With probability p, outcome A occurs; with probability 1 - p; outcome B occurs.

Axioms of Decision Theory

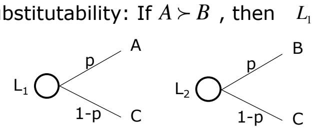
If you accept these conditions on your preferences, then decision theory should apply to you!

- Orderability: $A \succ B$ or $B \succ A$ or $A \sim B$
- Transitivity: If $A \succ B$ and $B \succ C$ then $A \succ C$
- Continuity: If $A \succ B \succ C$ then there exists p such that $L_1 \sim L_2$

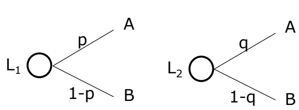


More Axioms of Decision Theory

- Substitutability: If $A \succ B$, then $L_1 \succ L_2$

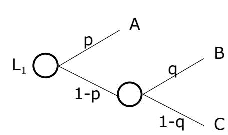


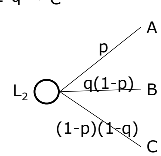
- Monotonicity: If $A \succ B$ and p > q, then $L_1 \succ L_2$



Last Axiom of Decision Theory

- Decomposibility: $L_1 \sim L_2$





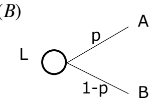
Main Theorem

If preferences satisfy these six assumptions, then there exists U (a real valued function) such that:

- If $A \succ B$, then U(A) > U(B)
- If $A \sim B$, then U(A) = U(B)

Utility of a lottery = expected utility of the outcomes

$$U(L) = p \cdot U(A) + (1 - p) \cdot U(B)$$



Survey Question 1

Which alternative would you prefer:

A. A sure gain of $240
B. A 25% chance of winning $1000 and a 75% chance of winning nothing

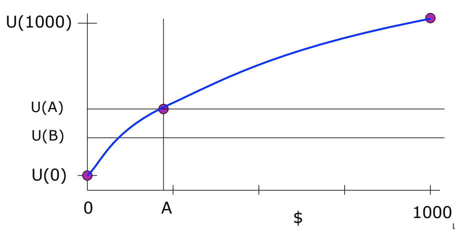
85% prefer option A to option B

- U(B) = .25 U($1000) + .75 U($0)
- U(A) = U($240)
- U(A) > U(B)

Utility of Money

- U(B) = .25 U($1000) + .75 U($0)
- U(A) = U($240)
- U(A) > U(B)

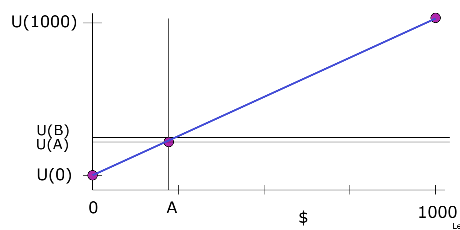
concave utility function
risk averse



Risk neutrality

- U(B) = .25 U($1000) + .75 U($0) = U($250)
- U(A) = U($240)

linear utility function
risk neutral



Why Play the Lottery?

Consider a lottery ticket:

- Expected payoff always less than price
- Is it ever consistent with utility theory to buy one?

It's kind of like preferring lottery B to A, below:

A. A sure gain of $260
B. A 25% chance of winning $1000 and a 75% chance of winning nothing

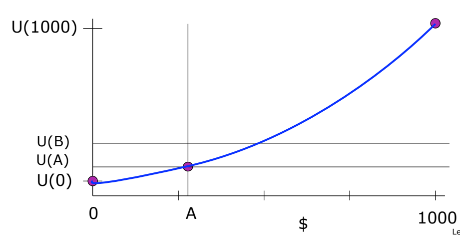
In utility terms:

- U(B) = .25 U($1000) + .75 U($0)
- U(A) = U($260)
- U(A) < U(B)

Risk seeking

- U(B) = .25 U($1000) + .75 U($0)
- U(A) = U($260)
- U(A) < U(B)

convex utility function
risk seeking



Survey Question 2

Which alternative would you prefer:

- C. A sure loss of $750
- D. A 75% chance of losing $1000 and a 25% chance of losing nothing

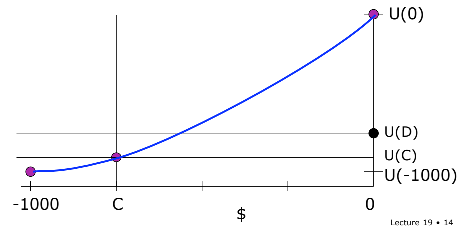
91% prefer option D to option C

- U(D) = .75 U(-$1000) + .25 U($0)
- U(C) = U(-$750)
- U(D) > U(C)

Risk seeking in losses

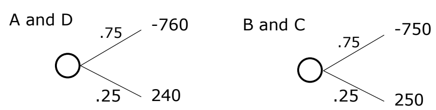
- U(D) = .75 U(-$1000) + .25 U($0)
- U(C) = U(-$750)
- U(D) > U(C)

convex utility function
risk seeking

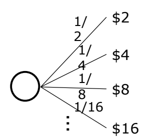


Human irrationality

Most people prefer A in question 1 and D in question 2.



St. Petersburg Paradox

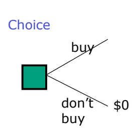


Expected value = 1 + 1 + 1 + … = ∞

Buying a Used Car

- Costs $1000
- Can sell it for $1100, $100 profit
- Every car is a lemon or a peach
- 20% are lemons
- Costs $40 to repair a peach, $200 to repair a lemon
- Risk neutral

decision tree



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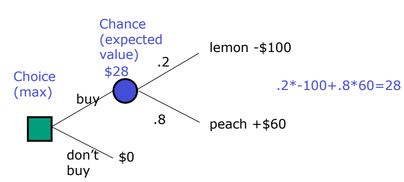

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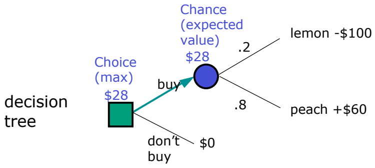
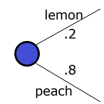


Expected Value of Perfect Info

How much should you pay for information of what type of car it is (before you buy it)?

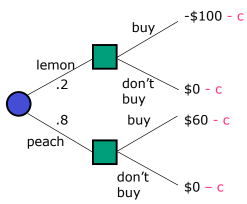
- Reverse the order of the chance and action nodes. The chance node represents uncertainty on what the information will be, once we get it.



Expected Value of Perfect Info

How much should you pay for information of what type of car it is (before you buy it)?

- Reverse the order of the chance and action nodes. The chance node represents uncertainty on what the information will be, once we get it.
- C is cost you have to pay for information



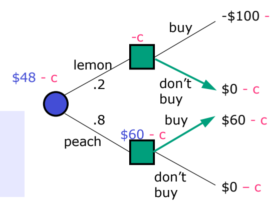
Expected Value of Perfect Info

How much should you pay for information of what type of car it is (before you take it to be repaired)?

- Reverse the order of the chance and action nodes. The chance node represents uncertainty on what the information will be, once we get it.
- C is cost you have to pay for information

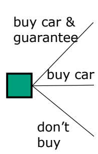
c = $20 [EVPI]

ties the expected value with no information



Guarantee?

- Costs $60
- Covers 50% of repair costs
- If repairs > $100, covers them all



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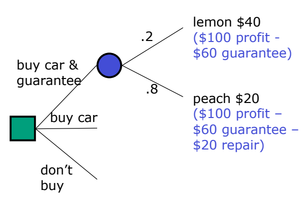

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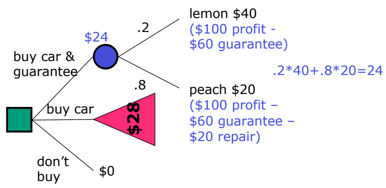

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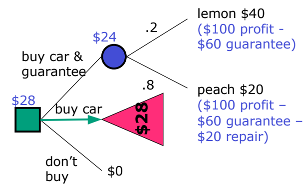


Most people would buy the guarantee in this situation, due to risk aversion.

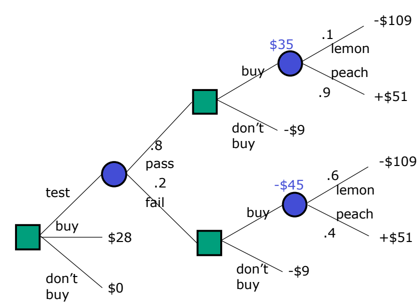
Inspection?

We can have the car inspected for $9
P(“pass” | peach) = 0.9P(“fail” | peach) = 0.1
P(“pass” | lemon) = 0.4 P(“fail” | lemon) = 0.6

P(“pass”) =
P(“pass” | lemon)P(lemon) + P(“pass” | peach)P(peach)
P(“pass”) = 0.4*0.2 + 0.9*0.8 = 0.8
P(“fail”) = 0.2

P(lemon | “pass”) = P(“pass” | lemon) P(lemon)/P(“pass”)
P(lemon | “pass”) = 0.4 * (0.2 / 0.8) = 0.1
P(lemon | “fail”) = P(“fail” | lemon) P(lemon)/P(“fail”)
P(lemon | “fail”) = 0.6 * (0.2 / 0.2) = 0.6

Inspection Tree



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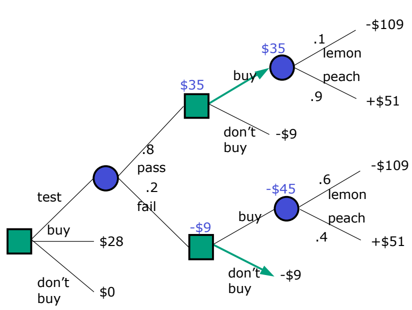

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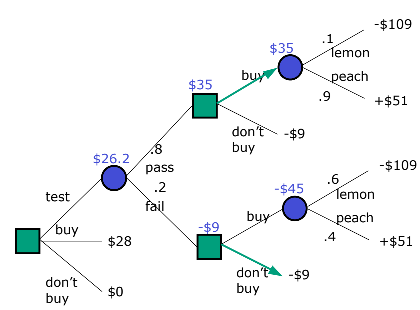

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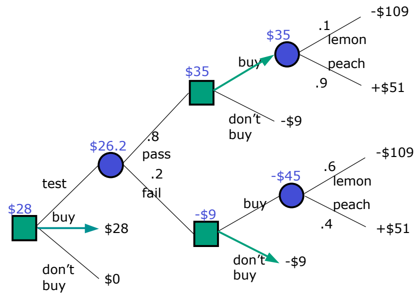

Recitation Problem

Let's consider one last scenario in the purchase of used cars. We are going to have the car inspected, and then use the result of the inspection to decide if we will:

- buy the car without a guarantee
- buy the car with a guarantee
- not buy the car

Calculate the decision tree for this scenario. Use all the costs and probabilities from the previous scenarios. What is the expected value? Is it better than just buying the car ($28)?

Another Recitation Problem

Is it ever useful (in the sense of resulting in higher utility) to pay for information, but take the same action no matter what information you get?