

Recitation 14

Solutions

December 16, 2001

15 I: d-separation

1. No, there is a path from A-D-B-E-C-F
2. Yes
3. Yes
4. Yes
5. A and G are d-separated given nothing
6. A and G are d-connected given L, M

15 II: More Watson and Holmes

1. $P(H) = P(H|SR)P(S)P(R) + P(H|S\bar{R})P(S)P(\bar{R}) + P(H|\bar{S}R)P(\bar{S})P(R) + P(H|\bar{S}\bar{R})P(\bar{S})P(\bar{R}) = .344$
2. $P(R|H) = \frac{P(H|R)P(R)}{P(H)} = .58$
3. $P(S|H) = \frac{P(H|S)P(S)}{P(H)} = .267$
4. $P(W|H) = P(W|R)P(R|H) + P(W|\bar{R})P(\bar{R}|H) = .664$
5. $P(R|WH) = \frac{P(RWH)}{P(WH)}$
 $P(R, W, H) = P(RWH|S)P(S) + P(RWH|\bar{S})P(\bar{S}) = .2$
 $P(W, H) = P(WH|SR)P(S)P(R) + P(WH|S\bar{R})P(S)P(\bar{R}) + P(WH|\bar{S}R)P(\bar{S})P(R) + P(WH|\bar{S}\bar{R})P(\bar{S})P(\bar{R}) = .2288$
6. $P(S|WH) = \frac{P(SWH)}{P(WH)}$
 $P(S, W, H) = P(SWH|R)P(R) + P(SWH|\bar{R})P(\bar{R}) = .0344$
 $P(W, H) = .2288$

16 I

1. Start with $p(D) = \sum_A P(A) \sum_B P(B|A) \sum_C P(C|B) P(D = d|C)$
 $= \sum_A P(A) \sum_B P(B|A) f_1(B) = \text{sum}_A P(A) f_2(A)$
2. Start with $p(D) = \sum_A P(A) \sum_B P(B|A) \sum_C P(C|B) P(D|C)$
 $= \sum_A P(A) \sum_B P(B|A) f_1(B, D) = \text{sum}_A P(A) f_2(A, D)$

16II

Eliminate in this order: B, C, J, D, E, K, F, G, A, H, I, L, M, O, N, P
the formula looks like

$$\sum_P \sum_N \sum_O P(P|NO) \sum_M \sum_L P(O|LM) \sum_I \sum_H P(M|HI) \\
\sum_A P(A) P(H|A) P(I|A) \sum_G P(G|A) \sum_F P(F|A) P(L|FG) \sum_K \\
\sum_E P(E|A) \sum_D P(D|A) P(K|DE) \sum_J P(J|BC) P(N|JK) \sum_C P(C|A) \sum_B P(B|A) P(J|BC)$$

this yields factors like: $f_1(JCA)f_2(J, A)f_3(NKA)f_4(NKAE)f_5(NKA)$
 $f_6(NA)f_7(NALG)f_8(NAL)f_9(NLHI)f_{10}(NLMI)$
 $f_{11}(NLM)f_{12}(MON)f_{13}(ON)f_{14}(PN)f_{15}(P)$

16III

in the early stages of variable elimination, each variable that you eliminate adds 1 or 2 new variables (the variables to the 'north' and 'west' of the one you eliminated). In other words, the variables stored in each factor will keep growing larger, meaning that the amount of storage and computation will grow exponentially.