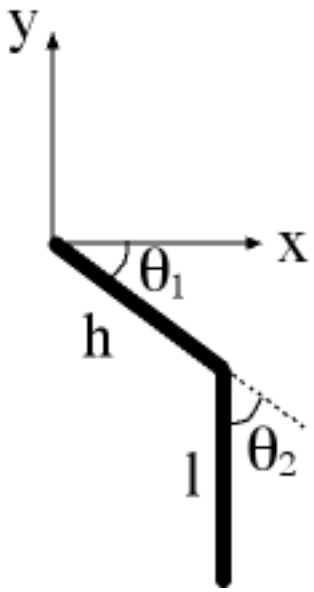
6.836 Embodied Intelligence, 2001
Problem Set 2
Issued February 23, due March 9.

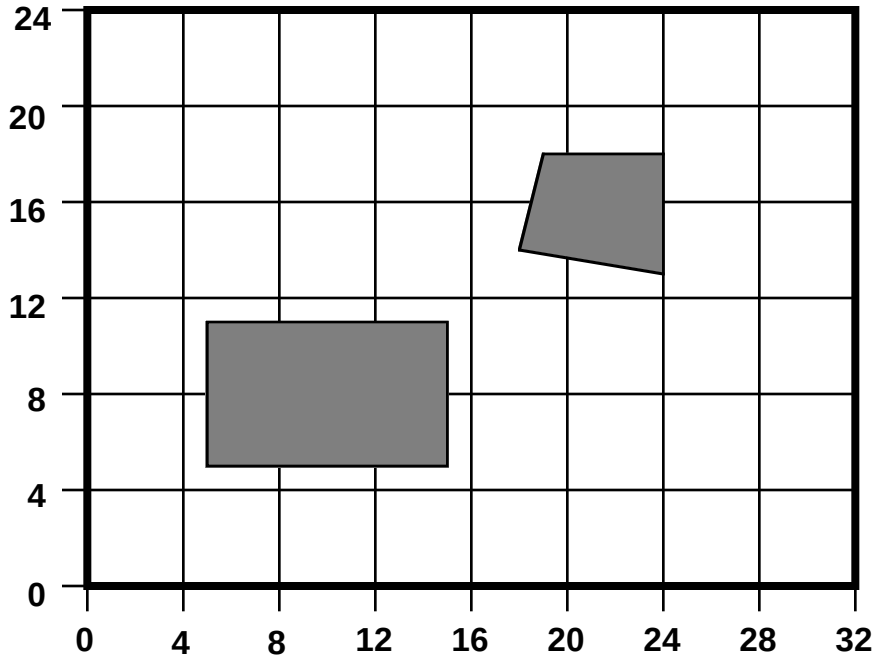
A. Consider the 2 DOF leg below.



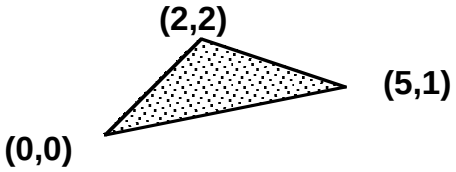
It has two revolute joints. The first joint is centered at the origin, and the second is on the knee. Its foot has coordinate $[x, y]$ in the plane. Angle θ_1 is positive in the downward direction for the thigh where zero means that the thigh is straight out parallel to the ground. Angle θ_2 is how bent the knee is, positive is down, from the straight knee position. The length of the thigh is h, and the length of the lower leg is l. Compute the following:

1. The forward kinematics, $[x, y]^T = f([\theta_1, \theta_2]^T)$, or $X = f(\Theta)$.
2. The inverse kinematics, $[\theta_1, \theta_2]^T = g([x, y]^T)$, or $\Theta = g(X)$.
3. The Jacobian $J(\Theta)$, where $\dot{X} = J(\Theta)\dot{\Theta}$.
4. The determinant of the Jacobian, $|J(\Theta)|$. Determine whether it can ever be zero and explain the physical interpretations in that case.
5. The inverse Jacobian, $J^{-1}(\Theta)$.
6. Functions of time for θ_1 and θ_2 for the endpoint of the manipulator to follow the line $x(t) =$ `startx` $+$ `speed` $\times t$, $y(t) =$ `height`.
7. Suppose you have a processor with only 16 bit `int`s, signed and unsigned, but no floating point, and so no `sin` or `cos` procedures. Outline the code, including the description of the contents of any lookup tables and their sizes, but don't bother with the initialization of those tables, that would let you implement `stroke(int height, int startx, int endx, int speed)` where each of the arguments is in some appropriate scaled coordinate system. Assume you have a primitve `wait(int ticks)` which waits for `ticks` milliseconds, and a procedure called `setmotors(int theta1, int theta2)` which drives the motors to some target angles, again in an integer scaled coordinate system. You will have to use the idea of leading the motors to make this all work.

B. Consider the following two dimensional workspace (all vertices of obstacles have integer coordinates):



Suppose we wish to move the following object (not drawn to scale–the coordinates give the right scale) around this workspace, with no rotations allowed.



1. Draw the configuration space diagram (graph paper will make it easy to see for you and me) of that problem. Use the origin of the moving object as the reference point.
2. Overlay a cell decomposition that is made of 2 by 2 cells and mark them as full, mixed, or empty.
3. Illustrate a successive refinement strategy to find a path from coordinate $(4, 16)$ to $(26, 16)$.