6.867 Machine learning and neural networks

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Lecture 19: representation, graph models

Topics

- Representations of model structure
  - properties of representations
  - state diagrams vs. graph models
  - Bayesian networks

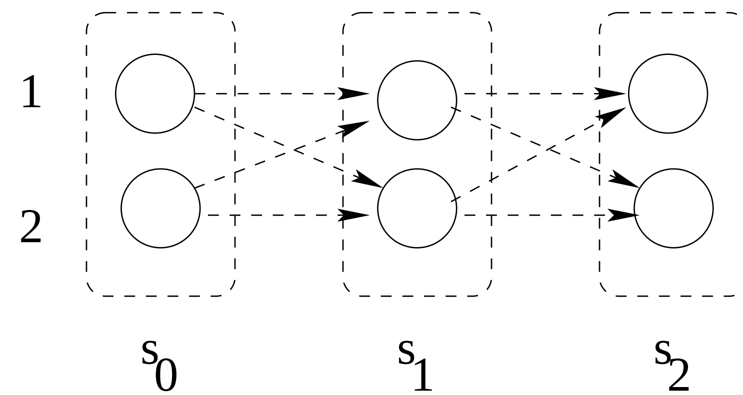
What is a good representation?

- Properties of good representations
  1. Explicit
  2. Compact
  3. Modular
  4. Permits efficient computation
  5. etc.

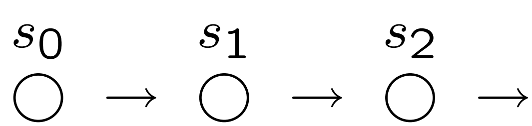
Representing the model structure

- Two possible representations of Markov models:

  1. in terms of state diagrams (nodes in the graph correspond to the possible values of the states)



  2. in terms of variables (nodes in the graph are variables):

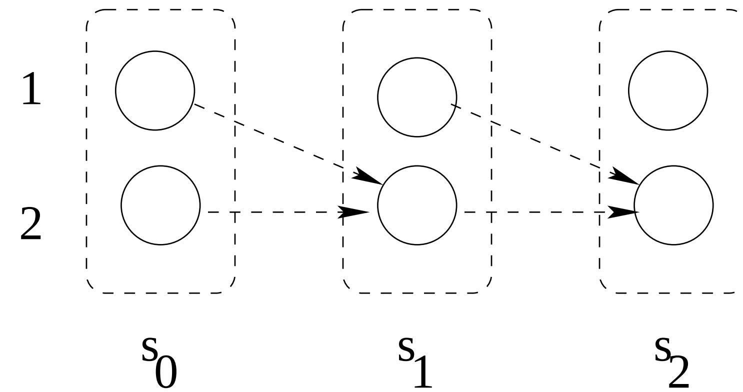


- The representations differ in terms of what aspects of the model are made *explicit*

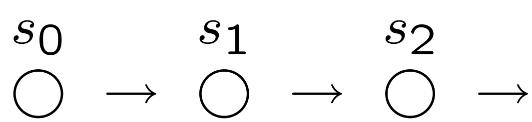
Model structure cont'd

- Case 1: *sparse transition* structure

  1. State transition diagram is *explicit*



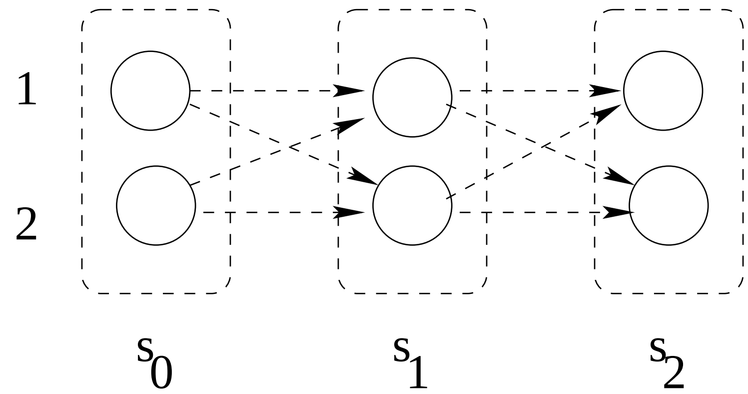
  2. Representation in terms of variables leaves this *implicit*



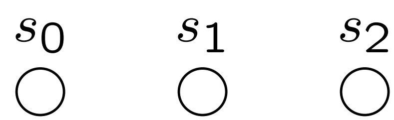
Model structure cont'd

- Case 2: successive states are *independent of each other*

  1. State transition diagram is fully connected



  2. Representation in terms of variables is *explicit*



Model structure cont'd

- Case 3: time series signals such as music may involve multiple relatively independent underlying processes operating at different *time scales*

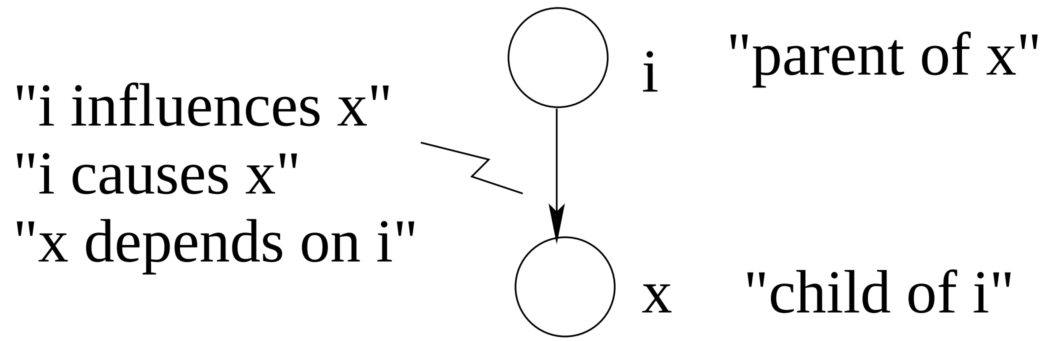
  1. State transition diagram (argh #$& ...)
  2. In terms of variables (graph model)

$$
\begin{array}{ccccccccccc}
s_0^{(1)} & & & & s_2^{(1)} & & & & s_4^{(1)} & & \\
\bigcirc & & \longrightarrow & & \bigcirc & & \longrightarrow & & \bigcirc & & \\
\downarrow & & & & \downarrow & & & & \downarrow & & \\
\bigcirc & \longrightarrow & \bigcirc & \longrightarrow & \bigcirc & \longrightarrow & \bigcirc & \longrightarrow & \bigcirc & \longrightarrow & \\
s_0^{(2)} & & s_1^{(2)} & & s_2^{(2)} & & s_3^{(2)} & & s_4^{(2)} & &
\end{array}
$$

Graphical models

- Graph representions of probability models in terms of *variables* are known as *graphical models*

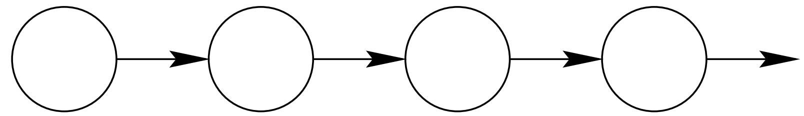
A mixture model as
a graphical model



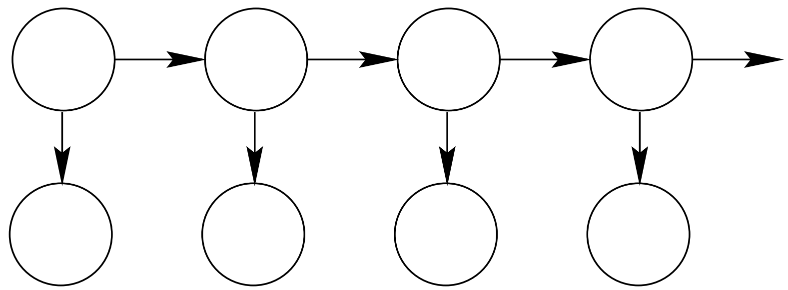
- Different types of graph models differ in terms of how we represent *dependencies* and *independencies* among the variables
  1. Bayesian networks (natural for "causal" relations)
  2. Markov random fields (natural for physical or symmetric relations)
  3. etc.

Bayesian networks: examples

A Markov chain:

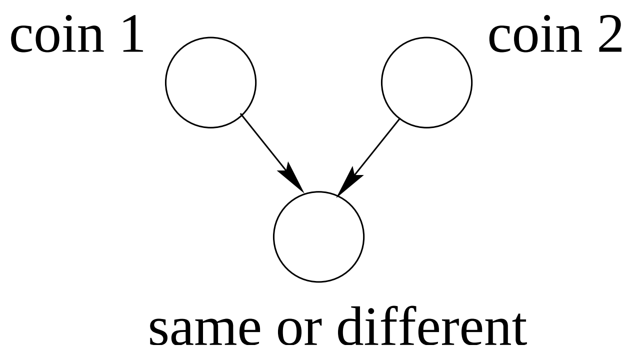


A hidden Markov model:



Qualitative inference

- The graph provides a qualitative description of the domain



$x_1 =$ first coin toss

$x_2 =$ second coin toss

$x_3 =$ same?

Qualitative inference

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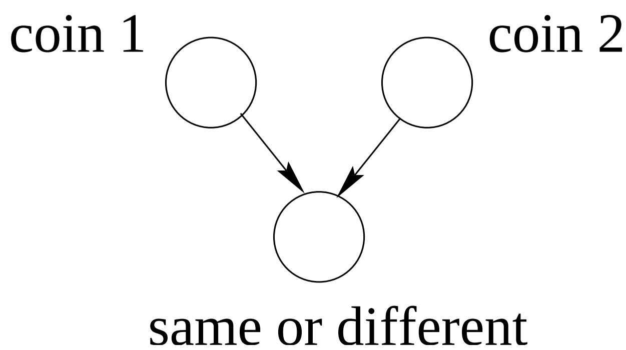

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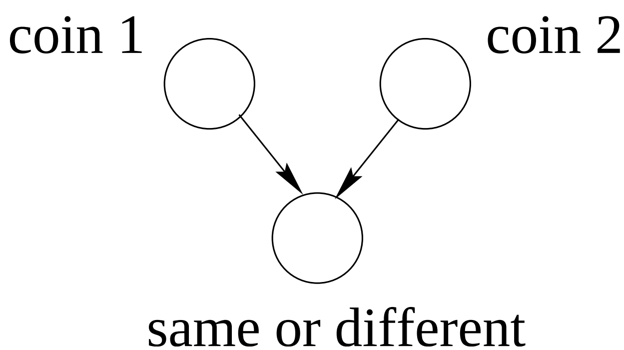
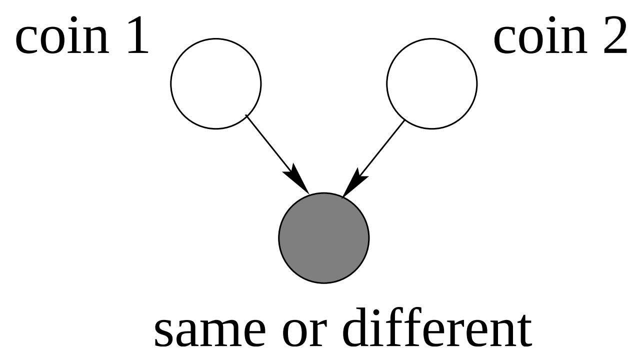


Marginal independence

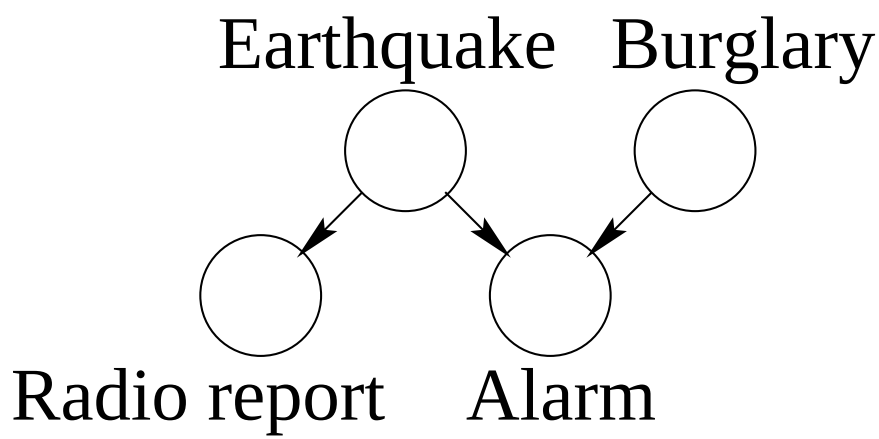


Induced dependence

Qualitative inference cont'd

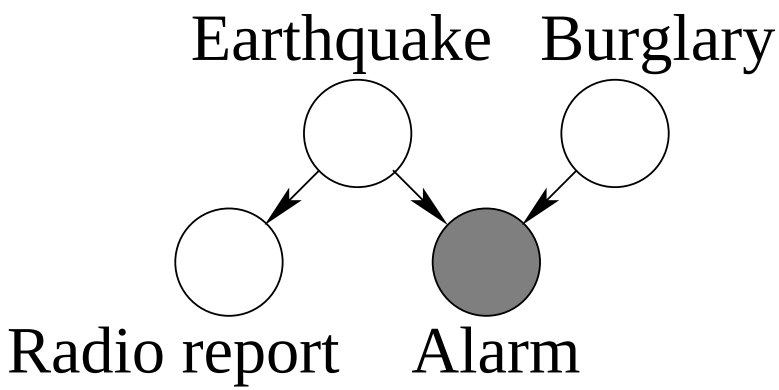
- Just by looking at the graph, we can determine what we can and cannot ignore (why important?)

  Marginal independence of "Earthquake" and "Burglary"

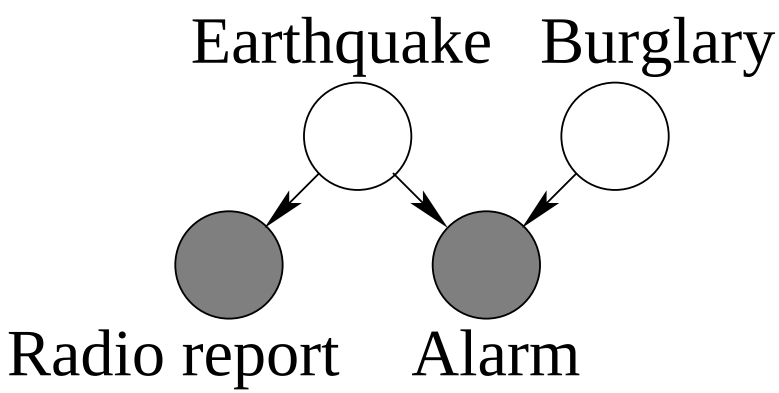


Qualitative inference cont'd

- Induced dependence:



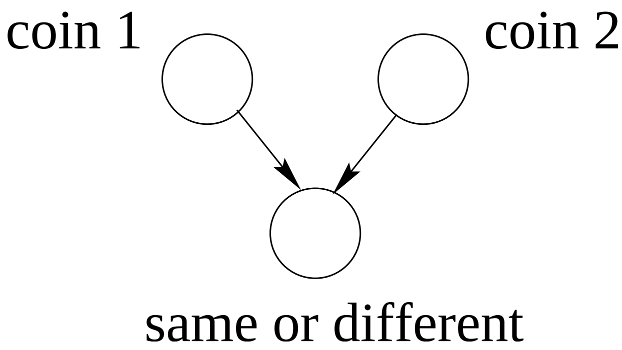
- Explaining away:



Two levels of description

- Graphical models need two levels of specification

  1. Qualitative properties captured by a graph



x_1 = first coin toss
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  2. Quantitative properties specified by the associated probability distribution

$$P(x_1, x_2, x_3) = P(x_1)\,P(x_2)\,P(x_3|x_1, x_2)$$

where, e.g.,

$$P(x_1 = heads) = 0.5$$
$$P(x_3 = same|x_1 = heads, x_2 = tails) = 0$$

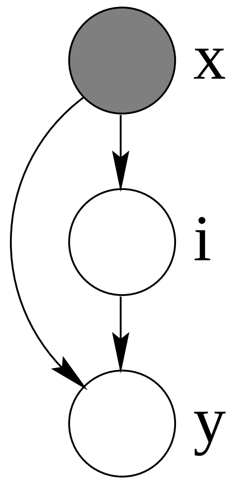
More examples

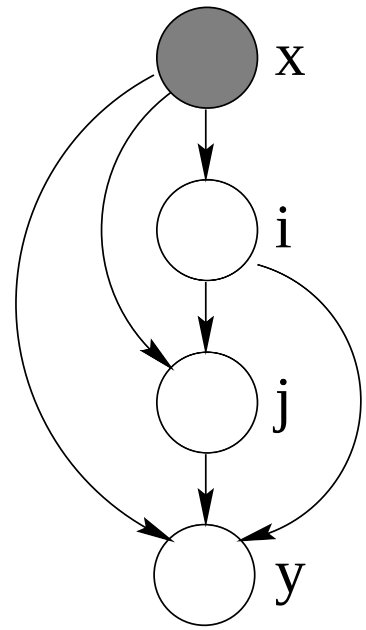
Mixture model   hierarchical mixture model

- i and j correspond to the discrete choices in the mixture model
- $\mathbf{x}$ is the (vector) variable whose density we wish to model

- We cannot tell what the component distributions $P(\mathbf{x}|i)$ are based on the graph alone

More examples cont'd



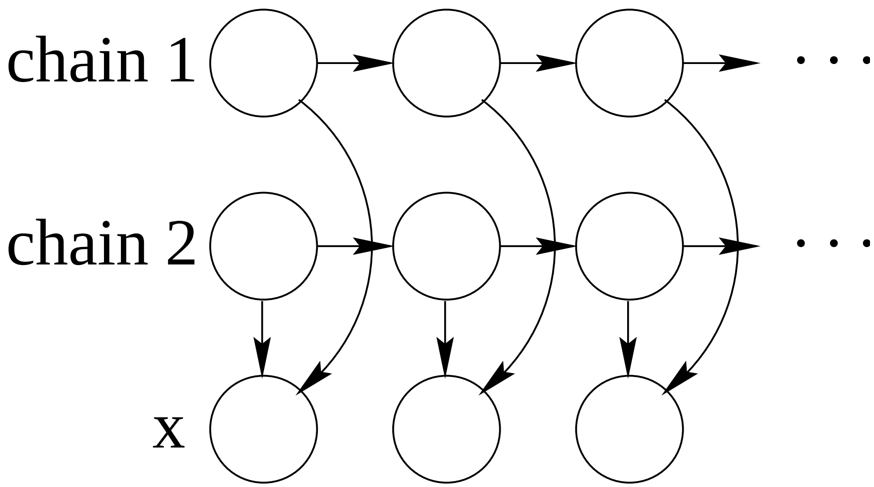


Mixture of experts   hierarchical mixture of experts

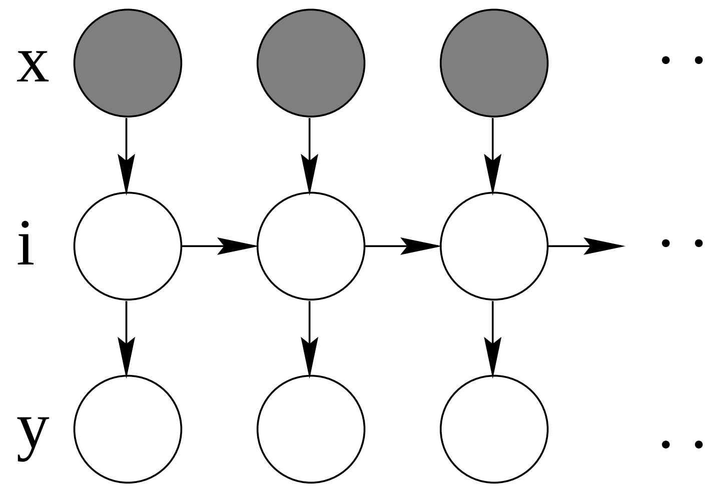
- In this case the choices of i and j and the output y depend on the input x

  (The shaded variables denote *observed* values; we do not need to model the density over $\mathbf{x}$)

More examples cont'd



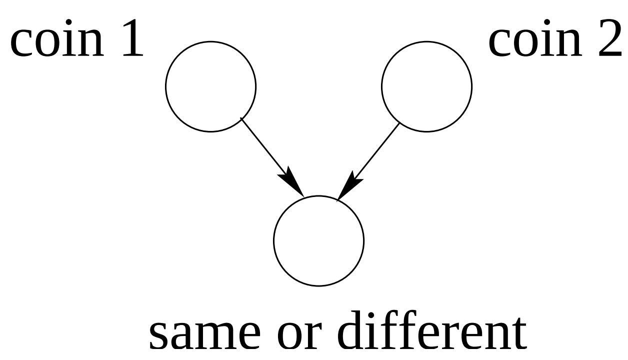
Factorial HMM



input-output HMM

- In factorial HMMs, independent processes conspire to generate the observed output sequence
- In input-output HMMs, any observed sequence of outputs y is accompanied by a corresponding sequence of *inputs* $\mathbf{x}$
  - the model tranforms any input sequence into an output sequence (markov?)

Graph model specification



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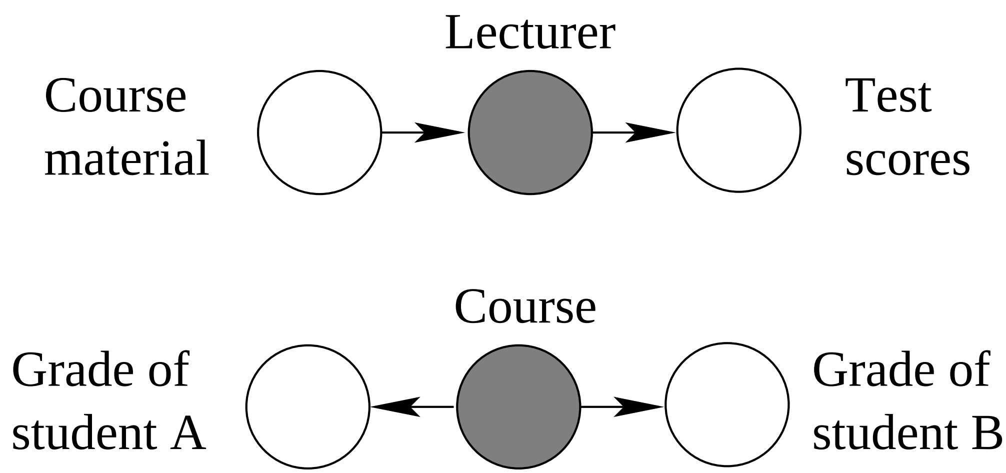
$x_3 =$ same?

- We need to address the following questions
  1. What is the graph semantics?
  2. What type of probability distribution can be associated with any specific graph?
  3. How can we exploit the graph in making quantitative inferences?

Graph semantics

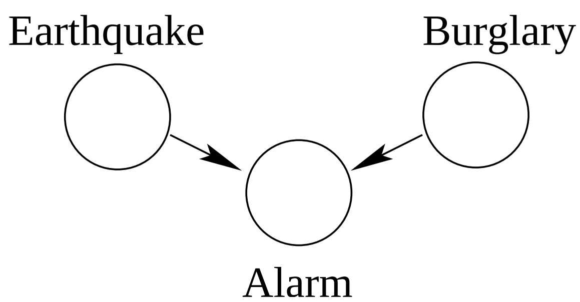
- The graph captures *independence properties* among the variables
- The independences can be read from the graph based on some notion of *graph separation*



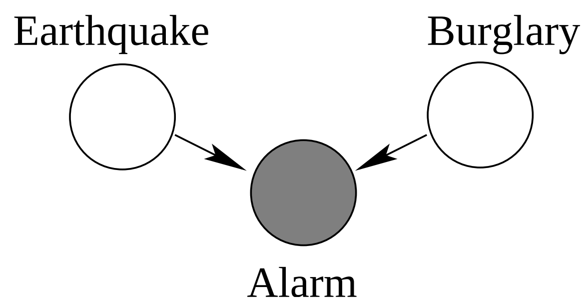
conditional independence

Graph semantics cont'd

- We have already seen the interesting cases...



marginal independence



induced dependence

- Note that the formal "graph separation" measure here must pay attention to the direction of the edges

Graph separation criterion (briefly)

- D-separation criterion (D for Directed edges):

  Definition: variables x and y are D-separated (conditionally independent) given z if they are separated in the *moralized ancestral graph*

- Example: