Machine learning: lecture 22

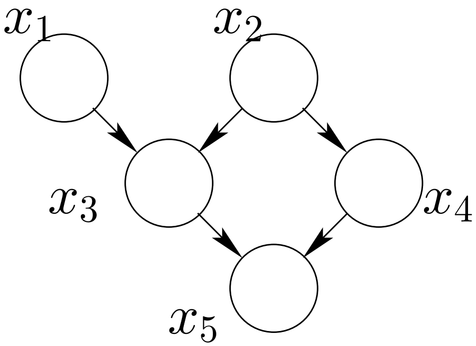
Tommi S. Jaakkola
MIT CSAIL
tommi@csail.mit.edu

Outline

- Exact inference
  - graph operations
  - cliques and potential functions
  - message passing (junction tree) algorithm
- Approximate inference

Exact inference: key steps

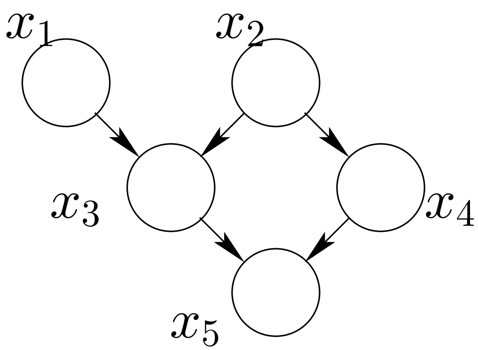
- Moralization



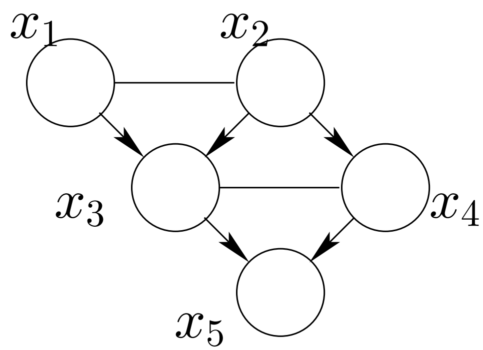
original graph

Exact inference: key steps

- Moralization



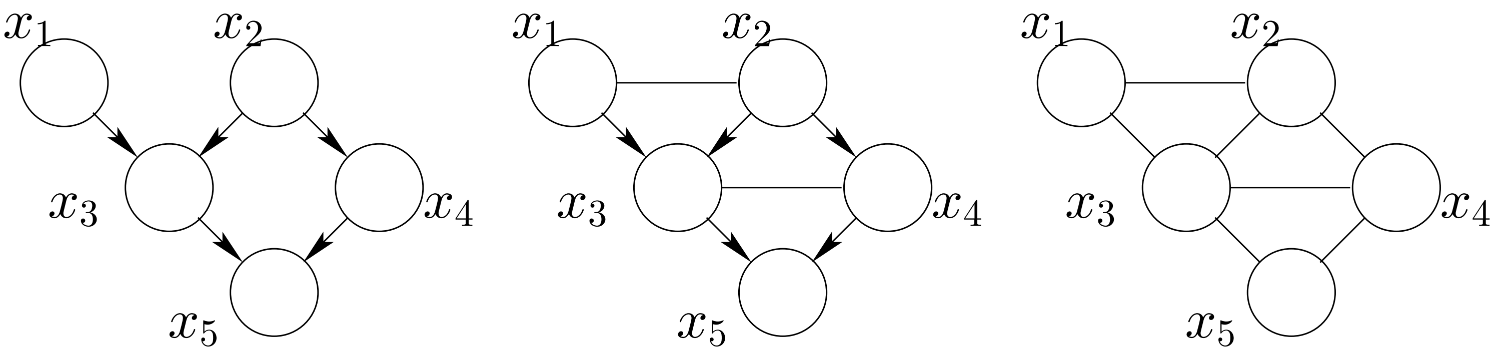
original graph



“marry” parents

Exact inference: key steps

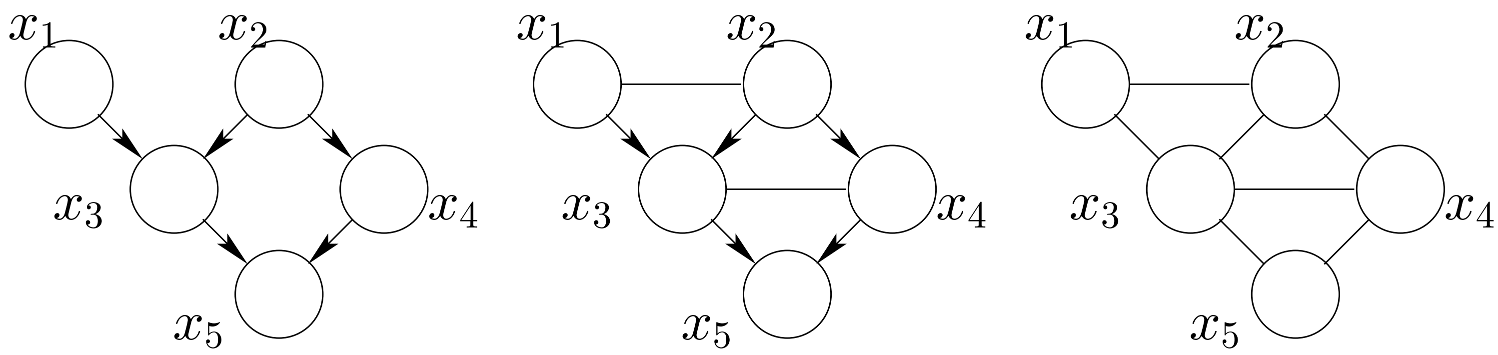
- Moralization



original graph "marry" parents moral graph

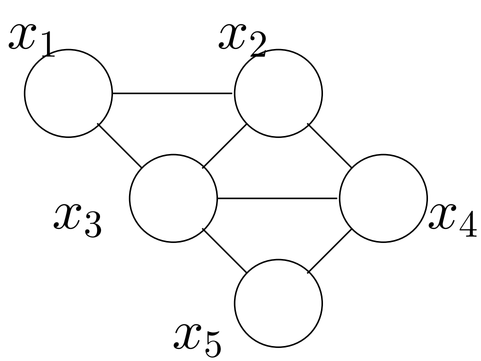
Exact inference: key steps

- Moralization



original graph “marry” parents moral graph

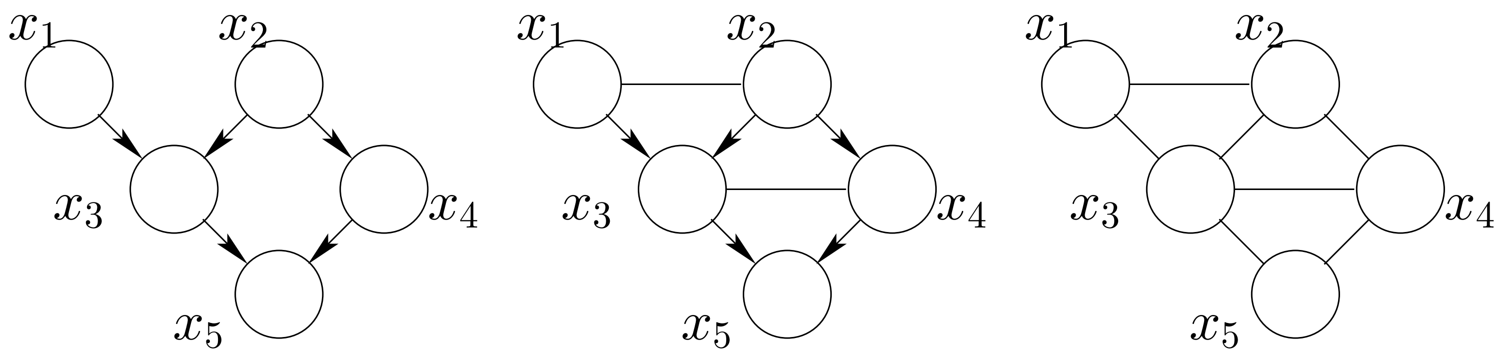
- Triangulation (add edges so that any cycle of four or more nodes has a “chord”)



already triangulated

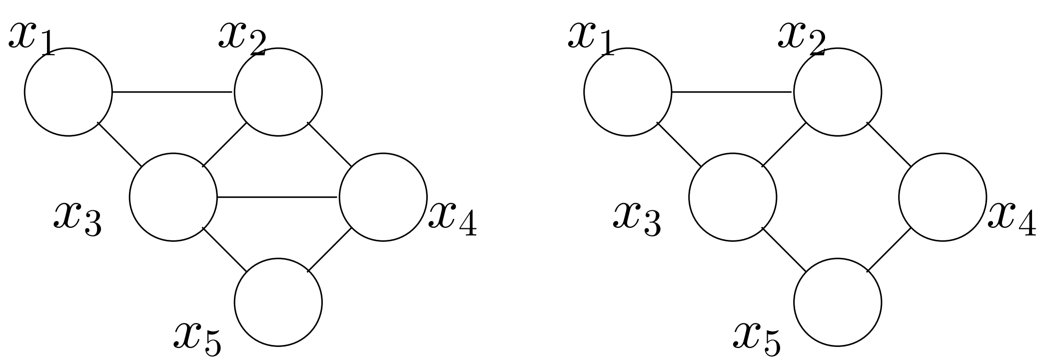
Exact inference: key steps

- Moralization



original graph "marry" parents moral graph

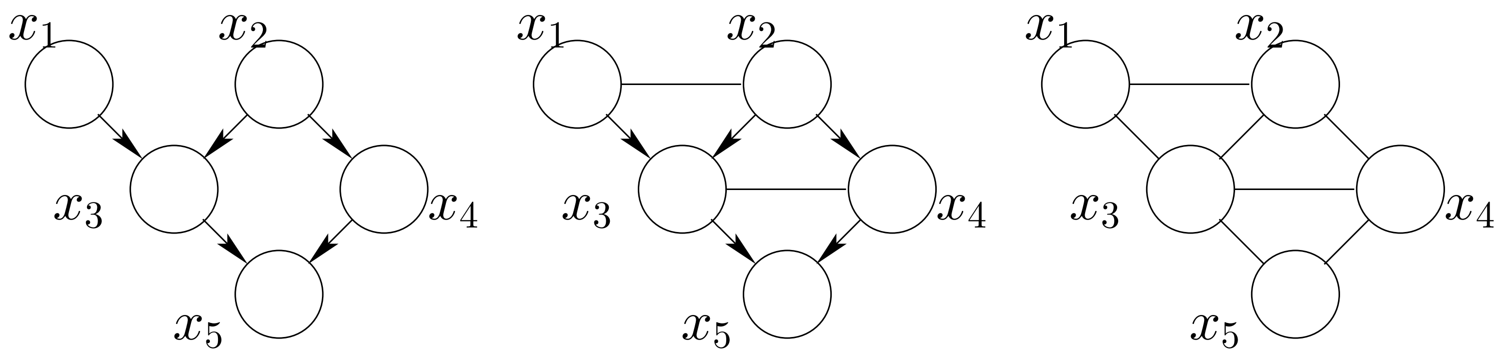
- Triangulation (add edges so that any cycle of four or more nodes has a "chord")



already triangulated not triangulated

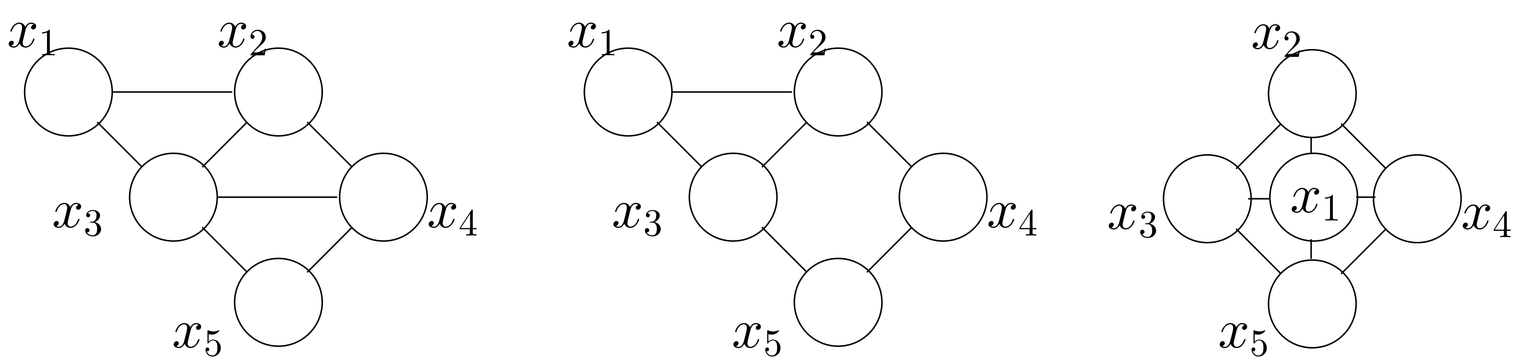
Exact inference: key steps

- Moralization



original graph "marry" parents moral graph

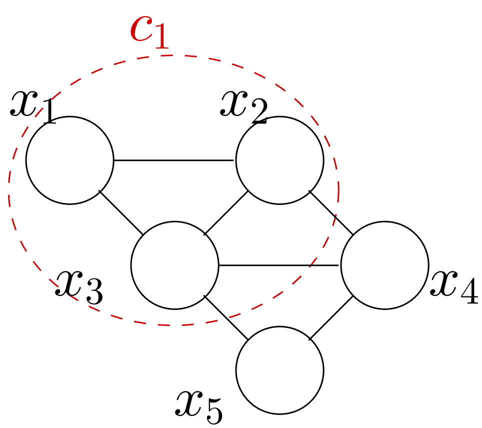
- Triangulation (add edges so that any cycle of four or more nodes has a "chord")



already triangulated not triangulated not triangulated

Exact inference: key steps cont'd

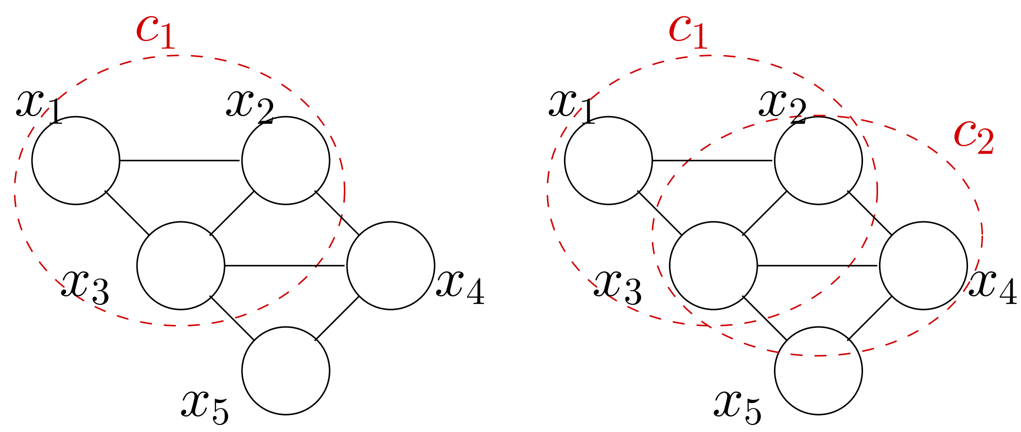
- Find the maximal cliques of the triangulated graph



$$c_1 = \{x_1, x_2, x_3\}$$

Exact inference: key steps cont'd

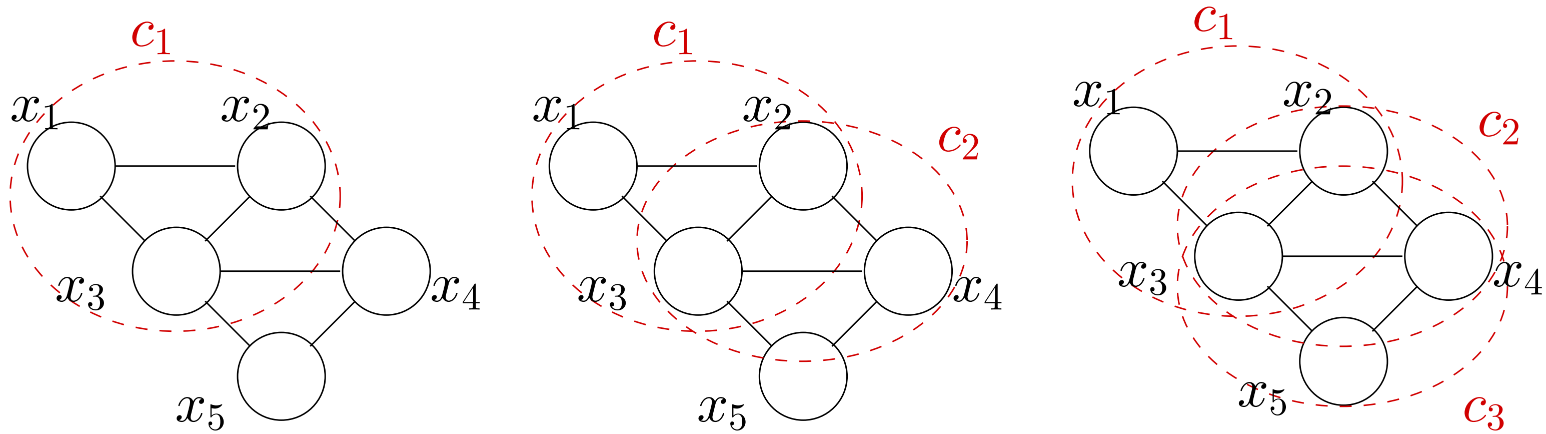
- Find the maximal cliques of the triangulated graph



$$c_1 = \{x_1, x_2, x_3\} \quad c_2 = \{x_2, x_3, x_4\}$$

Exact inference: key steps cont'd

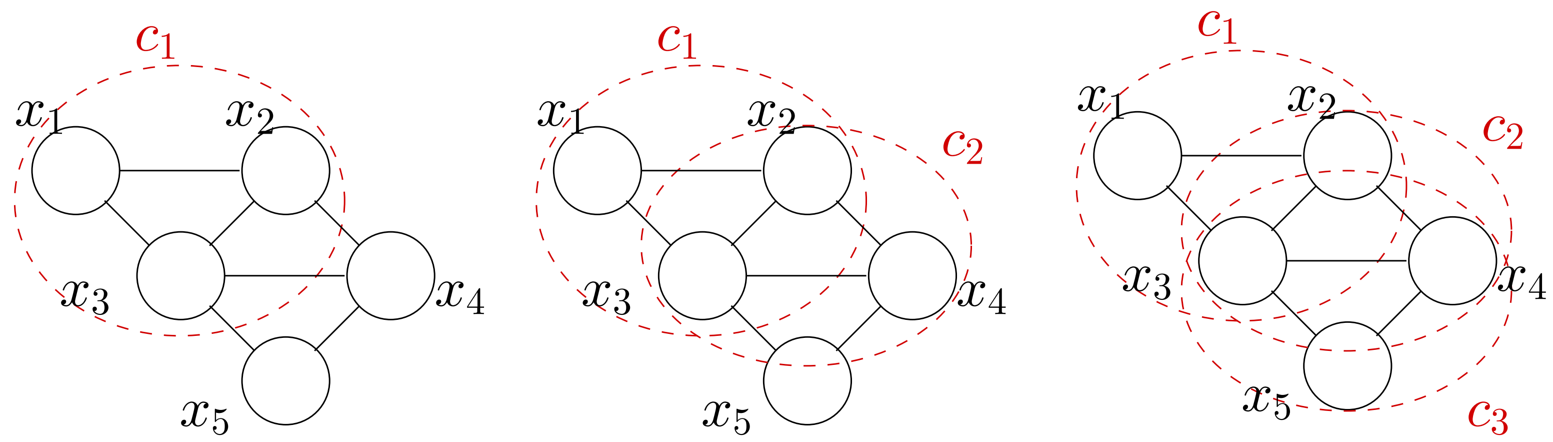
- Find the maximal cliques of the triangulated graph



$$c_1 = \{x_1, x_2, x_3\} \quad c_2 = \{x_2, x_3, x_4\} \quad c_3 = \{x_3, x_4, x_5\}$$

Exact inference: key steps cont'd

- Find the maximal cliques of the triangulated graph



$c_1 = \{x_1, x_2, x_3\}$ $\quad c_2 = \{x_2, x_3, x_4\}$ $\quad c_3 = \{x_3, x_4, x_5\}$

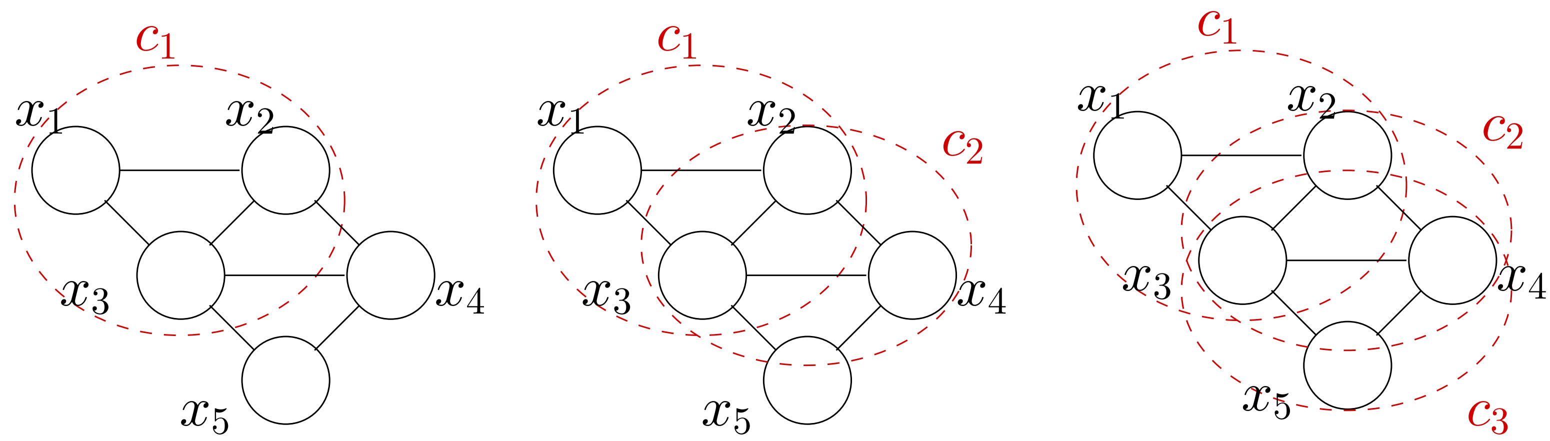
Theorem: [Hammersley-Clifford] Any distribution consistent with an undirected graph must factor according to the (maximal) cliques in the graph

$$P(\mathbf{x}) = \prod_{c \in \mathcal{C}} \psi_c(\mathbf{x}_c)$$

where $\mathbf{x}_c$ are the set of variables associated with clique c.

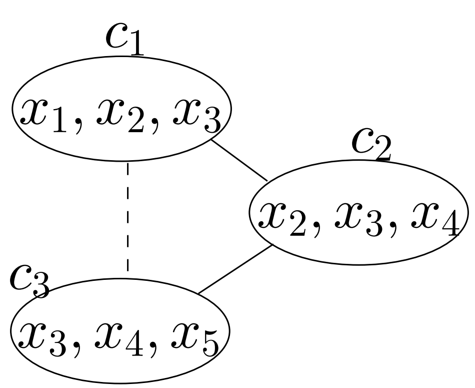
Exact inference: key steps cont'd

- Find the maximal cliques of the triangulated graph



$$c_1 = \{x_1, x_2, x_3\} \quad c_2 = \{x_2, x_3, x_4\} \quad c_3 = \{x_3, x_4, x_5\}$$

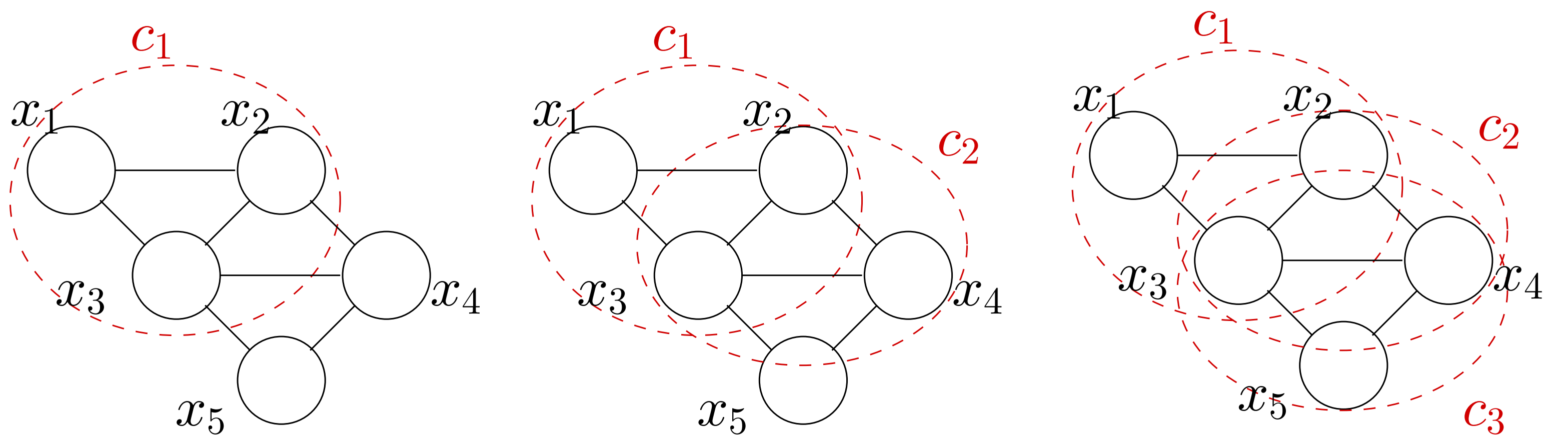
- Clique trees and junction trees



clique tree

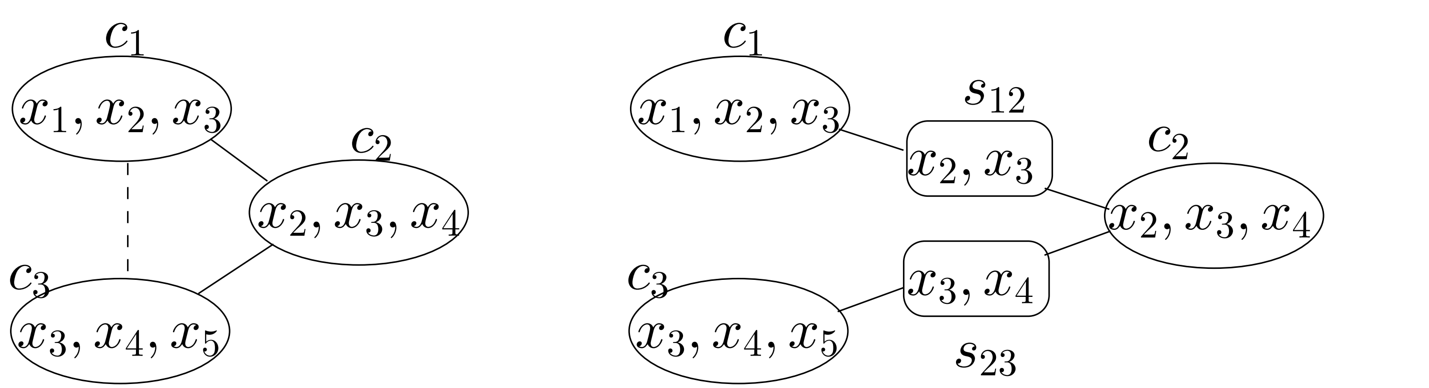
Exact inference: key steps cont'd

- Find the maximal cliques of the triangulated graph



$$c_1 = \{x_1, x_2, x_3\} \quad c_2 = \{x_2, x_3, x_4\} \quad c_3 = \{x_3, x_4, x_5\}$$

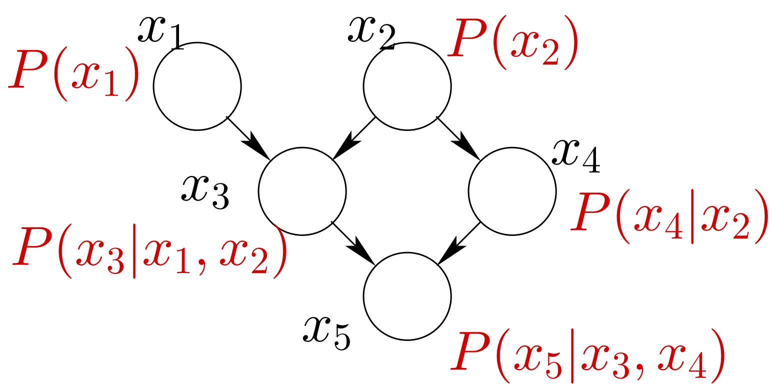
- Clique trees and junction trees



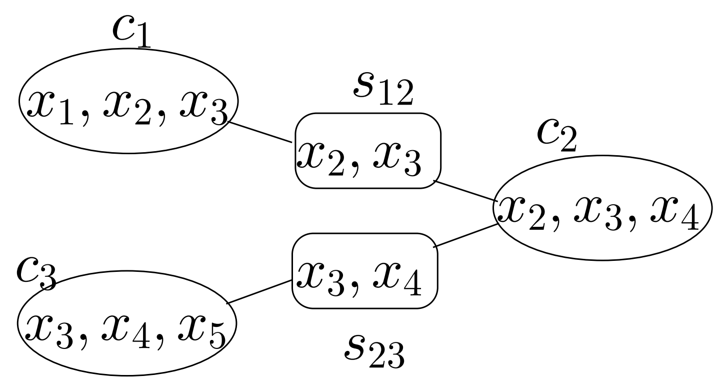
clique tree junction tree (with separators)

Exact inference: potentials

- Associating graphs and potentials



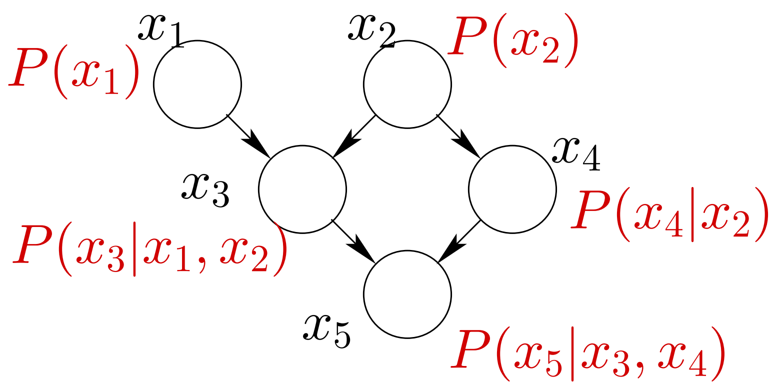
original graph w/ probs



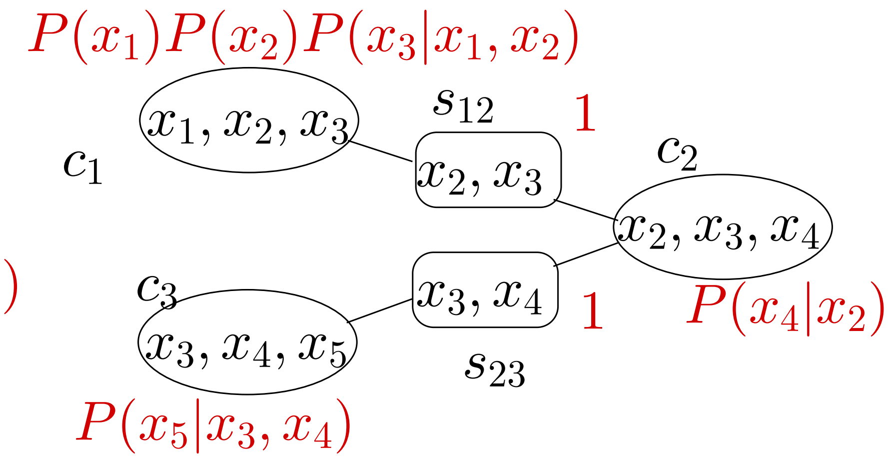
junction tree

Exact inference: potentials

- Associating graphs and potentials



original graph w/ probabilities

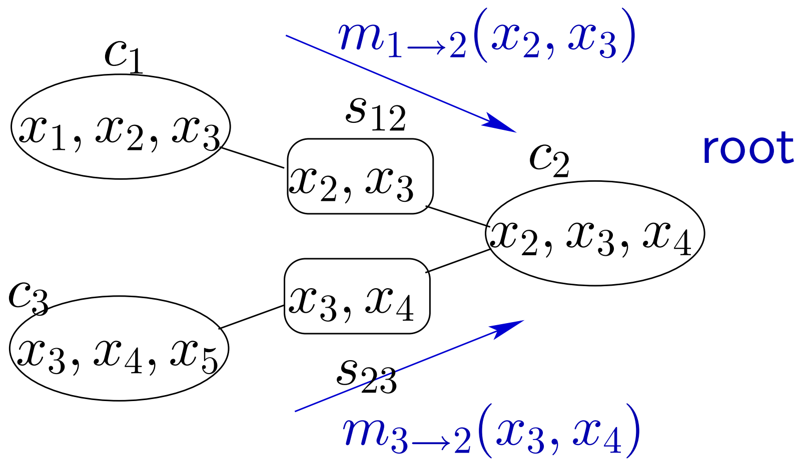


junction tree w/ probs

$$
\begin{aligned}
\psi_{c_1}(x_1,x_2,x_3) &= P(x_1)P(x_2)P(x_3|x_1,x_2)\\
\psi_{c_2}(x_2,x_3,x_4) &= P(x_4|x_2)\\
\psi_{c_3}(x_3,x_4,x_5) &= P(x_5|x_3,x_4)\\
\psi_{s_{12}}(x_2,x_3) &= 1 \quad \text{(separator)}\\
\psi_{s_{23}}(x_3,x_4) &= 1 \quad \text{(separator)}
\end{aligned}
$$

Exact inference: message passing

- Select a root clique
- Collect evidence



- Distribute evidence