6.891

Computer Vision and Applications

Prof. Trevor. Darrell

Lecture 11: Model-based vision

- Hypothesize and test
- Interpretation Trees
- Alignment
- Pose Clustering
- Geometric Hashing

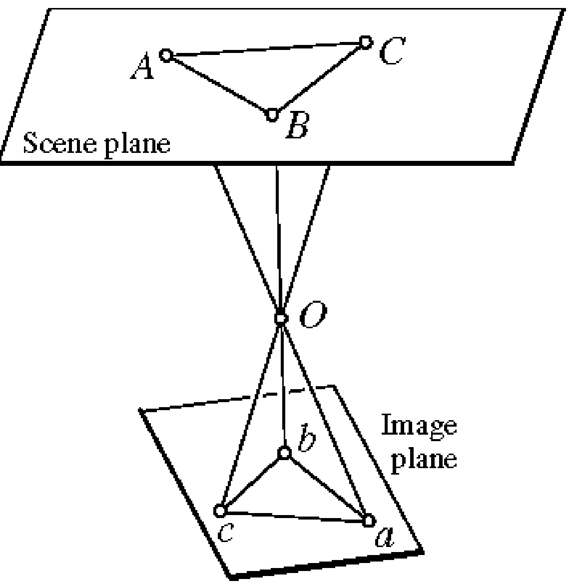
Readings: F&P Ch 18.1-18.5

Last time

Projective SFM

- Projective spaces
- Cross ratio
- Factorization algorithm
- Euclidean upgrade

Projective transformations



<u>Definition:</u>

A *projectivity* is an invertible mapping h from P^2 to itself such that three points x_1,x_2,x_3 lie on the same line if and only if $h(x_1),h(x_2),h(x_3)$ do.

<u>Theorem:</u>

A mapping $h:P^2 \rightarrow P^2$ is a projectivity if and only if there exist a non-singular 3x3 matrix $\mathbf{H}$ such that for any point in P^2 reprented by a vector x it is true that $h(x)=\mathbf{H}x$

<u>Definition:</u> Projective transformation

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{or} \quad x' = \mathbf{H}\,x$$

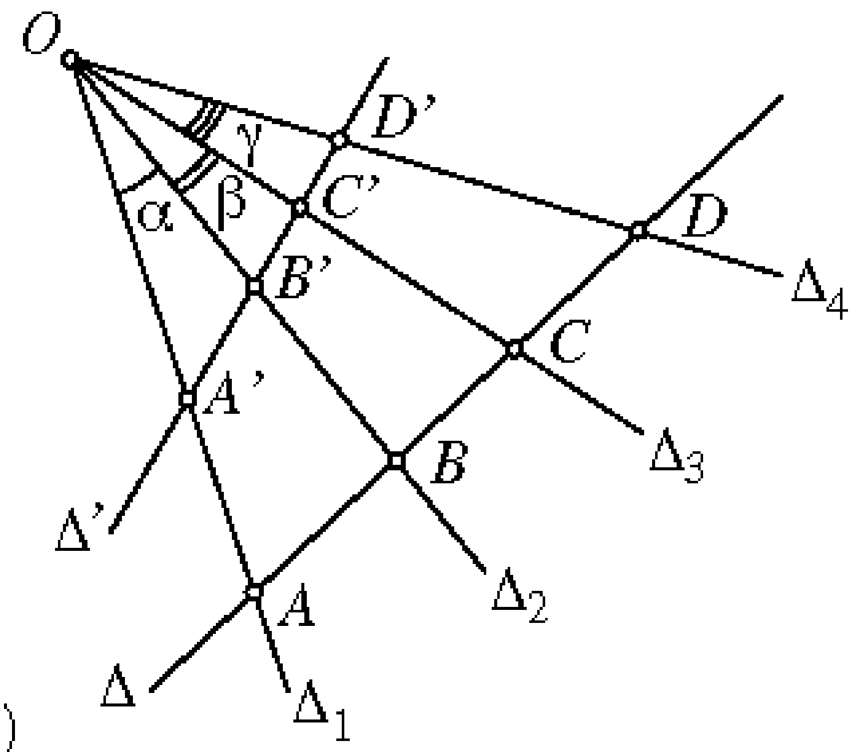
8DOF

projectivity=collineation=projective transformation=homography

[F&P, www.cs.unc.edu/~marc/mvg]

$$\{A, B; C, D\} \stackrel{\text{def}}{=} \frac{\overline{CA}}{\overline{CB}} \times \frac{\overline{DB}}{\overline{DA}}$$

The value of this cross ratio is independent of the intersecting line or plane:



(a)

[F&P]

Two-frame reconstruction

(i) Compute F from correspondences
(ii) Compute camera matrices from F
(iii) Compute 3D point for each pair of corresponding points

computation of F

use $x'_i F x_i = 0$ equations, linear in coeff. F
8 points (linear), 7 points (non-linear), 8+ (least-squares)
(more on this next class)

computation of camera matrices

Possible choice:

$$P = [I \mid 0] \quad P' = [[e']_\times F \mid e']$$

triangulation

compute intersection of two backprojected rays

[www.cs.unc.edu/~marc/mvg]

Perspective factorization

$$\lambda_{ij}\mathrm{m}_{ij} = \mathbf{P}_i\,\mathrm{M}_j\,, i = 1,\ldots,m,\ j = 1,\ldots,m$$

All equations can be collected for all i as

$$\mathbf{m} = \mathbf{PM} \quad \text{where,} \quad \mathbf{m} = \begin{bmatrix} \mathbf{m}_1\Lambda_1 \\ \mathbf{m}_2\Lambda_2 \\ \cdots \\ \mathbf{m}_n\Lambda_n \end{bmatrix},\ \mathbf{P} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \cdots \\ \mathbf{P}_m \end{bmatrix}$$

with:

$$\mathbf{m}_i = [m_{i1}, m_{i2}, \ldots, m_{im}],\ \mathbf{M} = [\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_m]$$

$$\Lambda_i = \mathrm{diag}(\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{im})$$

m are known, but Λ_i, **P** and **M** are unknown…

Observe that **PM** is a product of a $3m$x4 matrix and a 4xn matrix, i.e. it is a rank 4 matrix

Iterative perspective factorization

When Λ_i are unknown the following algorithm can be used:

1. Set λ_{ij}=1 (affine approximation).

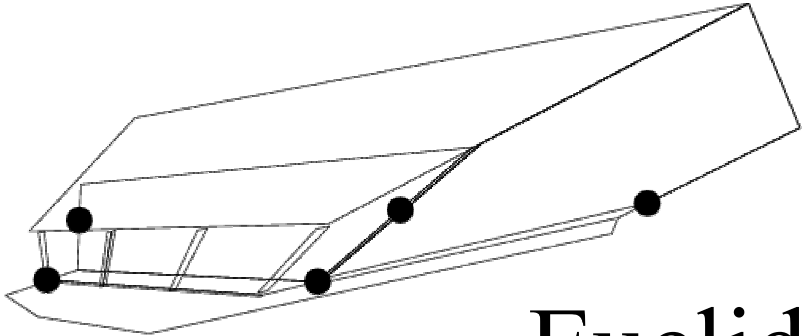
2. Factorize **PM** and obtain an estimate of **P** and **M**. If s_5 is sufficiently small then STOP.

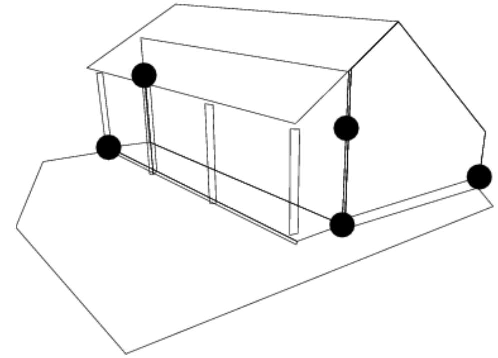
3. Use **m**, **P** and **M** to estimate Λ_i from the camera equations (linearly) $\mathbf{m}_i\,\Lambda_i=\mathbf{P}_i\mathbf{M}$

4. Goto 2.

In general the algorithm minimizes the *proximity measure* $P(\Lambda,\mathbf{P},\mathbf{M})=s_5$

Structure and motion recovered up to an arbitrary projective transformation





Euclidean upgrade

Given a camera with known intrinsic parameters, we can take the calibration matrix to be the identity and write the perspective projection equation in some Euclidean world coordinate system as

$$\boldsymbol{p} = \frac{1}{z}(\mathcal{R} \quad \boldsymbol{t})\begin{pmatrix}\boldsymbol{P}\\1\end{pmatrix} = \frac{1}{\lambda z}(\mathcal{R} \quad \lambda\boldsymbol{t})\begin{pmatrix}\lambda\boldsymbol{P}\\1\end{pmatrix}$$

for any non-zero scale factor λ. If $\mathcal{M}_i$ and P_j denote the shape and motion parameters measured in some Euclidean coordinate system, there must exist a 4 ×4 matrix $\mathcal{Q}$ su

$$\hat{\mathcal{M}}_i = \mathcal{M}_i\mathcal{Q} \text{ and } \hat{\boldsymbol{P}}_j = \mathcal{Q}^{-1}\boldsymbol{P}_j.$$

[F&P]

Today: “Model-based Vision”

Still feature and geometry-based, but now with moving objects rather than cameras…

Topics:

- Hypothesize and test
- Interpretation Trees
- Alignment
- Pose Clustering
- Invariances
- Geometric Hashing

Approach

- Given
  - CAD Models (with features)
  - Detected features in an image
- Hypothesize and test recognition…
  - Guess
  - Render
  - Compare

Hypothesize and Test Recognition

- Hypothesize object identity and correspondence
  - Recover pose
  - Render object in camera
  - Compare to image
- Issues
  - where do the hypotheses come from?
  - How do we compare to image (verification)?

Features?

- Points

but also,

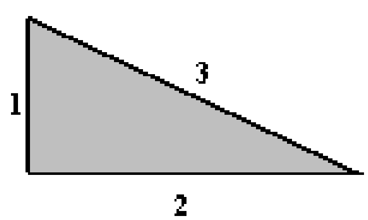
- Lines
- Conics
- Other fitted curves
- Regions (particularly the center of a region, etc.)

How to generate hypotheses?

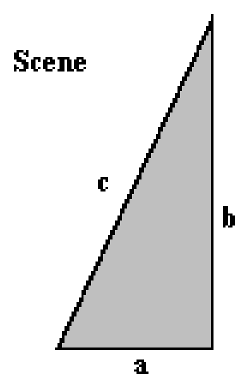
- Brute force
  - Construct a correspondence for all object features to every correctly sized subset of image points
  - Expensive search, which is also redundant.
  - L objects with N features
  - M features in image
  - $O(LM^N)$!
- Add geometric constraints to prune search, leading to *interpretation tree search*
- Try subsets of features (frame groups)…

Interpretation Trees

- Tree of possible model-image feature assignments
- Depth-first search
- Prune when unary (binary, …) constraint violated
  - length
  - area
  - orientation



Model

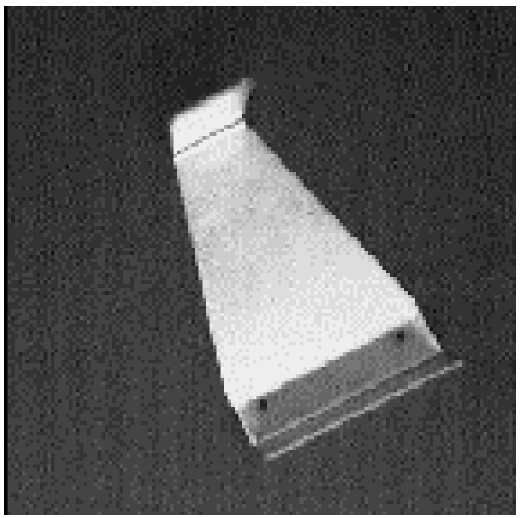


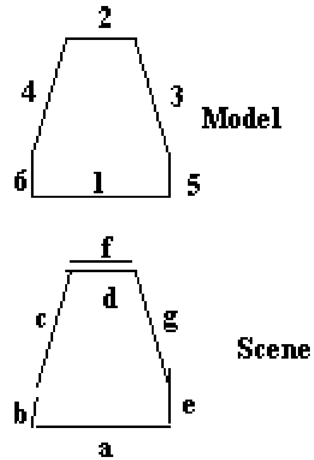
(a,1)

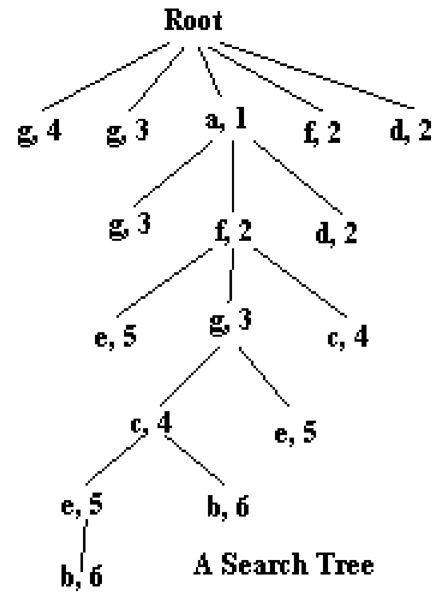
(b,2) …

…

Interpretation Trees







"Wild cards" handle spurious image features

[A.M. Wallace. 1988.]

Adding constraints

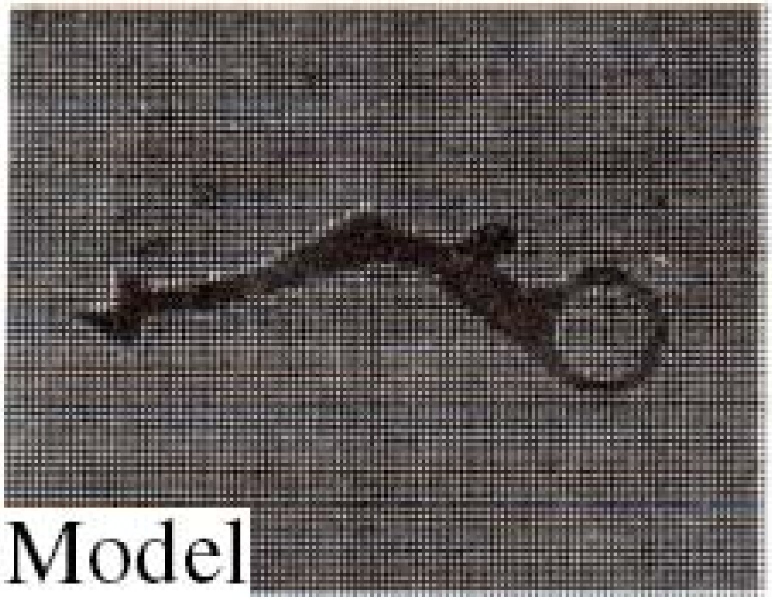
- Correspondences between image features and model features are not independent.
- A small number of good correspondences yields a reliable pose estimation --- the others must be consistent with this.
- Generate hypotheses using small numbers of correspondences (e.g. triples of points for a calibrated perspective camera, etc., etc.)

Pose consistency / Alignment

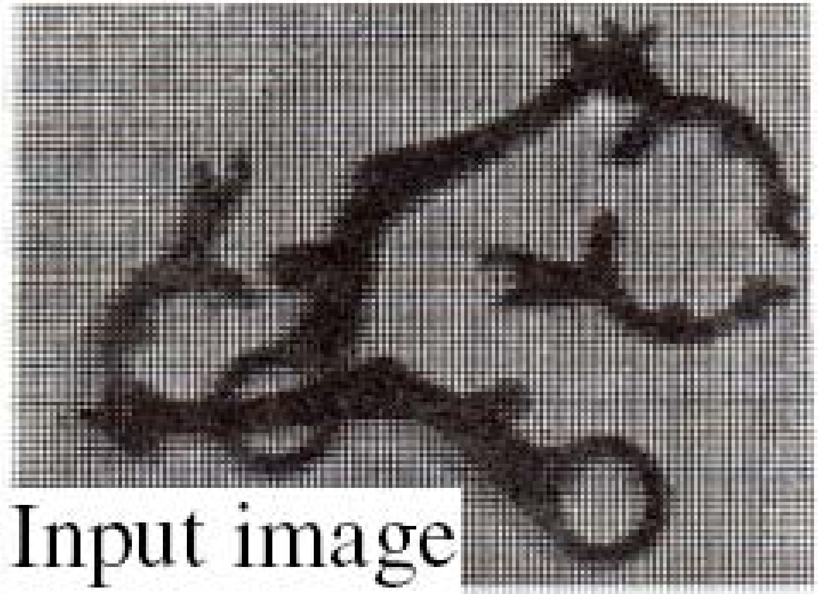
- Given known camera type in some unknown configuration (pose)
  - Hypothesize configuration from set of initial features
  - Backproject
  - Test
- "Frame group" -- set of sufficient correspondences to estimate configuration, e.g.,
  - 3 points
  - intersection of 2 or 3 line segments, and 1 point

Alignment

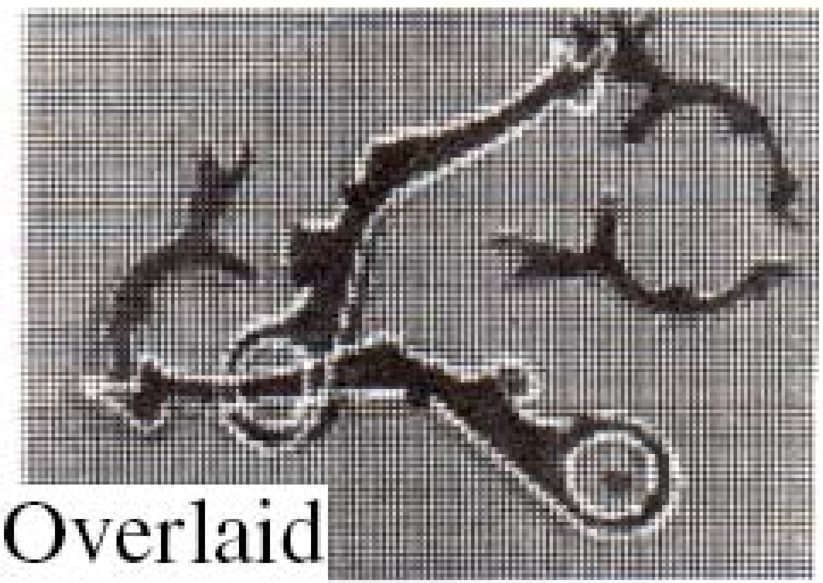
```
For all object frame groups O
  For all image frame groups F
    For all correspondences C between
      elements of F and elements
      of O

      Use F, C and O to infer the missing parameters
      in a camera model

      Use the camera model estimate to render the object

      If the rendering conforms to the image,
        the object is present
    end
  end
end
```



Model



Input image



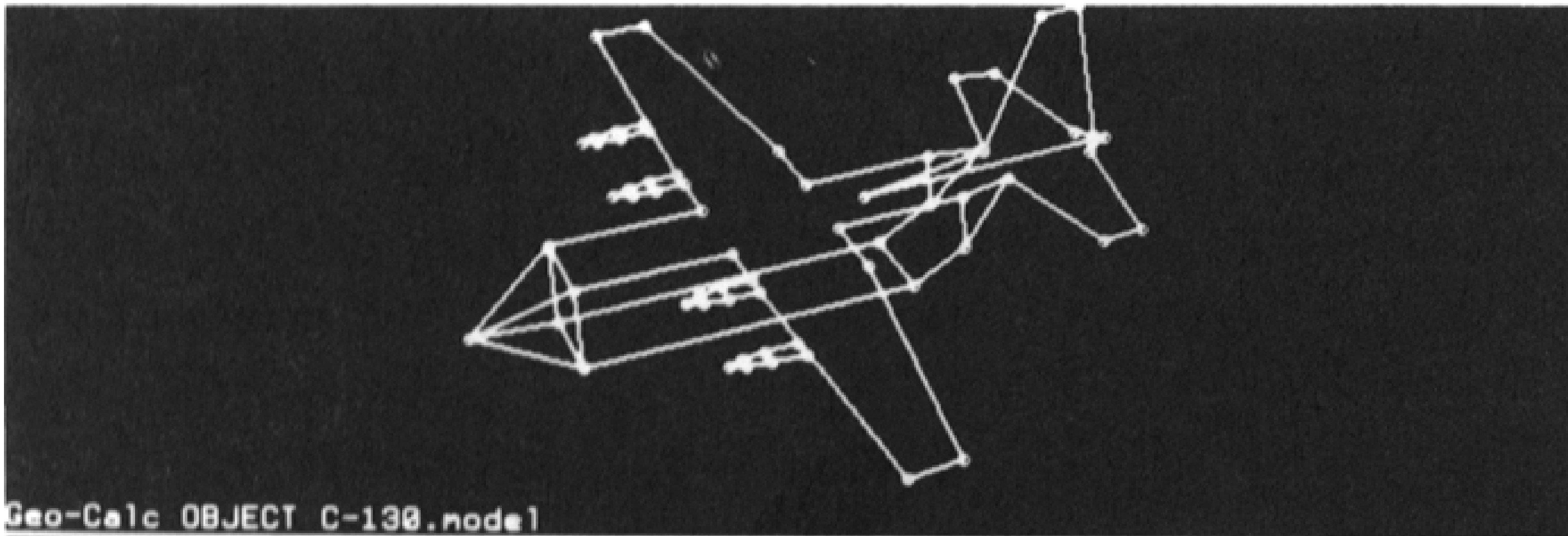
Overlaid

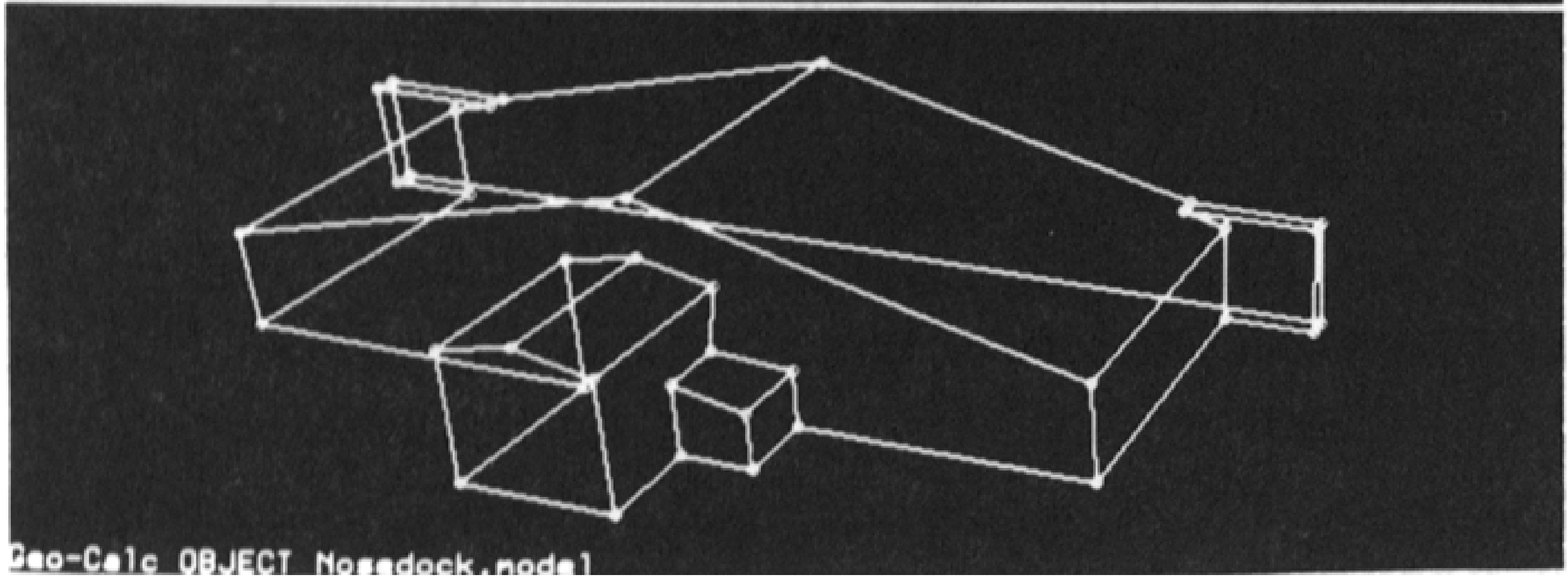
Pose clustering

- Each model leads to many correct sets of correspondences, each of which has the same pose
- Vote on pose, in an accumulator array (per object)

Pose Clustering

```
For all objects O
  For all object frame groups F(O)
    For all image frame groups F(I)
      For all correspondences C between
        elements of F(I) and elements
        of F(O)

        Use F(I), F(O) and C to infer object pose P(O)

        Add a vote to O's pose space at the bucket
        corresponding to P(O).
      end
    end
  end
end
For all objects O
  For all elements P(O) of O's pose space that have
    enough votes

    Use the P(O) and the
    camera model estimate to render the object

    If the rendering conforms to the image,
    the object is present
  end
end
```


Geo-Calc OBJECT C-130.model


Geo-Calc OBJECT Nosedock.model

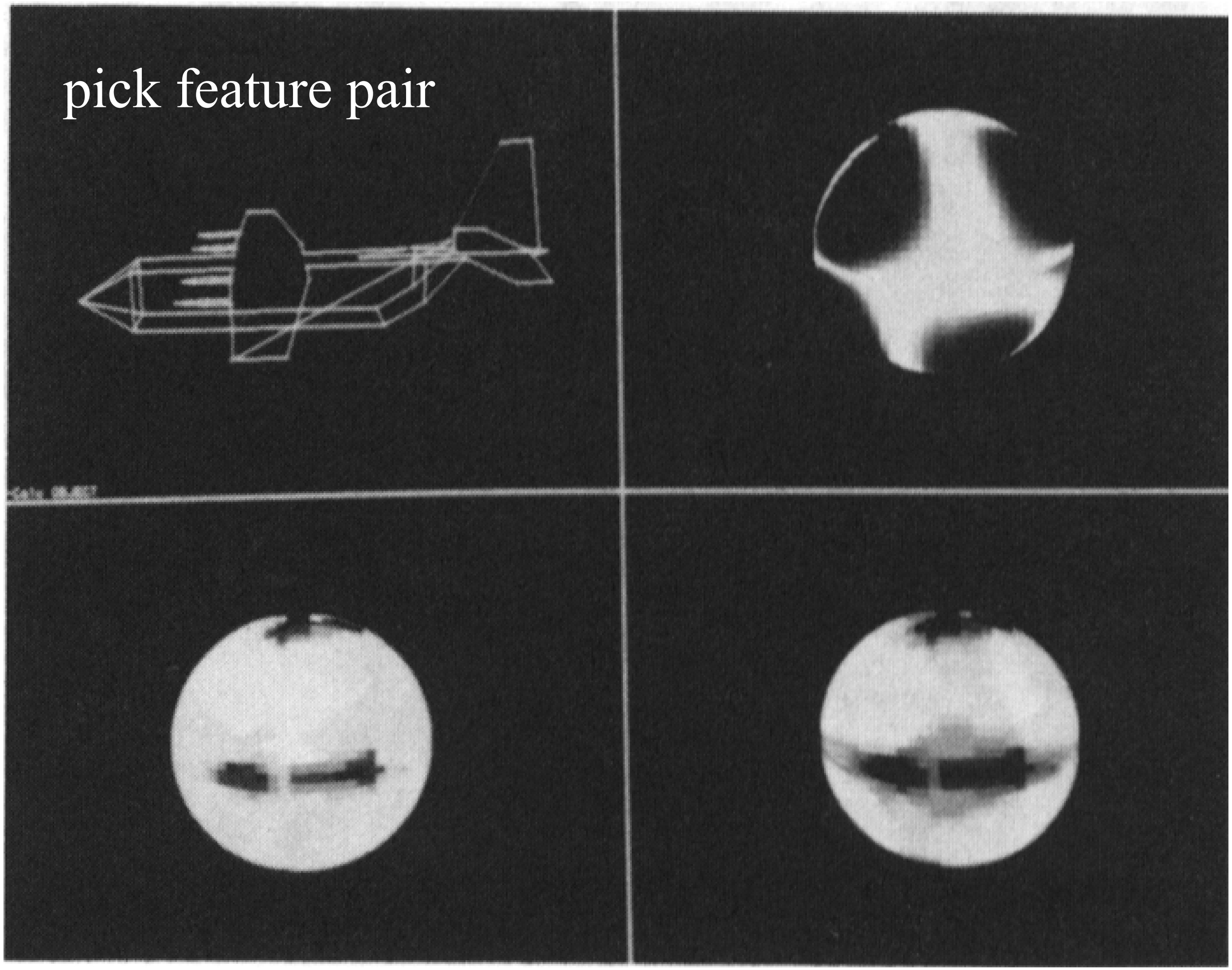
Pose clustering

Problems

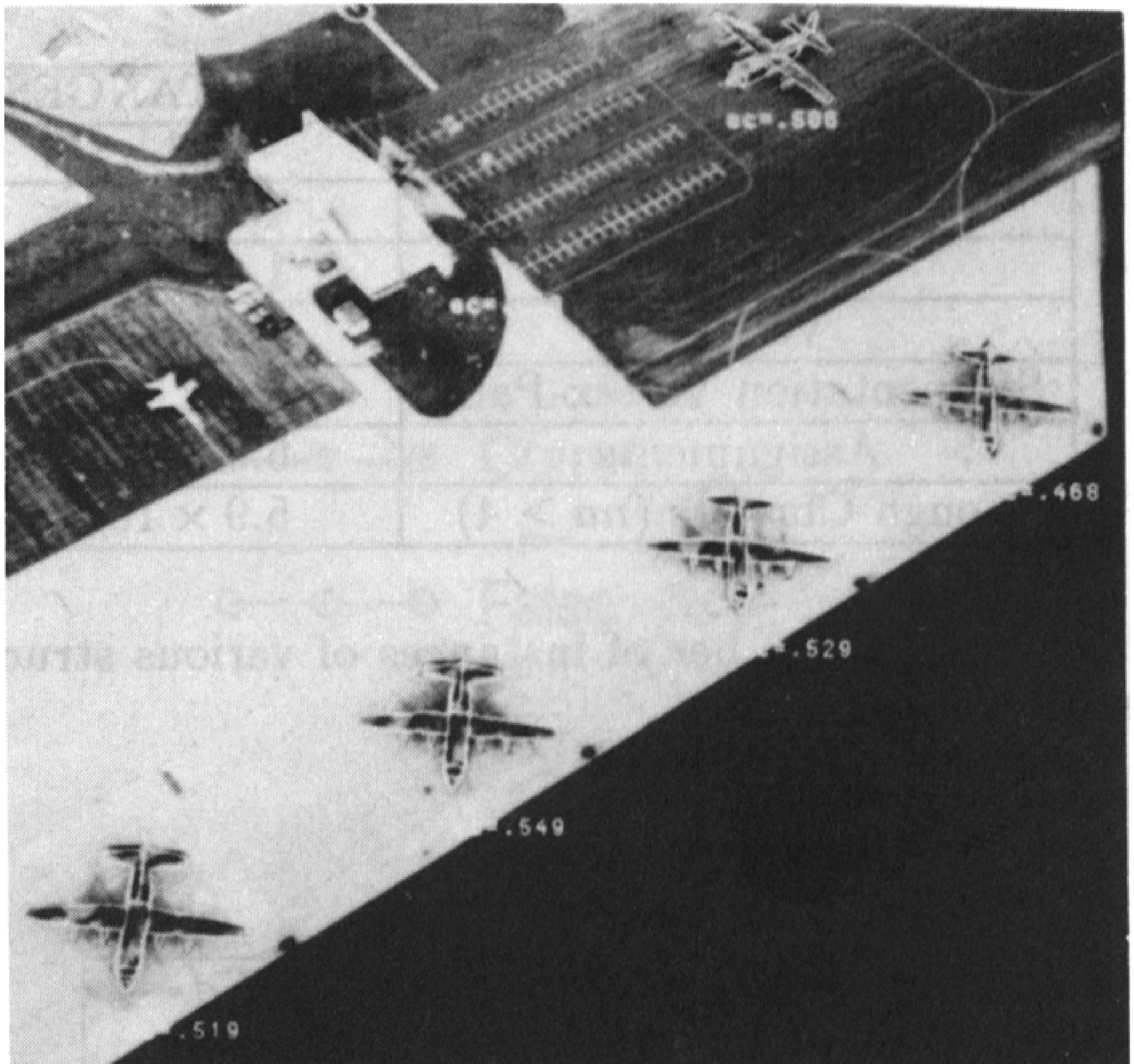
- Clutter may lead to more votes than the target!
- Difficult to pick the right bin size

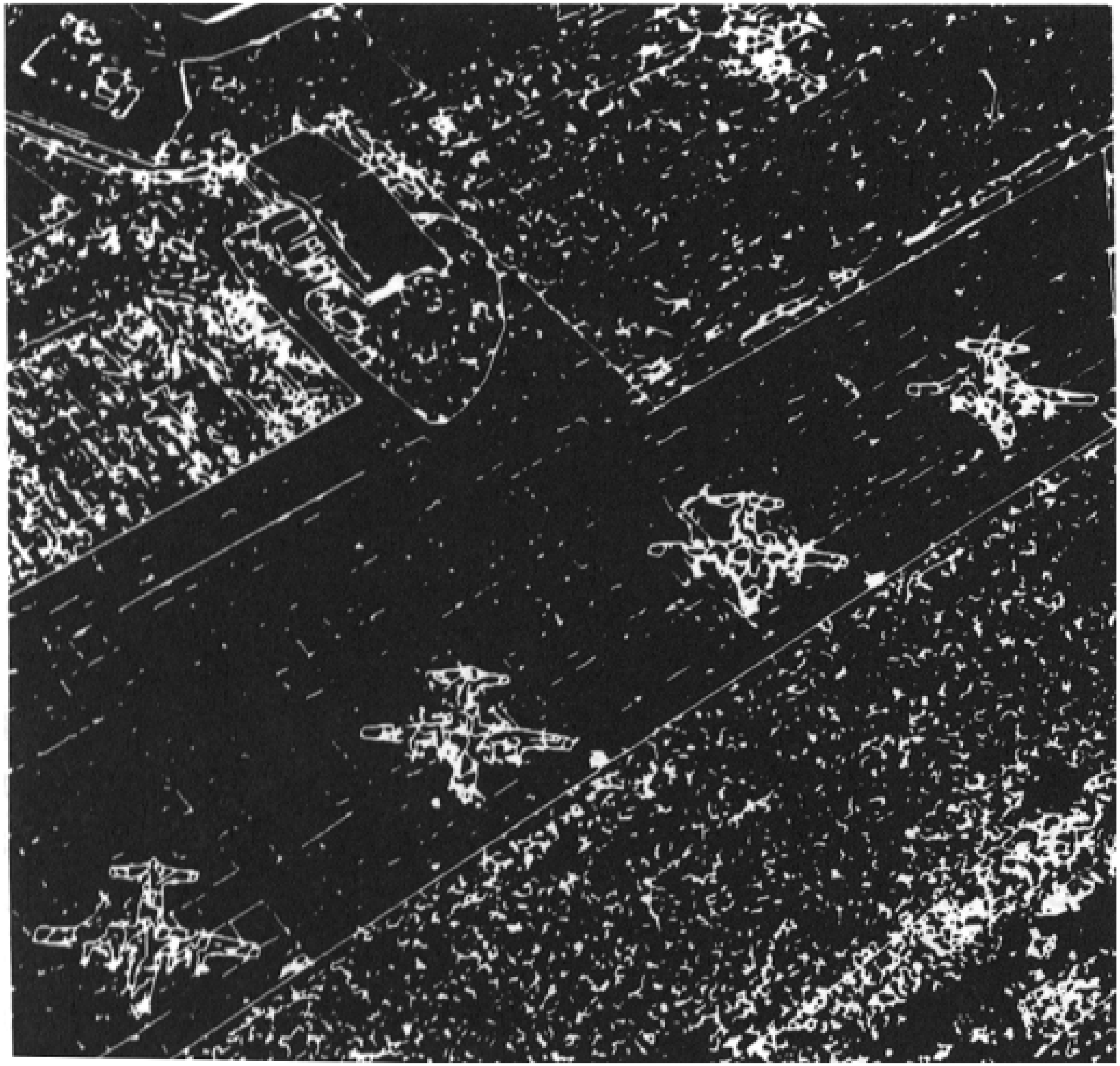
Confidence-weighted clustering

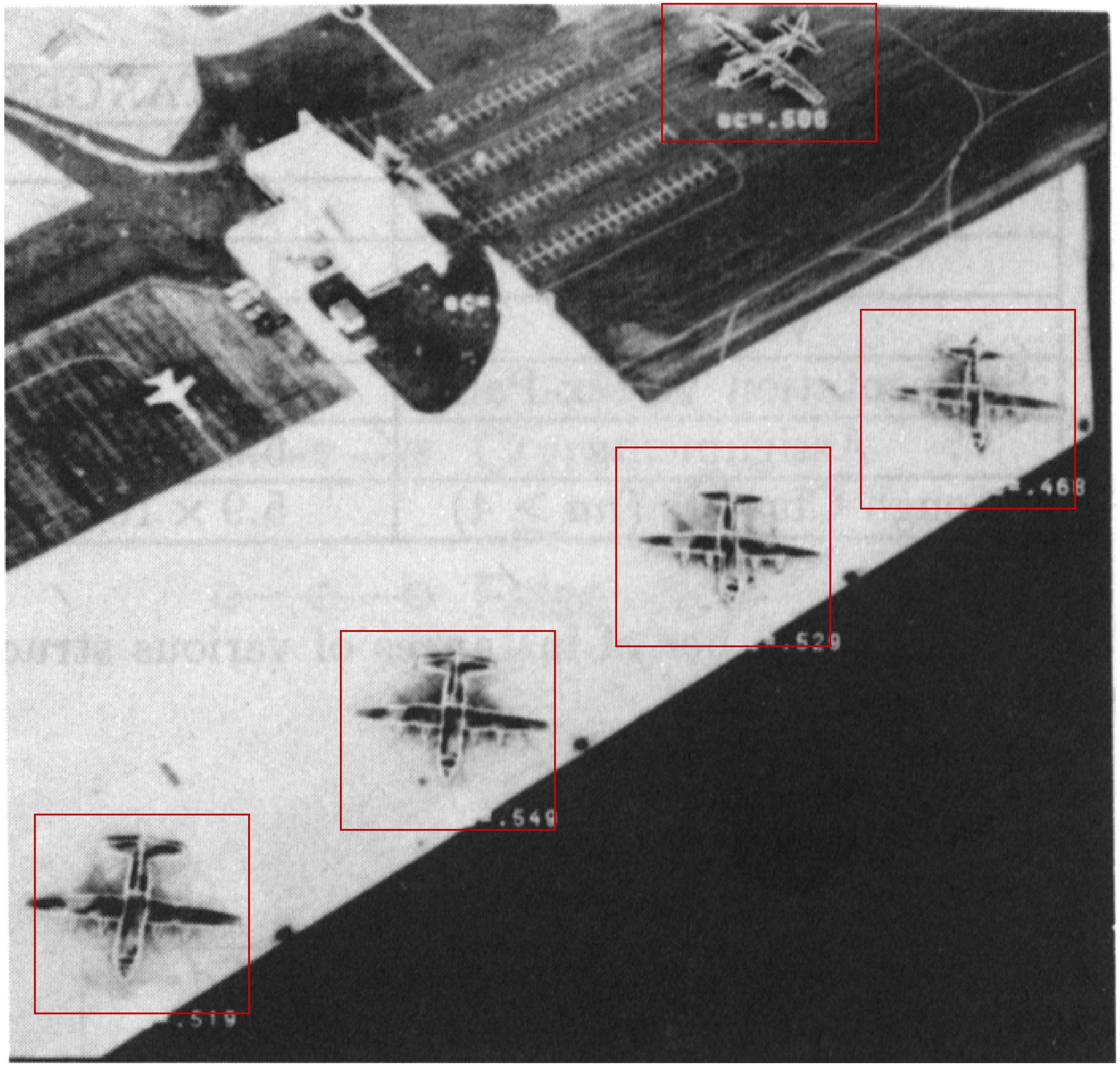
- See where model frame group is reliable (visible!)
- Downweight / discount votes from frame groups at poses where that frame group is unreliable…



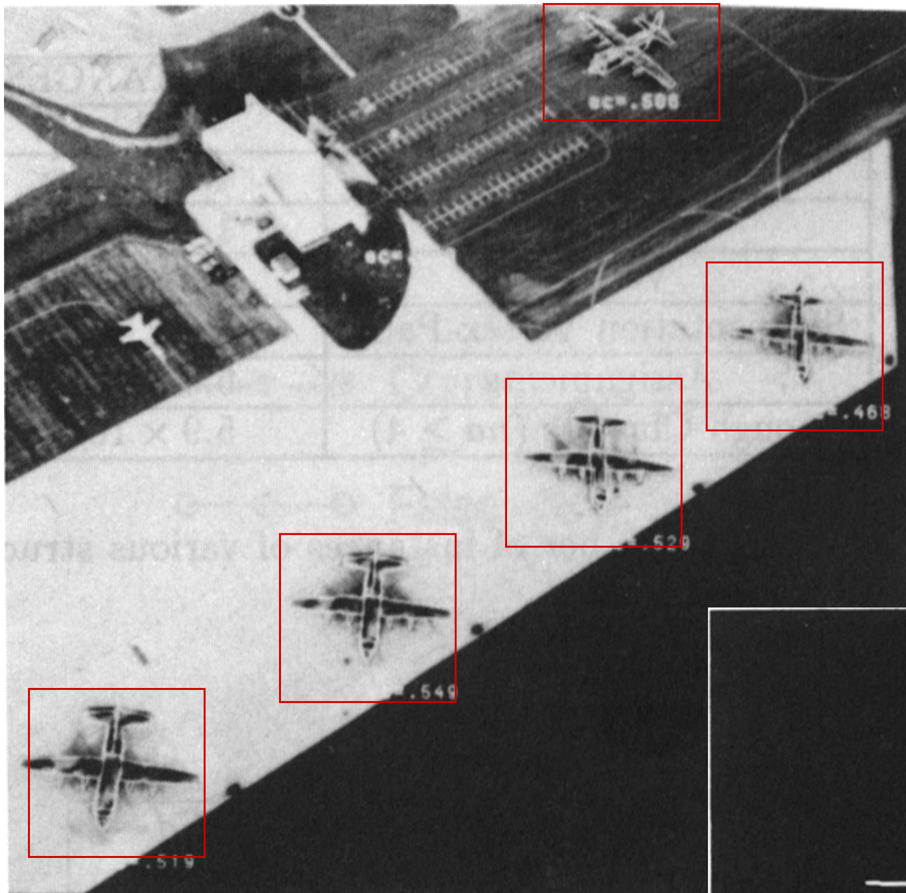
dark regions show reliable views of those features

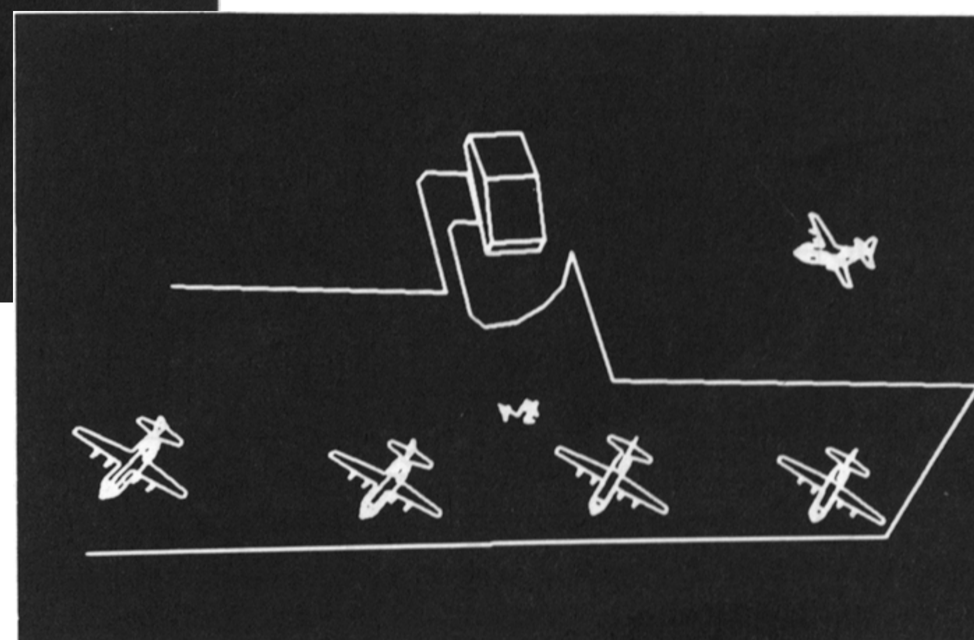

sc=.506
sc=
.468
.529
.549
.519





.468
.529
.549
.519


sc=.508
.468
.529
.549
.519



Detecting 0.1% inliers among 99.9% outliers?

- Example: David Lowe's SIFT-based Recognition system
- Goal: recognize clusters of just 3 consistent features among 3000 feature match hypotheses
- Approach
  - Vote for each potential match according to model ID and pose
  - Insert into multiple bins to allow for error in similarity approximation
  - Using a hash table instead of an array avoids need to form empty bins or predict array size

[Lowe]

Lowe's Model verification step

- Examine all clusters with at least 3 features
- Perform least-squares affine fit to model.
- Discard outliers and perform top-down check for additional features.
- Evaluate probability that match is correct
  - Use Bayesian model, with probability that features would arise by chance if object was *not* present
  - Takes account of object size in image, textured regions, model feature count in database, accuracy of fit (Lowe, CVPR 01)

[Lowe]

Solution for affine parameters

- Affine transform of [x,y] to [u,v]:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- Rewrite to solve for transform parameters:

$$\begin{bmatrix} x & y & 0 & 0 & 1 & 0 \\ 0 & 0 & x & y & 0 & 1 \\ & & \cdots & & & \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_x \\ t_y \end{bmatrix} = \begin{bmatrix} u \\ v \\ \vdots \end{bmatrix}$$

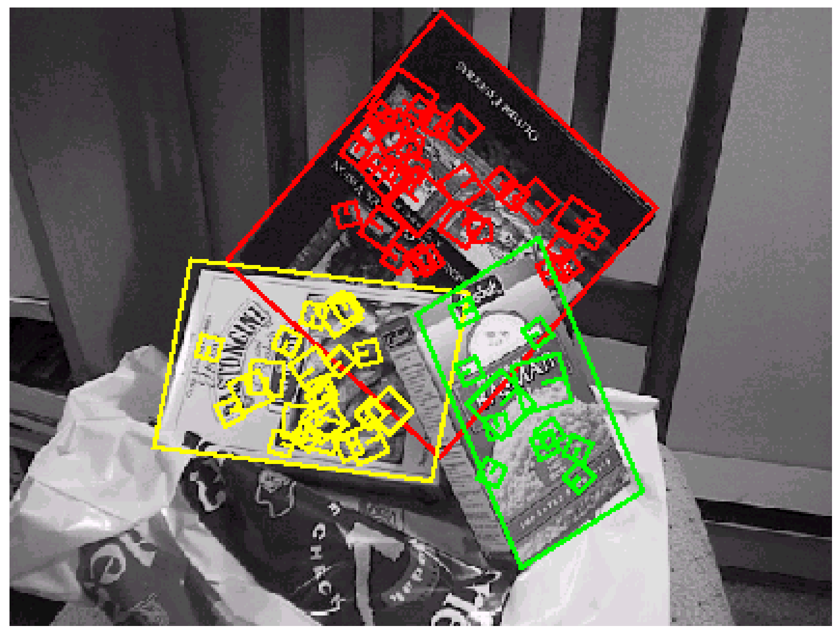
[Lowe]

Models for planar surfaces with SIFT keys:

[Lowe]

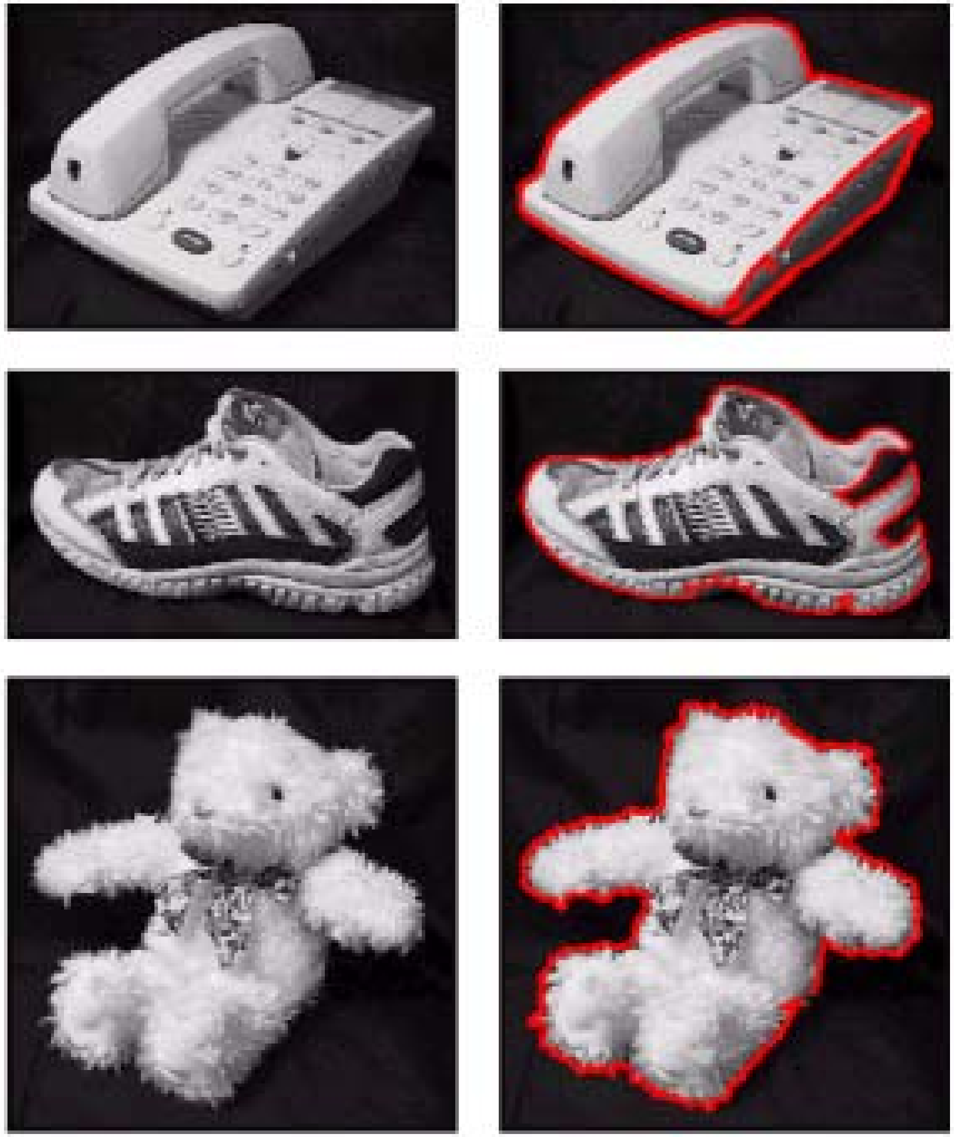
Planar recognition

- Planar surfaces can be reliably recognized at a rotation of 60° away from the camera
- Affine fit approximates perspective projection
- Only 3 points are needed for recognition



[Lowe]

3D Object Recognition



- Extract outlines with background subtraction

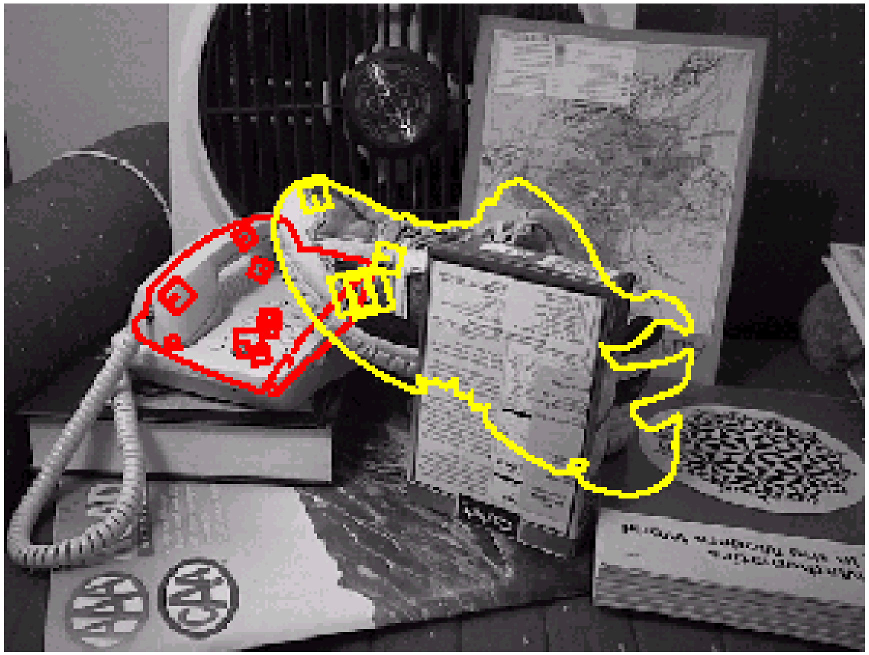
[Lowe]

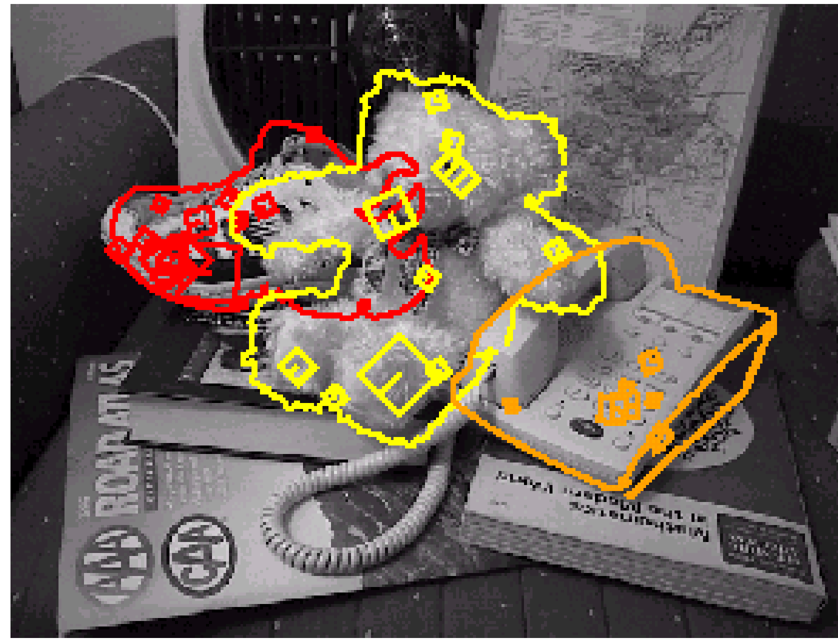
3D Object Recognition

- Only 3 keys are needed for recognition, so extra keys provide robustness
- Affine model is no longer as accurate

[Lowe]

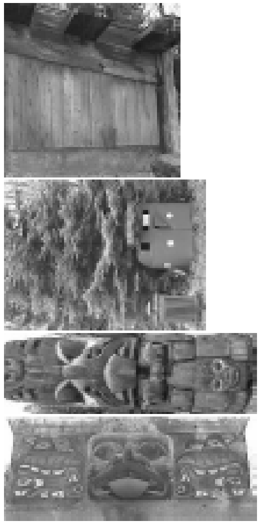
Recognition under occlusion

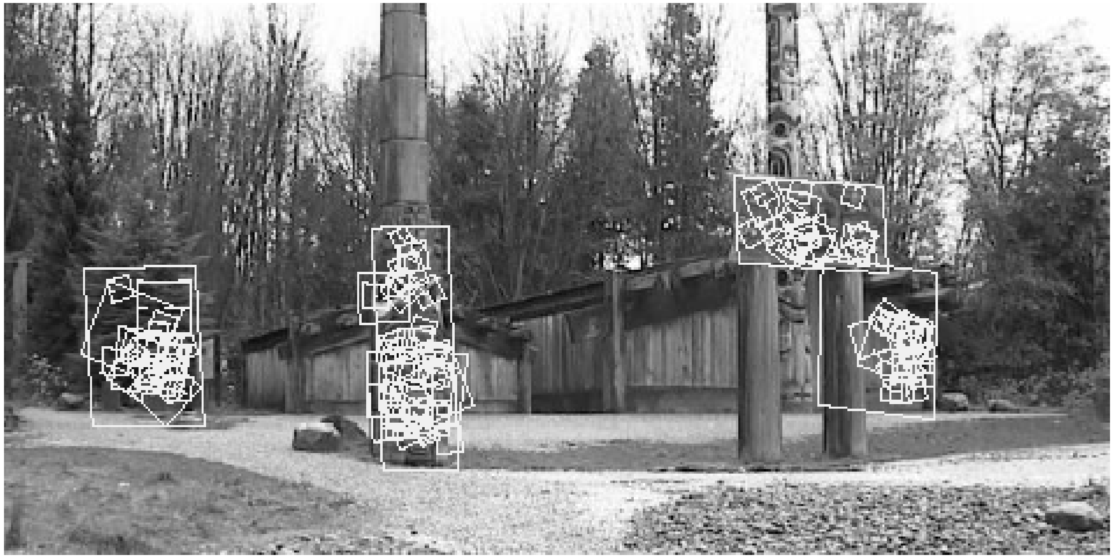




[Lowe]

Location recognition

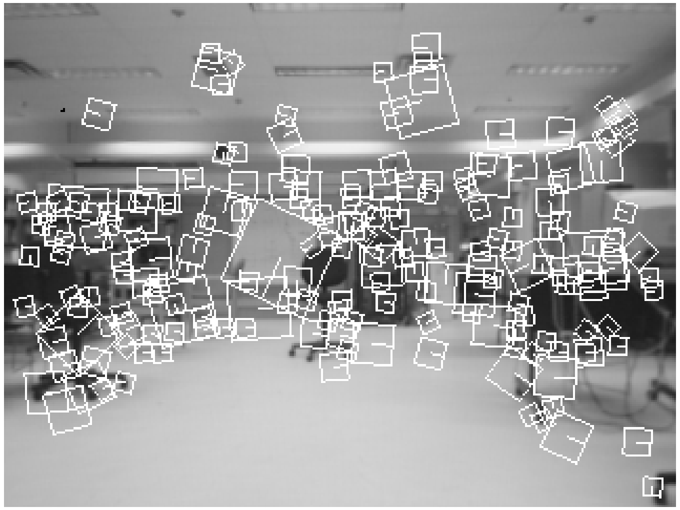




[Lowe]

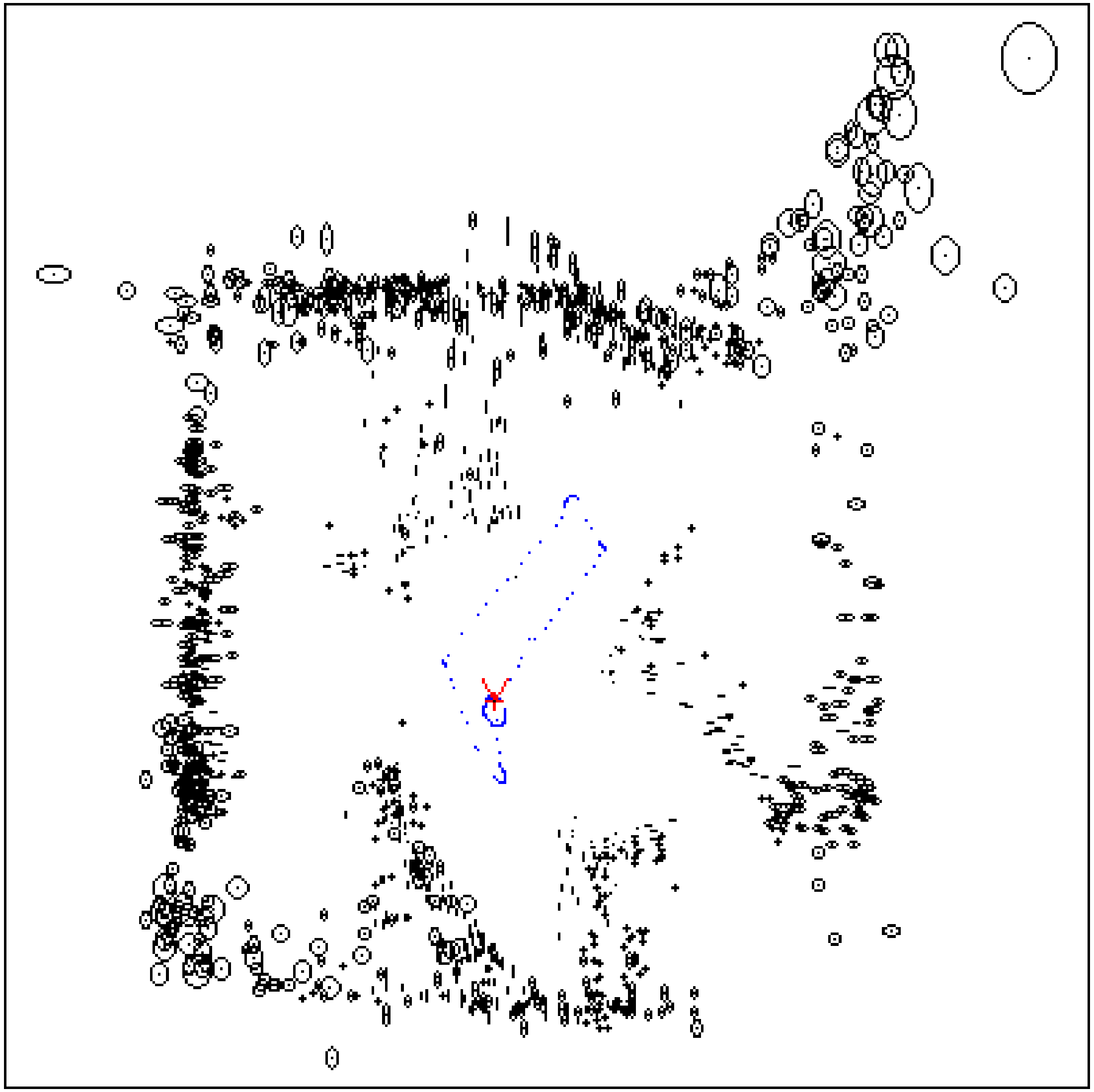
Robot Localization

- Joint work with Stephen Se, Jim Little



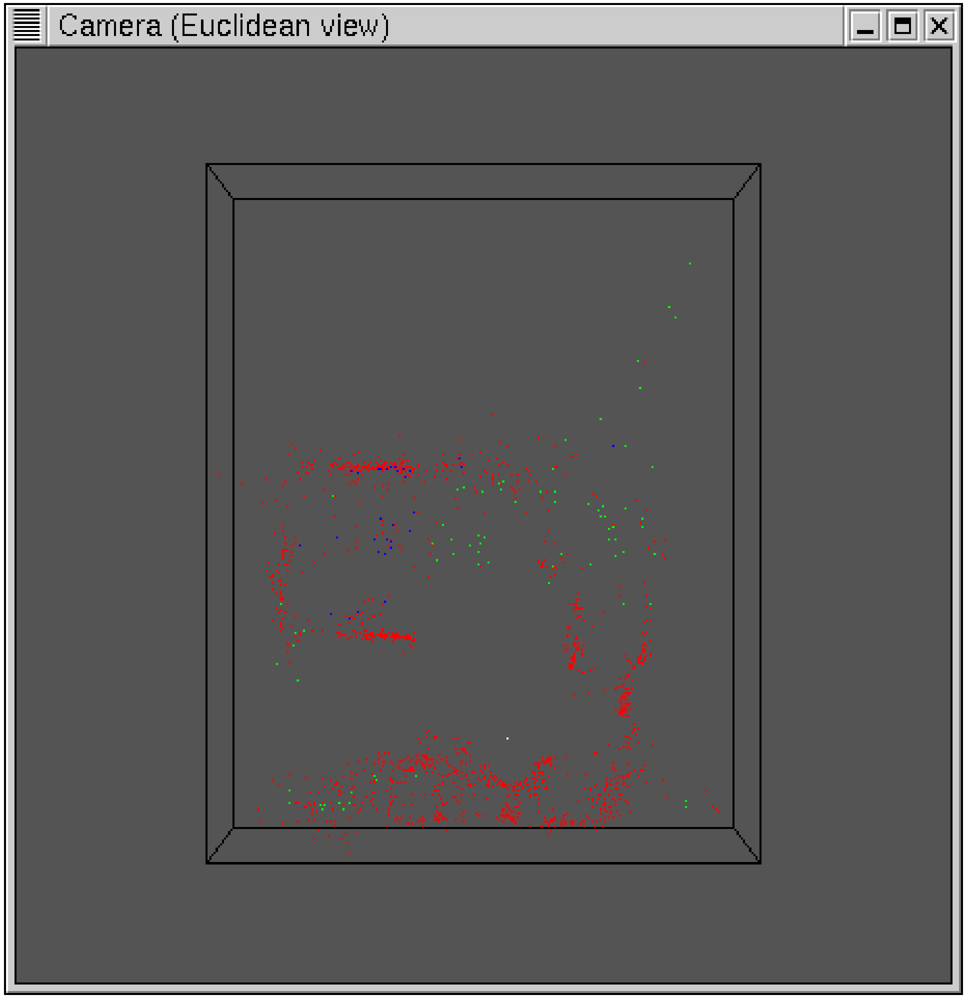
[Lowe]

Map continuously built over time



[Lowe]

Locations of map features in 3D



[Lowe]

Invariant recognition

- Affine invariants
  - Planar invariants
  - Geometric hashing
- Projective invariants
  - Determinant ratio
- Curve invariants

Invariance

- There are geometric properties that are invariant to camera transformations
- Easiest case: view a plane object in scaled orthography.
- Assume we have three base points P_i on the object
  - then any other point on the object can be written as

$$P_k = P_1 + \mu_{ka}\left(P_2 - P_1\right) + \mu_{kb}\left(P_3 - P_1\right)$$

Invariance

- Now image points are obtained by multiplying by a plane affine transformation, so

$$
\begin{aligned}
p_k &= AP_k \\
&= A\left(P_1 + \mu_{ka}\left(P_2 - P_1\right) + \mu_{kb}\left(P_3 - P_1\right)\right) \\
&= p_1 + \mu_{ka}\left(p_2 - p_1\right) + \mu_{kb}\left(p_3 - p_1\right)
\end{aligned}
$$

Invariance

$$P_k = P_1 + \mu_{ka}(P_2 - P_1) + \mu_{kb}(P_3 - P_1)$$

$$\begin{aligned} p_k &= AP_k \\ &= A(P_1 + \mu_{ka}(P_2 - P_1) + \mu_{kb}(P_3 - P_1)) \\ &= p_1 + \mu_{ka}(p_2 - p_1) + \mu_{kb}(p_3 - p_1) \end{aligned}$$

Given the base points in the image, read off the μ values for the object

- they're the same in object and in image --- **invariant**
- search correspondences, form μ's and vote

Geometric Hashing

- Objects are represented as sets of “features”
- Preprocessing:
  - For each tuple *b* of features, compute location (μ) of all other features in basis defined by *b*
  - Create a table indexed by (μ)
  - Each entry contains *b* and object ID

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

GH: Identification

- Find features in target image
- Choose an arbitrary basis *b*'
- For each feature:
  - Compute (μ') in basis *b*'
  - Look up in table and vote for (Object, *b*)
- For each (Object, *b*) with many votes:
  - Compute transformation that maps *b* to *b*'
  - Confirm presence of object, using all available features

S. Rusinkiewicz

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

Geometric Hashing

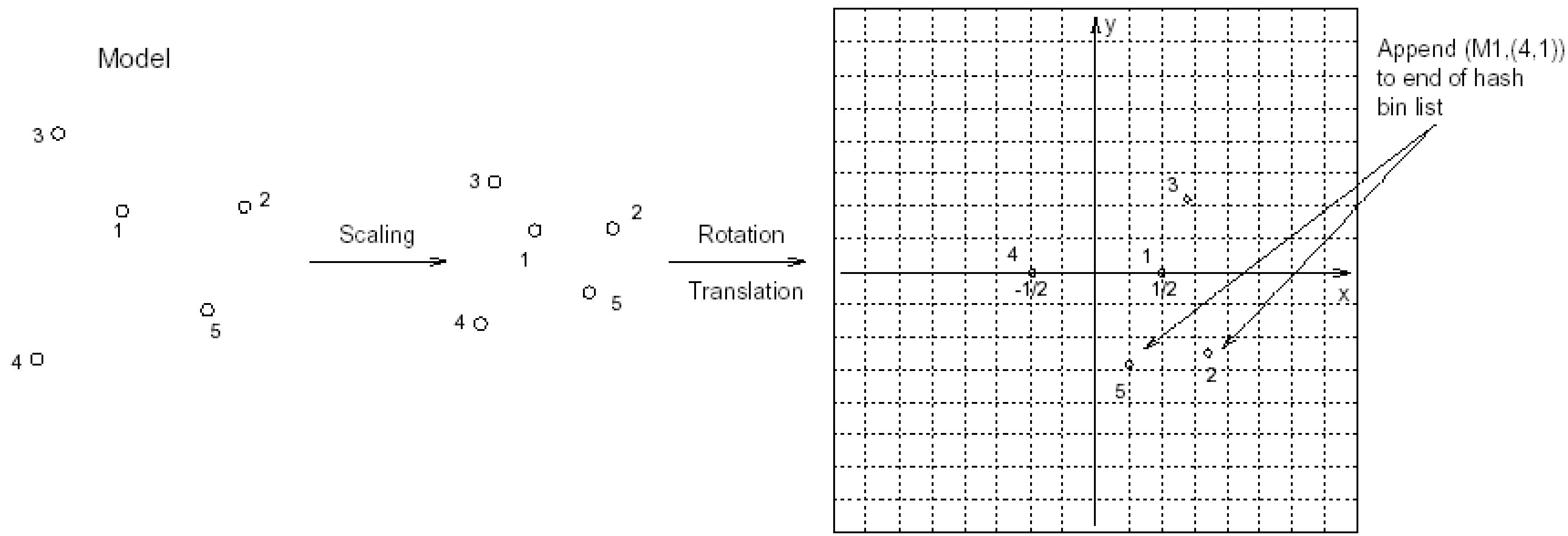
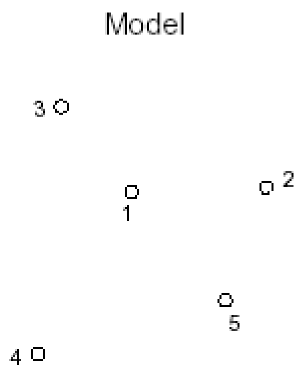


Figure 1. Determining the hash table entries when points 4 and 1 are used to define a basis. The models are allowed to undergo rotation, translation, and scaling. On the left of the figure, model M_1 comprises five points.

Wolfson and Rigoutsos, *Geometric Hashing, an Overview,* 1997

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

Basis Geometric Hashing



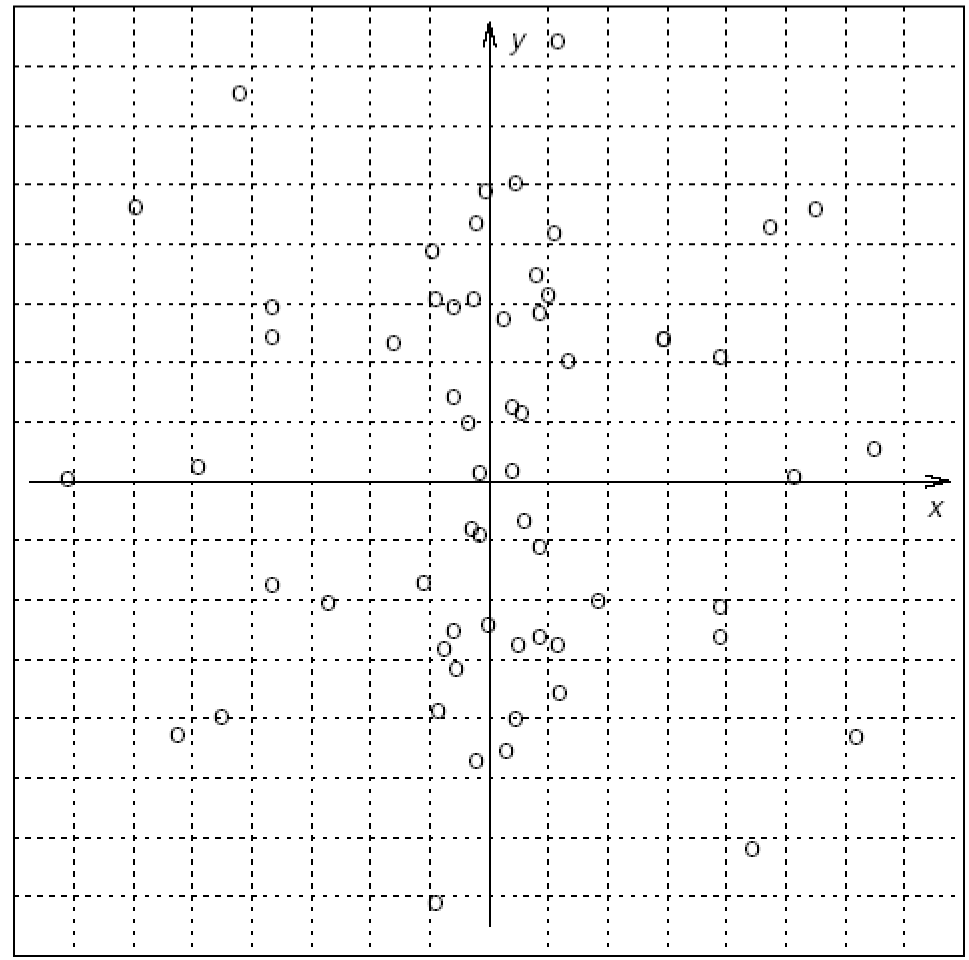


Figure 2. The locations of the hash table entries for model M_1. Each entry is labeled with the information "model M_1" and the basis pair (i, j) used to generate the entry. The models are allowed to undergo rotation, translation, and scaling.

Wolfson and Rigoutsos, *Geometric Hashing, an Overview,* 1997

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

Geometric Hashing

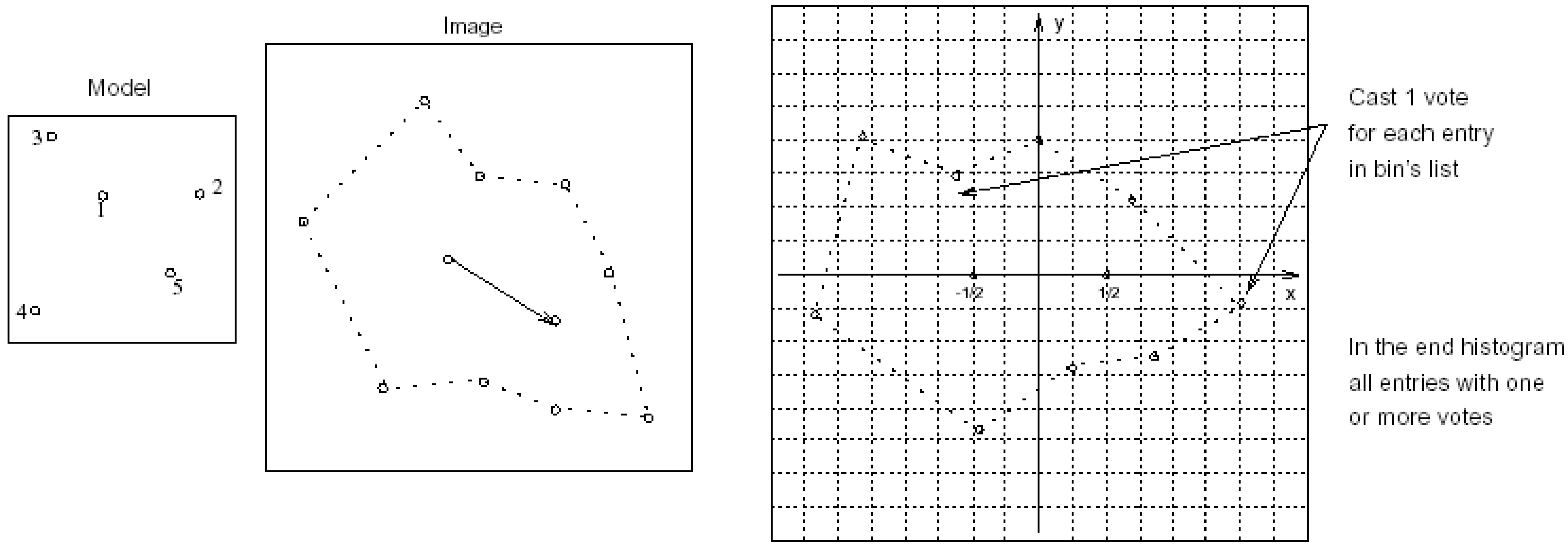


Figure 3. Determining the hash table bins that are to be notified when two arbitrary image points are selected as a basis. Similarity transformation is allowed.

Wolfson and Rigoutsos, *Geometric Hashing, an Overview,* 1997

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

Algorithm 18.3: Geometric hashing: voting on identity and point labels

For all groups of three image points $T(I)$ ==b
  For every other image point $\boldsymbol{p}$
    Compute the μ's from $\boldsymbol{p}$ and $T(I)$
    Obtain the table entry at these values
      if there is one, it will label the three points in $T(I)$
      with the name of the object
      and the names of these particular points.
    Cluster these labels;
      if there are enough labels, backproject and verify
    end
  end
end

Indexing with invariants

- Generalize to heterogeneous geometric features
- Groups of features with identity information invariant to pose – *invariant bearing groups*

Projective invariants

- Projective invariant for coplanar points
- Perspective projection of coplanar points is a plane perspective transform:

  p=MP $\longrightarrow$ p=AP, with 3x3 A

- determinant ratio of 5 point tuples is invariant

$$\frac{\det\left(\left[p_i p_j p_k\right]\right)\det\left(\left[p_i p_l p_m\right]\right)}{\det\left(\left[p_i p_j p_l\right]\right)\det\left(\left[p_i p_k p_m\right]\right)}$$

$$\frac{\det([p_i p_j p_k])\det([p_i p_l p_m])}{\det([p_i p_j p_l])\det([p_i p_k p_m])} = \frac{\det([AP_i AP_j AP_k])\det([AP_i AP_l AP_m])}{\det([AP_i AP_j AP_l])\det([AP_i AP_k AP_m])}$$

$$= \frac{\det(A\ [P_i P_j P_k])\det(A\ [P_i P_l P_m])}{\det(A\ [P_i P_j P_l])\det(A\ [P_i P_k P_m])}$$

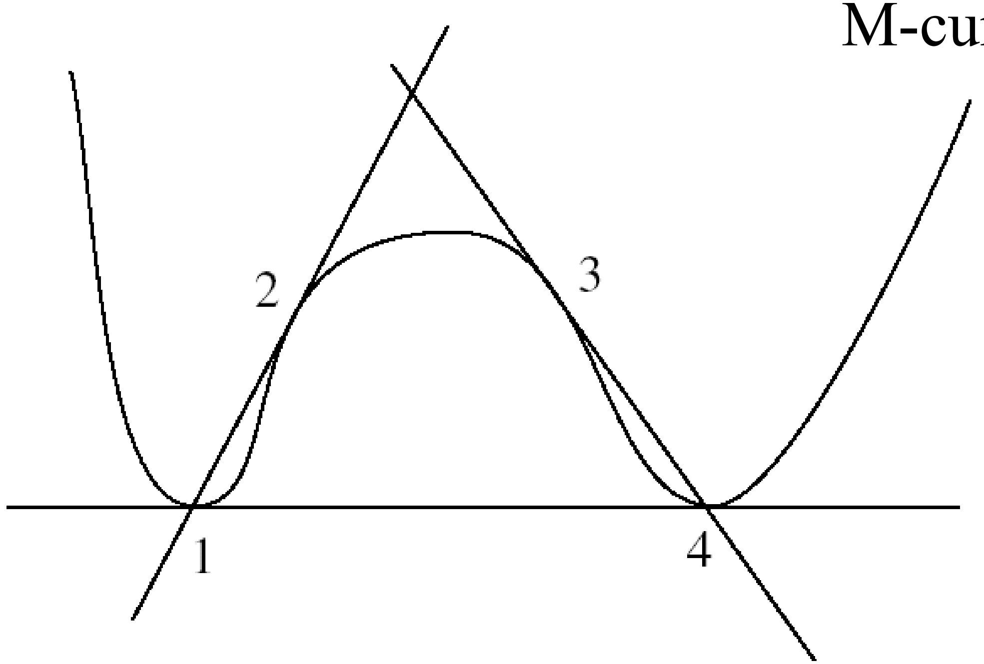
$$= \frac{(\det(A)^2)}{(\det(A)^2)}\frac{\det([P_i P_j P_k])\det([P_i P_l P_m])}{\det([P_i P_j P_l])\det([P_i P_k P_m])}$$

$$= \frac{\det([P_i P_j P_k])\det([P_i P_l P_m])}{\det([P_i P_j P_l])\det([P_i P_k P_m])}$$

Tangent invariance

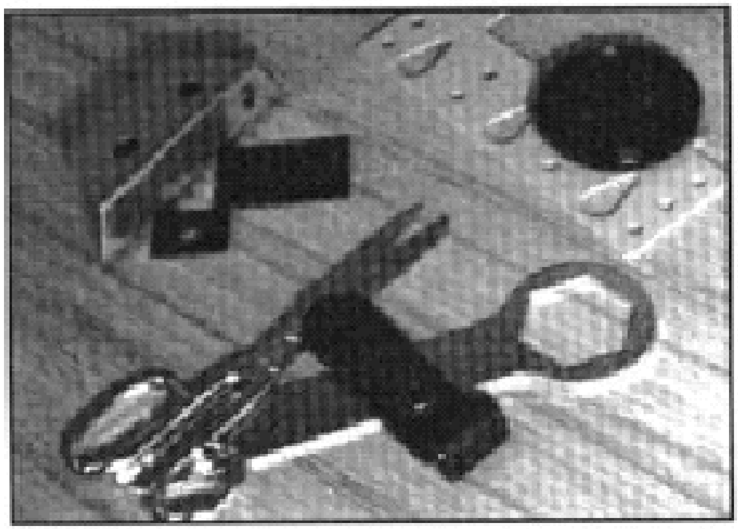
- Incidence is preserved despite transformation

M-curve construction

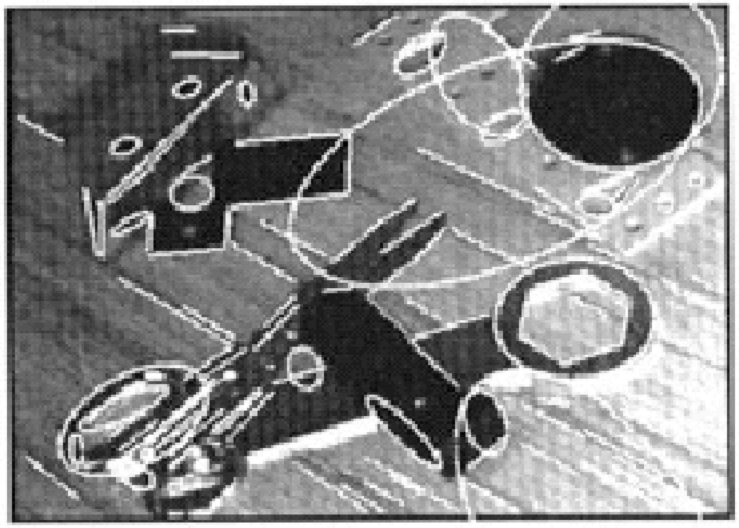


- Transform four points above to unit square: measurements in this canonical frame will be invariant to pose.

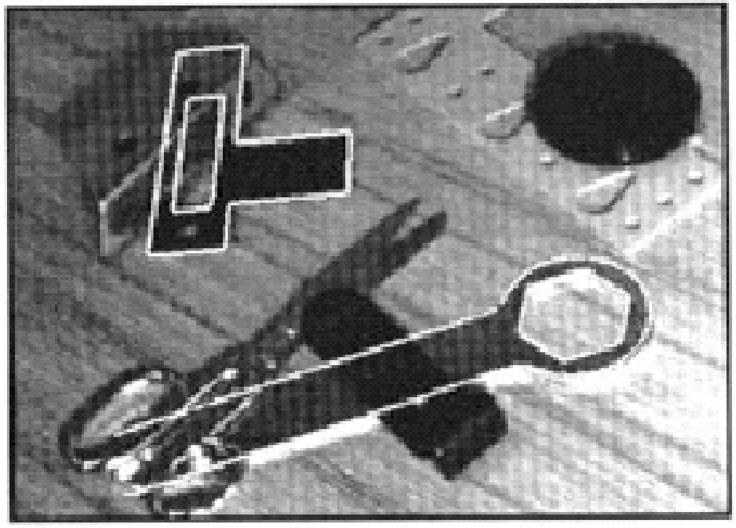
```
For each type T of invariant-bearing group
  For each image group G of type T

  Determine the values V of the invariants of G

    For each model feature group M of type T whose invariants
    have the values V

      Determine the transformation that takes M to G

      Render the model using this transformation

      Compare the result with the image, and accept if
      similar
    end
  end
end
```



a

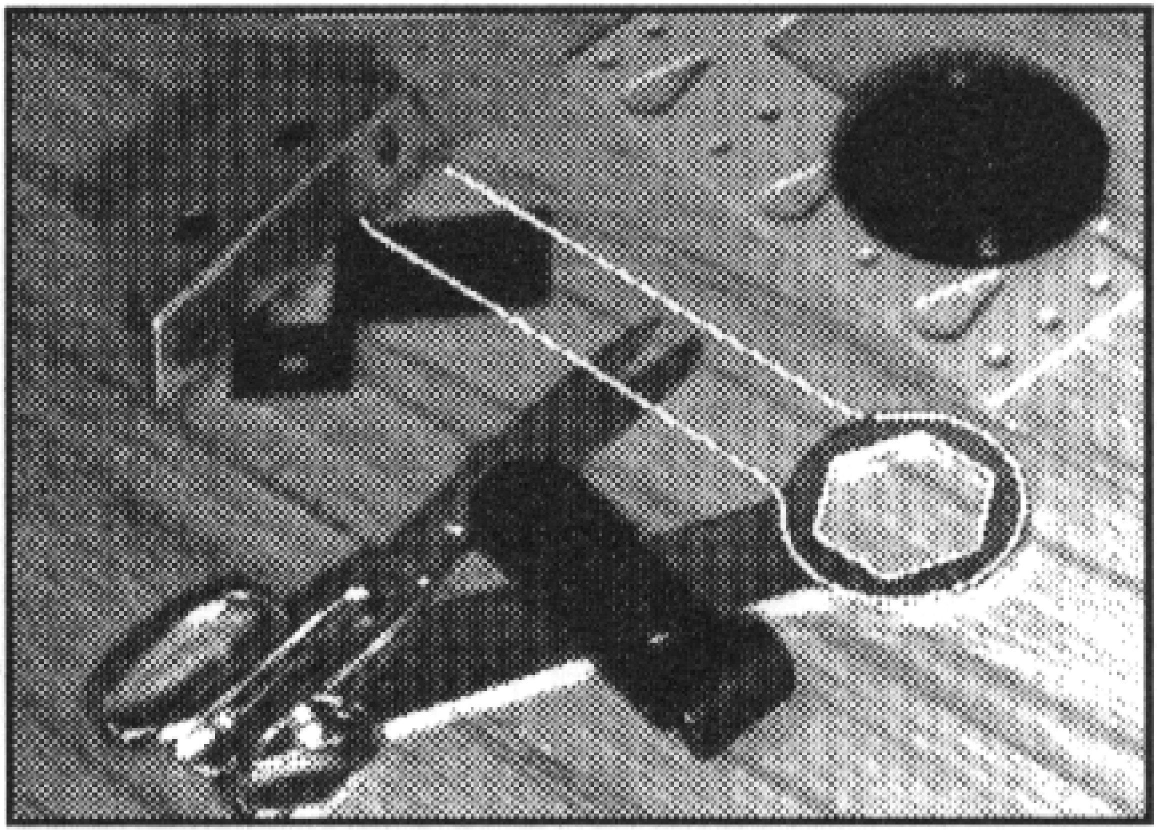


b



Verification?

- Edge score
  - are there image edges near predicted object edges?
  - very unreliable; in texture, answer is usually yes
- Oriented edge score
  - are there image edges near predicted object edges with the right orientation?
  - better, but still hard to do well (see next slide)
- Texture largely ignored [Forsythe]
  - e.g. does the spanner have the same texture as the wood?



Algorithm Sensitivity

- Geometric Hashing
  - A relatively sparse hash table is critical for good performance
  - Method is not robust for cluttered scenes (full hash table) or noisy data (uncertainty in hash values)
- Generalized Hough Transform
  - Does not scale well to multi-object complex scenes
  - Also suffers from matching uncertainty with noisy data

Grimson and Huttenlocher, 1990

[http://www.cs.princeton.edu/courses/archive/fall03/cs597D/lectures/rigid_registration.pdf]

Comparison to template matching

- Costs of template matching
  - 250,000 locations x 30 orientations x 4 scales = 30,000,000 evaluations
  - Does not easily handle partial occlusion and other variation without large increase in template numbers
  - Viola & Jones cascade must start again for each qualitatively different template
- Costs of local feature approach
  - 3000 evaluations (reduction by factor of 10,000)
  - Features are more invariant to illumination, 3D rotation, and object variation
  - Use of many small subtemplates increases robustness to partial occlusion and other variations

[Lowe]

Today: “Model-based Vision”

- Hypothesize and test
- Interpretation Trees
- Alignment
- Pose Clustering
- Invariances
- Geometric Hashing
- ***Tuesday: Project previews!***