6.892 Machine learning seminar

http://www.ai.mit.edu/courses/6.892-ml

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This assignment is due **April 28**. You can either email the assignment to me as plain text (use subject "HOMEWORK 9") or give it to me in class.

1. Let G be a finite group with elements $g_1, \ldots, g_n$ and let $\Gamma : G \to G'$ be a mapping from group G to another group G' such that

$$\Gamma(gg') = \Gamma(g)\Gamma(g') \tag{1}$$

   for all $g, g' \in G$. The mapping need not be one to one. If for each $g \in G$, $\Gamma(g)$ is a d by d matrix and the above multiplication operation is the matrix multiplication then Γ is called a *representation* of G. d is the dimensionality of this matrix representation.

   a) Provide three different representations of dimensionalities $1, 1$ and 2 for the group corresponding to the reflection transformation: $x \to -x$.

   b) Provide a 1-dimensional *faithful* representation, i.e, Γ is one to one, for the group resulting from all $n\pi/2$, $n = 0, 1, \ldots$, rotations in 2-D.

2. a) Pick a non-trivial symmetry/invariance property relevant to machine learning (not a simple translation, rotation, or permutation) and express it as a group.

   b) Find a meaningful kernel which is invariant to these transformations.

3. **Research level:** Provide an efficient mechanism for deducing certain invariance properties from data. This problem is akin to estimating the structure of a graphical model (deducing conditional independence relations between the variables). Invariances like independencies are not (necessarily) directly visible from a few examples but need to be deduced from observed statistics. The key aspects of this problem are 1) representational organization of invariances and 2) the objective function used to assess the validity of a set of hypothesized invariances. You'll need to get into the theory of group representations to make progress...