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Analog Computation

Analog computation derives its name from the fact that many physical systems are *analogous* to problems we would like to solve. If we can set up an experiment that exploits this similarity, measuring its results will provide us with data that, by analogy, solve our original problems. This is in sharp contrast to digital computation, which relies on a physical system emulating a completely abstract mathematical system.

The first large-scale computational devices were analog machines, designed to solve two large classes of problems: solving differential equations and performing linear algebra. The physical analogies exploited divide into two categories: mechanical and electrical.

Lord Kelvin proposed one mechanical system in 1878.[1] Designed to solve sets of linear equations, it operated on a tilling-plate mechanism—essentially a collection of pulleys spaced along balanced plates and connected with rope. Coefficients of the linear equations would correspond to pulley placements, and the ropes would be tugged to balance the beams and hence the equations. Results would be read off as the lengths of loose ends of rope.

Kelvin's scheme had several advantages. First, it was based on rudimentary technology—within the capabilities of 19th century craftsmanship. Second, it was possible to increase the precision of a calculation through successive applications of the device, taking advantage of the characteristics of the linear equations it was designed to solve. Third, its mechanical nature made it ideal for "tweaking"—slight adjustments in inputs could be made quickly, and results obtained rapidly, without substantially reconfiguring the device.

The first large-scale implementation of Kelvin's machine was not built until 1936, when Wilbur at MIT constructed a 10-plate machine using runners and metal tape running along ball bearings in place of pulleys and rope.[2] The machine validated Kelvin's approach, solving sets of 9 simultaneous equations in roughly a third of the time required by a human calculator. Wilbur also noted the speed advantage of the machine would increase with the number of simultaneous equations. Arbitrary accuracy could be achieved,

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[1] Thomson
[2] Wilbur

as Kelvin had suggested, through successive approximations—these approximations were made easier because, as noted above, the machine could be reconfigured quickly to solve similar problems.

One of the first major mechanical devices for deriving solutions to differential equations was also constructed at MIT in the 1930's. Deriving inspiration from Lord Kelvin's work as well, Bush and Hazen built the Differential Analyzer from 1930-1931. The Differential Analyzer was substantially more intricate than the tilling-plate machine. Consisting of hundreds of gearboxes, rotator shafts and torque amplifiers, it could crank out reasonably precise solutions to 6th order differential equations. With its added complexity came an added flexibility—Shannon demonstrated that, properly configured, the Differential Analyzer could solve a wide range of mathematical problems.[3]

These mechanical devices had two major disadvantages in common: speed and size. Physical mechanisms can only move so fast within material tolerances (and ultimately are limited by the speed of sound), and can only be readily constructed to a certain degree of precision in any given size. The machines built by Wilbur and Bush each consumed an entire room, and the next iteration of the Differential Analyzer, funded by the Rockefeller foundation, weighed in at 100 tons while improving upon the original's precision by only one order of magnitude. As we will see, scalability is a major problem for all analog machines, but in the case of mechanical devices, the issue becomes preclusive.

Electronic analog computers overcome these difficulties to a great degree. By taking advantage of the fundamental laws of electromagnetism, these devices create an electrical analog to linear and differential equations without requiring the physical motion of components. One system built by Mallock in the early 1930's solved the same problems as the MIT tilling-plate machine by using variable-coil transformers.[4] Other systems used networks of resistors and inductors to achieve solutions to both linear and differential problems.[5] These systems had the advantages of speed and size missing in mechanical devices, and eventually dominated the field of analog computing.

That field, however, was to shrink rapidly in importance with the emergence of digital computing. Digital computers superceded their analog counterparts for several reasons—chiefly speed, accuracy and

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[3] Shannon
[4] Mallock
[5] Soroka

flexibility. These advantages are derived from a single source that is the bane of analog systems: scalable precision.

It is a straightforward engineering exercise to increase the precision of a digital system—you simply add more bits; a 2-bit adder is not fundamentally different from a 500-bit adder. The same increase in accuracy in an analog system requires massively better materials and measuring devices—try buying a 1-ohm resistor accurate to a nano-ohm. Tricks that worked for linear computation to increase accuracy through successive approximation become useless in nonlinear systems. As digital systems push for smaller and smaller components that can safely interact, analog systems must contend with increasing error due to interference. In discussion, Tom Knight pointed out that the transistors used in analog logic are much larger than their digital counterparts, and for good reason: without the digital abstraction, the transistors must be substantially more reliable to achieve the required precision.

Despite this issue, some systems are still designed to take advantage of analog computation. A typical example is the NeuroClassifier, an analog VLSI chip designed for rapid pattern matching of particle traces in high-energy physics.[6] Essentially a hardware neural network, the chip multiplies hundreds of 5-bit-precision voltage quantities per "cycle"—essentially the time it takes for a signal to propagate from one end of the chip to the other. Running with a power dissipation of only 600mW, the chip can perform 20 billion mathematical operations per second, classifying particles within nanoseconds. These raw speeds are achievable because by using strictly analog logic (with the exception of on-chip SRAM to store inputs and outputs), silicon can be devoted almost exclusively to computational elements. Also, at the low precision required, analog multipliers can be constructed out of far fewer transistors than their digital counterparts—the NeuroClassifier is built with merely 40,000 transistors.[7]

Clearly, however, the NeuroClassifier is a one-trick pony, and while it can provide impressive performance now, digital logic remains far more scalable, for all the reasons discussed earlier. Digital logic also has a massive advantage in flexibility. It is very hard to design "general-purpose" analog computers comparable in functionality to the digital CPUs we take for granted today. As was the case with the early analog machines, setup time is the bottleneck: pulleys must be adjusted, rotators must be moved and

[6] Mesa

[7] Yet it has a package approximately the size of an Intel Pentium-class chip—more evidence of the substantially larger transistors required for analog logic that Tom Knight talked about.

transformer coils must be looped. Essentially, a multiple-operation analog machine must provide a means by which it can be reconfigured, and this operation is usually expensive. An FPGA-style mass-wiring approach might provide assistance, but the components used by analog logic are dissimilar enough to make potential reconfiguration not worth the expense.

Ultimately, as discussed in a paper by Vergis, Steiglitz and Dickinson, analog and digital computers are equivalent,[8] so a decision to use analog computation is ultimately a pragmatic one. Clearly the tide has turned, perhaps for good, toward digital devices, as the advantages of scalability and accuracy are often decisive. However, analog devices may be more appropriate for specific applications, notably those that require direct interaction with analog systems or where precision can be safely discarded in favor of easier fabrication of massive numbers of computational units.

[8] Interestingly demonstrated through the proposition of analog devices to solve NP-complete problems in polynomial time: an amazing advantage over digital computers, it seems, until one realizes that the device would require infinite power to operate! (see Vergis et al.)

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