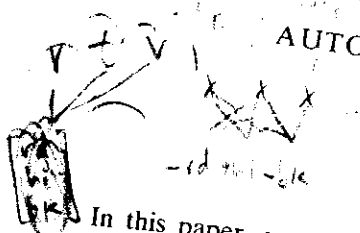


* KITAGAWA 1987 in Hack & Ojeda (eds.) 'Discontinuous Constituency'

THE "NO CROSSING CONSTRAINT" IN AUTOSEGMENTAL PHONOLOGY*

1. INTRODUCTION



In this paper, we examine a disquieting problem concerning the "constraining power" of the "No Crossing Constraint" with respect to multiplanar Autosegmental Phonological Representations. We argue that the "No Crossing Constraint" is not a constraint at all, since it does not reduce or restrict the class of well-formed Autosegmental Phonological Representations.

In order to make this argument and its conclusions precise, it is necessary to consider a number of more fundamental formal questions concerning Autosegmental Phonological Representations, such as: "What are Autosegmental Phonological Representations? - Are they simply diagrams? Mathematical objects of a previously unknown kind? What are the formal consequences of the No Crossing Condition? What is an association line? What is a tier? What are the differences between "uniplanar" and "multiplanar" representations? Do these differences have any consequences for the restrictiveness of Autosegmental Phonological theory?" Goldsmith's original (1976) formal description of Autosegmental Phonology, which underpins all subsequent work in Autosegmental Phonology, defines Autosegmental Phonological Representations to be *graphs*. Given this basis, our argument concerning the failure of the No Crossing Constraint to constrain Autosegmental Phonological Representations can be summarised as follows: Graphs which conform to the No Crossing Constraint are planar graphs. Some "multiplanar" Autosegmental Phonological Representations are planar graphs, but some are (necessarily) nonplanar graphs. The No Crossing Constraint does not restrict the class of nonplanar graphs, so by Occam's razor we are faced with two possibilities. Either we continue to recognize nonplanar Autosegmental Phonological

* We would like to thank Adrian Simpson, Anthony Warner, Ewan Klein and several anonymous reviewers for their extensive comments on this paper. We have also benefitted from valuable discussions on the content of this paper with Jim Scobbie, John Kelly, William Edmondson and Steven Bird. Their assistance has, we feel, improved this paper considerably. Those faults which remain are entirely ours.

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Representations as well-formed and drop or modify the No Crossing Constraint, or we retain the No Crossing Constraint, and thus cease to recognize nonplanar Autosegmental Phonological Representations as well-formed. We consider two sets of examples of Autosegmental Phonological Representations, one from the literature and one of our own, which are necessarily nonplanar, and thus conclude that the first alternative must be selected. We conclude that the No Crossing Constraint is not a constraint, since it does not apply to necessarily nonplanar Autosegmental Phonological Representations.

2. AUTOSEGMENTAL PHONOLOGY AND THE "NO CROSSING CONSTRAINT"

Autosegmental Phonology is a phonological theory which employs multi-tiered representations rather than strings as its central data structure (Goldsmith 1976, 1990; van der Hulst and Smith 1982). Phonological processes such as assimilation, harmony etc. are given derivational accounts similar to those employed in string-based approaches to phonology. But in line with the trend towards declarative formalisms in linguistic theory, Autosegmental Phonological Representations and derivations are sanctioned not by extrinsically ordered grammar rules, but by general 'principles' tempered by 'constraints', together with some language-specific rules. Well-formedness of an Autosegmental Phonological Representation and its derivation are assessed by their adherence to and satisfaction of these 'principles', 'constraints' and rules. In this paper we shall consider the principal 'constraint' of Autosegmental Phonology, the so-called 'No Crossing Constraint' (Hammond 1988), and show that it does not, in fact, constrain the class of well-formed Autosegmental Phonological Representations. (See Section 3 for a definition of the NCC.)

We are not alone in examining the basis of the No Crossing Constraint. Sagey (1988), for instance, examines two interpretations of the representations used in Autosegmental Phonology, and shows that the No Crossing Constraint follows as a necessary consequence of one of these interpretations. She concludes that the ill-formedness of Autosegmental Phonological Representations with crossing association lines derives from 'extralinguistic knowledge' about two timing relations, 'precedence' and 'overlap'.

Autosegmental Phonology is a relatively new and developing branch of linguistic theory. Work to date has been largely of an empirical nature, discovering and exploring the descriptive potential of tiered phonological

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representations. The development of a formal and rigorous presentation of Autosegmental Phonology is a project which now deserves attention, and work is in progress on several fronts. Building on the work of Sagey (1988), Bird and Klein (1990) have studied the temporal interpretation of Autosegmental Phonological Representations. Kay (1987) has examined Autosegmental Phonological Representations from a computational standpoint, and has elegantly demonstrated that n -tier Autosegmental Phonological Representations can be generated or parsed by $n + 1$ -tape finite state transducers. Our work in this paper explores the consequences of Goldsmith's (1976) original topological presentation of Autosegmental Phonology.

Autosegmental Phonology is a theory of grammar for a particular family of graphical languages (sets of Autosegmental Phonological Representations). In this paper, we are careful to distinguish the syntax of these languages (i.e., the *form* of phonological representations) from their semantics, that is, from possible *interpretations* of those representations. The fact that the No Crossing Constraint can be derived from a particular interpretation of Autosegmental Phonological Representations suggests that Sagey's hypothesis involves phenomena that are not strictly speaking 'extralinguistic' (as Sagey concludes), but rather 'extrasyntactic', i.e., semantic in the terms just defined.

As part of our work in constructing computational implementations of nonlinear phonology, we have independently duplicated Sagey's result. However, we have also developed the stronger *syntactic* argument that the No Crossing Constraint (NCC) is not a constraint at all, strictly speaking, since it does not restrict the class of well-formed phonological representations. The core of our argument can be briefly sketched as follows:

A distinction must be drawn between Autosegmental Phonological Representations, and diagrams of those Autosegmental Phonological Representations. Diagrams are not linguistic objects, but pictures of linguistic objects, and may have properties such as perspective, colour etc. which are of no relevance to linguistic theory. The NCC is a constraint on diagrams, not on Autosegmental Phonological Representations. When the conditions by which the NCC restricts the class of diagrams are examined and linguistically irrelevant factors such as width or straightness of lines are removed, it is apparent that the *intention* of the NCC is to enforce the following *planarity constraint*:

Autosegmental Phonological Representations are planar graphs. Thus, the planarity constraint is the defining distinction between the two varieties of Autosegmental Phonology, planar and nonplanar (i.e., multiplanar).

In planar Autosegmental Phonology the No Crossing Constraint has no place in *linguistic* theory, since it is *universally* the defining characteristic of planar graphs. In nonplanar Autosegmental Phonology the NCC is unrestrictive, because all graphs can be portrayed as 3-D diagrams with no lines crossing.

We consider our syntactic argument to be stronger than Sagey's semantic argument, since it is not dependent on a particular theory of the interpretation of phonological representations, but follows from general principles of graph theory alone.

Proving something which is widely thought to have been demonstrated already may seem like a strange activity, but if a precedent is called for we cite the following episode in the history of generative grammar. Throughout the 1960s and 1970s it was believed and taught by grammarians that Chomsky (1963, pp. 378-79) and others had proved that English was not a Context-Free Language. In the early 1980s, however, these 'proofs' were shown to be defective in various respects (Pullum and Gazdar 1980), and it was not until some years later that respectable proofs of this widely-believed fact were actually constructed (Manaster-Ramer 1983; Huybregts 1984; Shieber 1985; Culy 1985).

The rest of this paper is set out as follows. In Section 3 we consider the form of Autosegmental Phonological Representations and we introduce a few important basic definitions and principles of graph theory. (This is the most technical part of the paper, and readers may choose to skim through this section on their first reading.) In Section 4 we consider the veracity of the claim that the No Crossing Constraint restricts the class of representations in planar and nonplanar Autosegmental Phonology, and we examine the belief of some proponents of 3-D Autosegmental Phonology that the *necessity* of nonplanar representations has already been demonstrated. We show, by an exhaustive examination of arguments to this effect from the literature, that this belief has not in fact yet been proved. Finally, in Section 5 we present some data exemplifying a number of interacting harmonies in Guyanese English that in Autosegmental Phonology are amenable only to nonplanar representation, and we thus prove that such representations may indeed be necessarily non-planar (as Auto-

segmental phonology is non-restrictive.

3. AUTOSEGMENTAL

We proceed by considering Autosegmental Phonological Representations, interpretability, and their role in our subsequent discussion of linguistic objects, from the perspective of planar graphs (which are represented by APRs are, in fact, graphs).

Let us first consider this question is that first hypothesis can be necessarily flat perspective, that a instance, it is possible but with a three-dimensional Euclidean 3-space to perspectiveless dimensional perspective are not in themselves pictures of Autosegmental Phonological Representations.

A more sophisticated intermediate problems of Autosegmental Phonological Representation's 'properties' that hypothesis begs the question which are not. The

In Autosegmental Phonology a number of phonological slots) and a two-dimensional perspective. In addition, the planar representation are not. For example, the [+R] are autosegmental (1976, p. 28) defining set of sequences of levels', together

segmental phonologists suspected), but that as a consequence, the NCC is non-restrictive.

3. AUTOSEGMENTAL PHONOLOGY AND GRAPH THEORY

We proceed by considering the syntax of Autosegmental Phonological Representations, introducing definitions of the terminology which we employ in our subsequent argument. To begin, we must be careful to distinguish Autosegmental Phonological Representations (APRs), which are linguistic objects, from both diagrams (which are pictorial objects) and graphs (which are mathematical objects). Later, we shall conclude that APRs are, in fact, graphs.

Let us first consider the question 'What are APRs?' A naïve answer to this question is that they are diagrams i.e., pictures in journals, etc. This first hypothesis can easily be dismissed. Being pictorial objects, diagrams are necessarily flat. However, diagrams may have properties, such as perspective, that are not shared by phonological representations. For instance, it is possible to portray a two-dimensional object on a flat surface but with a three-dimensional perspective, e.g., by drawing a circle in Euclidean 3-space as an ellipse in the plane of the paper. (We shall refer to perspectiveless diagrams as 2-D diagrams, and to diagrams with three-dimensional perspective as 3-D diagrams.) Therefore, diagrams in journals are not in themselves Autosegmental Phonological Representations, but pictures of Autosegmental Phonological Representations. What, then, are Autosegmental Phonological Representations?

A more sophisticated hypothesis, which does not fall prey to the immediate problems of the naïve hypothesis, is 'an Autosegmental Phonological Representation is a mathematical object that has precisely the 'important' properties that Autosegmental Phonological diagrams have.' But this hypothesis begs the question as to which properties are 'important', and which are not. The resolution of this question is fundamental to this paper.

In Autosegmental Phonology, a phonological representation consists of a number of phonological objects (segments, autosegments and timing slots) and a two-place relation, called association (A), over those objects. In addition, the phonological objects in an Autosegmental Phonological Representation are partitioned into a number of well-ordered sets, called *tiers*. For example, in Fig. 1, the objects d , b and m are segments, $[+H]$ and $[+R]$ are autosegments, and the X es are timing slots. Goldsmith (1976, p. 28) defines an Autosegmental Phonological Representation as a set of sequences L^i of objects a_j^i (each of which is a tier, which Goldsmith calls 'levels'), together with an ordered sequence A of pairs of objects

of the type above on the title level.

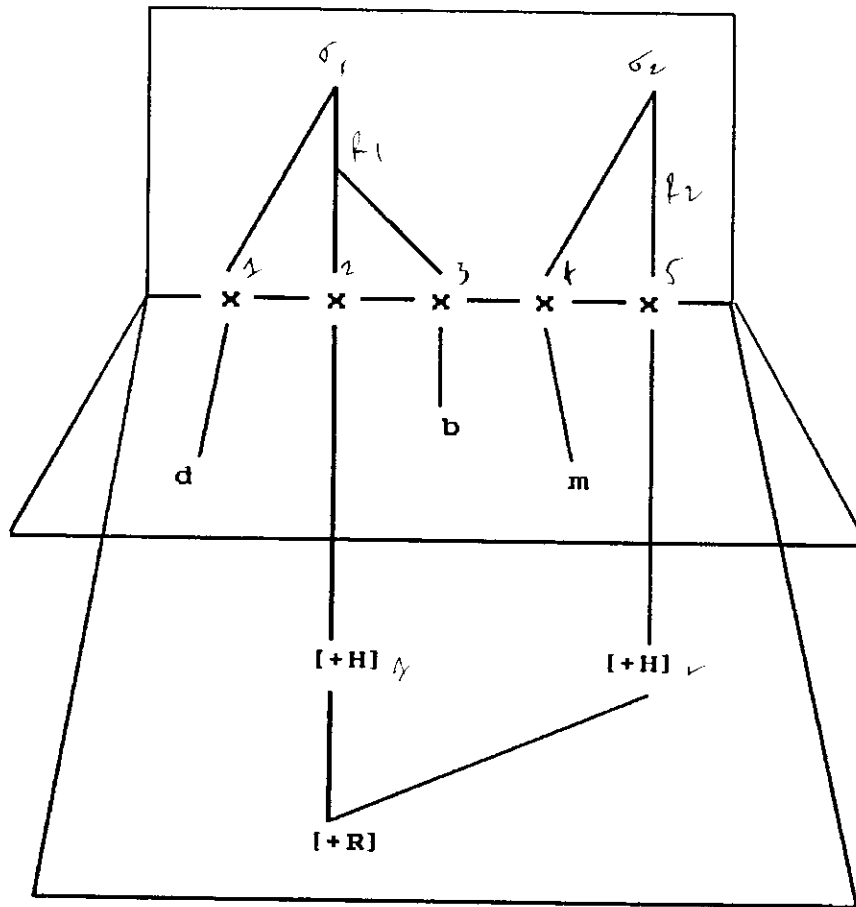


Fig. 1. A paddle-wheel graph.

whose first and second members are taken from disjoint tiers. In his own words, he states:

Each autosegmental level is a totally ordered sequence of elements, a_j^i ; this is the j^{th} element on the i^{th} level. Call the set of segments on the i^{th} level level L^i .

Since each L^i is a sequence, each tier is a function from the first n natural numbers to an unlabelled set of objects a_j^i , where $n = |L^i|$. Let us give each set of objects a_j^i in L^i the label O^i . Since the objects in each tier form a set, each a_j^i is unique and the function is one-to-one. The natural numbers are totally ordered by \leq , which maps one-to-one onto a total order \leq^i for each L^i . Each L^i , therefore, is a set of objects O^i for which there exists a total order \leq^i . From \leq^i it is possible to define $<^i$ in the usual way, viz.: $a <^i b \Leftrightarrow a \leq^i b$ and $\neg(b \leq^i a)$. $<^i$ provides the usual definition of

$O^i = n$ objects on L^i , from a_1^i to a_n^i
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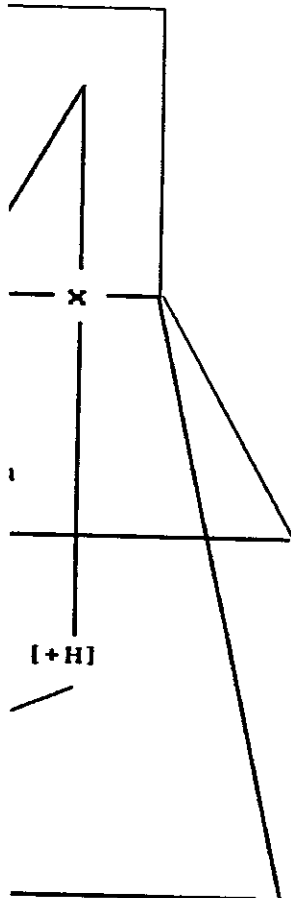
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¹ It is a logical possibility: "set", "pair" etc., he is but such a possibility is



adjacency, namely a is adjacent to b iff $a <^i b$ or $b <^i a$ and there is no c such that $a <^i c <^i b$ or $b <^i c <^i a$.

Following an illustration, Goldsmith continues with his formal presentation:

In addition to these . . . sequences of segments, there is a totally ordered sequence of pairs - essentially the association lines . . . Call the set of these pairs "A".

We shall use the symbol \sqsubseteq to denote the total ordering of A . Since there may be more than two tiers in an Autosegmental Phonological Representation, A can be partitioned into subsets $A^{i,j}$, each of which contains the set of pairs (association lines) linking L^i with L^j . The partitions $A^{i,j}$ are now called *charts* by Goldsmith (1990). The relationship between \sqsubseteq and \leq^i is as follows:

Let \sqsubseteq be the total ordering on A , let $\{(a, b), (c, d)\} \subset A$ and let $\{a, c\}$ and $\{b, d\}$ be in disjoint tiers. Goldsmith states (1976: 28) 'A in a sense organizes the other levels' (i.e. the endpoints of A). Formally

$$(a^i, b^j) \sqsubseteq (c^i, d^j) \Leftrightarrow a^i \leq^i c^i \text{ and } b^j \leq^j d^j \quad (1)$$

THEOREM 1: The ordering of association lines \sqsubseteq is not total for a whole Autosegmental Phonological Representation.

Proof. We shall show that there exists a pair of association lines which is not in \sqsubseteq . Suppose (a_1^1, a_2^1) is in $A^{1,2}$ and (a_2^2, a_3^2) is in $A^{2,3}$. If \sqsubseteq contains $(a_1^1, a_2^1), (a_2^2, a_3^2)$ then either $a_1^1 \leq^1 a_2^2$ or $a_2^2 \leq^2 a_1^1$. But this cannot be so, because \leq only holds within a tier, not across tiers. Therefore, \sqsubseteq does not contain $((a_1^1, a_2^1), (a_2^2, a_3^2))$, and thus \sqsubseteq is not total for the whole Autosegmental Phonological Representation. \square

When Goldsmith states that A is a set of pairs forming a totally ordered sequence, it must therefore be understood to mean that each *chart* $A^{i,j}$ is totally ordered. Consequently, \sqsubseteq is not total, but can be partitioned into total orderings $\sqsubseteq^{i,j}$.

According to Goldsmith's definitions, then, an Autosegmental Phonological Representation is a structure (O, A) , where O is the set of phonological objects and A is a subset of $O^i \times O^j, i \neq j$. Such a structure is a graph. [Note that we are not claiming that (O, A) can be or ought to be interpreted, formalised or reconstructed as a graph. Given Goldsmith's definitions, (O, A) is a graph.¹] This graph may be augmented with maps

¹ It is a logical possibility that when Goldsmith uses terms such as "totally ordered sequence", "set", "pair" etc., he intends something other than the standard mathematical definitions, but such a possibility is so fanciful that it does not merit consideration.

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π_1 and π_2 from A to O which pick out the endpoints of each association line. These are defined by Goldsmith as follows:

Define projection π_1 from 2^A (the set of subsets of A) to 2^{L^1} (the set of subsets of L^1) in the natural way:

$$\pi_1(\{(a_j^1, a_k^2), (a_l^1, a_m^2), (a_n^1, a_p^2), \dots\}) = \{a_j^1, a_l^1, a_n^1, \dots\}$$

That is, the 1st projection π_1 picks out the set of first elements of the pairs. Likewise, the second projection π_2 picks out the second element of each pair, and so forth.

Subsequent to Goldsmith's original account of Autosegmental Phonology (Goldsmith 1976), a number of extensions and refinements to the above account have been explored by various phonologists. These extensions are considered in detail below. However, Goldsmith's original definitions of APRs are applicable to all subsequent versions of Autosegmental Phonology. Consequently, APRs in more recent work can be factored into two parts: a part P which conforms to the general or universal properties of APRs presented above, and E , the specific parochial extensions of a particular version of Autosegmental Phonology, such as the requirement that all Association lines are anchored in a distinguished skeletal tier. Consequently, we shall employ the term APR to refer just to the P (general, universal) component of Autosegmental Phonological Representations.

In Autosegmental *diagrams*,² phonological objects are represented by alphabetic symbols, features or vectors of features, and the association relation by straight lines connecting each pair of objects that is in the association relation. Tiers are portrayed in Autosegmental diagrams by sequences of objects separated by spaces. The No Crossing Constraint is the statement that in a well-formed Autosegmental diagram, lines of association may not cross.

We shall not consider what phonological representations in generative phonology³ denote. A large number of views concerning this question have been advanced over the years, and it seems unlikely to us that agreement will ever be reached. Some phonologists (e.g., Dogil 1984, Browman and Goldstein 1986, Goldsmith 1976, 16) believe they denote artulatory scripts. Some (e.g., Halle 1985) believe they denote mental scripts. Some believe they denote abstract, purely phonological objects (Foley 1977). Some believe they are just convenient fictions, ephemeral constructions to aid in the development of theoretical ideas.

Despite the ongoing debate about the semantics of APRs, it is possible

² Figures 4 and 1 are typical Autosegmental diagrams.

³ Autosegmental Phonology is but one variety of generative phonology.

to demonstrate the no consideration of the this, we first set out theory.

In mathematics, a over those objects (c and π_2 , though these cf. Rosen 1977) is ca

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⁴ APRs with a single at least two tiers.

to demonstrate the nonrestrictiveness of the No Crossing Constraint from consideration of the *syntax* (i.e., form) of APRs alone. In order to do this, we first set out some elementary definitions and theorems of graph theory.

In mathematics, a collection of objects and a two-place relation defined over those objects (often with the explicit inclusion of endpoint maps π_1 and π_2 , though these are usually omitted if multiple arcs are not permitted, cf. Rosen 1977) is called a *graph*.

Formally, a graph G is a tuple (V, E) , (optionally with the addition of endpoint maps π_1, π_2) where V is any set of objects, called *vertices* in graph theory, and E a set of pairs of vertices, called *edges*.

The term 'vertex' is a general term for primitive objects in whatever domain is being modelled, and the term 'edge' is a general term for each pair of objects in a relation. The *degree* of a vertex is the number of edges of which that vertex is a member.

The definition of a graph and the terminology of graph theory are completely independent of any particular drawing conventions that may be used to represent a graph diagrammatically. Graphs are abstract mathematical entities with no unique visible manifestation. In particular 'graph' is not synonymous with 'diagram', 'vertex' is not synonymous with 'point' and 'edge' is not synonymous with 'line'. We must thus be careful to distinguish graphs from their various possible diagrammatic instantiations. Vertices can be any type of object whatsoever, edges are simply pairs of vertices, and graphs are simply pairs of sets of vertices and edges.

A *bipartite* graph is one in which the set of vertices can be partitioned into two disjoint subsets L^1 and L^2 , such that the set of edges is a subset of $L^1 \times L^2$. Since in Autosegmental Phonology the association relation holds only over objects on separate tiers L^1 and L^2 (i.e. an object may not be associated with any other object on the same tier), the existence of tiers in Autosegmental Phonology ensures that its graphs are (at least) bipartite.⁴ For instance, L^1 might be the set of timing slots, and L^2 the set of melody segments.

Graphs in which the set of nodes can likewise be divided into n disjoint subsets are called *n-partite* graphs. An Autosegmental Phonological Representation with an anchor tier and m melody tiers is thus *maximally* an $(m + 1)$ -partite graph, though it might, depending on what conditions are imposed on the association relation, be *minimally* merely a bipartite graph. This is the case if melody units can only be associated to anchor units,

⁴ APRs with a single tier are just segmental strings. To merit the name, an APR must have at least two tiers.

vertex =
objects modelled,
edge =
relation
degree =
of relations
per object

bipartite =
sets of
objects in
relations
 $L^1 \times L^2$

and not to each other. Pulleyblank (1986: 14) advances this condition. It is also the case in the theory of tier organisation proposed by Clements (1985), in which each segment is a tree of melody-units, which makes trees are bipartite graphs). In multi-tier 2-D Autosegmental diagrams, association lines do not usually 'meet' tiers except at a node. The graphs denoted by these diagrams are also (at most) bipartite. Since m -partite graphs where $m > 2$ are therefore prohibited in one way or another from occurring in the several varieties of Autosegmental Phonology, Autosegmental diagrams portray (amongst other things) bipartite graphs.

A complete bipartite graph of the form $K_{1,n}$ is called a *star graph*. Such a graph has one node (the *root*) linked to each of the others (the *leaves*).

A *circuit* graph is a connected graph in which every node is of degree two. We define a *chain* to be a circuit graph with one arc removed. In a chain, every node is linked to two others except for two end-nodes, which are of degree one. A *wheel* is a circuit graph to which is added an additional "hub" node and an arc linking every node in the circuit to the "hub" node. A *tree* graph is a connected graph with no circuits. In linguistic theory, trees are a very common type of graph, although they are practically always augmented by an ordering on the leaf nodes (nodes of degree 1). In this paper, however, we shall make no such additional stipulation. The only mention of trees we make is to the unordered trees which Clements (1985) proposes as a theory of the internal structure of segments. A *directed graph* is a graph in which arcs are *ordered* pairs, not simply pairs, of nodes. A directed acyclic graph or *dag* is a directed graph which does not contain any directed circuits (cycles).⁵

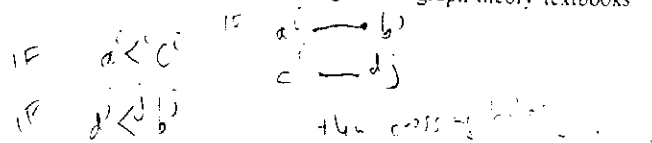
Having examined the structural properties of APRs and other graphs in a little detail, we shall now focus our attention on one of the defining properties of well-formed APRs, the NCC. We shall first show that the NCC follows directly from the total ordering of A .

Two association lines (a^i, b^j) and (c^i, d^j) are said to *cross* iff $a^i <^i c^i$ and $d^j <^j b^j$. The NCC is the statement that in an APR, there is no pair of association lines which cross.

THEOREM 2 Within a chart, the NCC follows from the total ordering of $A^{i,j}$.

Proof. If 'lines cross' (i.e. if $a^i <^i c^i$ but $d^j <^j b^j$) then by (1) neither $(a^i, b^j) \trianglelefteq (c^i, d^j)$ nor $(c^i, d^j) \trianglelefteq (a^i, b^j)$, in which case \trianglelefteq is not total and

⁵ Further details of these and other kinds of graphs are given in graph-theory textbooks such as Wilson (1985).



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3.1. Planarity

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A^{ij} is not well-defined. A^{ij} is only well-defined if lines do not cross, or in other words, if the NCC holds. \square

Since the NCC follows from mathematical properties of Autosegmental Phonological Representations, it is not a specifically *linguistic* constraint. Additionally, since the NCC follows from the total ordering of A^{ij} , it is not necessary for Autosegmental Phonology to contain both the NCC and the total ordering of A^{ij} . Either one or the other statements could be dropped, and so, by Occam's razor, should be.

Since all Autosegmental Phonological Representations are graphs on which some further restrictions have been placed (such as total ordering of tiers), all of the *universal* properties of graphs hold of Autosegmental Phonological Representations, together with some *special* properties. Autosegmental Phonological Representations are a *special* kind of graph, but they are also subject to all the universal properties of graphs.

The formal definition of APRs proposed by Goldsmith and explored in detail by us above captures the following necessary structural properties of all varieties of Autosegmental Phonological Representations: division into tiers, total ordering of tiers, adjacency of neighbouring elements within a tier, and the Association relation.

We shall now consider the relationship between the two kinds of Autosegmental diagrams (2-D and 3-D), and two kinds of graphs, planar graphs and Euclidean (nonplanar) graphs.

3.1. Planarity

A *Jordan curve* in the plane is a continuous curve which does not intersect itself. A graph G can be embedded in the plane if it is isomorphic to a graph which can be portrayed in the plane with points representing the vertices of G and Jordan curves representing edges in such a way that there are no crossings. A *crossing* is said to occur if either

- (1) the Jordan curves corresponding to two edges intersect at a point which corresponds to no vertex, or
- (2) the Jordan curve corresponding to an edge passes through a point which corresponds to a vertex which is not one of the two vertices which form that edge.

A *planar* graph is a graph which can be embedded in a plane surface.

A *Euclidean* graph is a graph which can be embedded in Euclidean 3-space, that is, normal, three-dimensional space. All planar graphs are Euclidean, but not all Euclidean graphs are planar. That is, there are

A^{ij} are sets which contain the set of pairs $\langle L_i, L_j \rangle$

Jordan curve
Jordan curve
they cannot intersect at a point which is not a vertex

A planar graph is one without crossings

X_5 which we shall label R_2 . The two $[+H]$ nodes must be disambiguated, so we shall subscript them from left to right $[+H]_1$ and $[+H]_2$. (We shall use these unambiguous labels to refer to the nodes in the embedding of the graph.) The embedded graph has an axis of fifteen nodes in any order. Because the order of nodes in the axis is irrelevant to the construction, we can always make the order of nodes in the axis conform to their order in the tiers of the APR. The embedding operation never requires the order of nodes within tiers to be altered. Suppose that the order of nodes in the axis is $\sigma_1, \sigma_2, R_1, R_2, X_1, \dots, X_5, d, b, m, [+H]_1, [+H]_2, [+R]$. There are fourteen association lines in the APR, so the construction will have fourteen 'paddles' intersecting at the axis. On each 'paddle' draw a Jordan curve between the two vertices of the axis which correspond to the pair of nodes linked by the association line corresponding to that 'paddle'. Viewed one-by-one, the paddles are as follows:

- $\sigma_1, \sigma_2, R_1, R_2, X_1, X_2, X_3, X_4, X_5, d, b, m, [+H]_1, [+H]_2, [+R]$
- $\sigma_1, \sigma_2, R_1, R_2, X_1, X_2, X_3, X_4, X_5, d, b, m, [+H]_1, [+H]_2, [+R]$
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- $\sigma_1, \sigma_2, R_1, R_2, X_1, X_2, X_3, X_4, X_5, d, b, m, [+H]_1, [+H]_2, [+R]$

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Given an embedding of a graph G in Euclidean 3-space, we can draw the embedding in a 3-D diagram by projecting it into the plane using projective geometry (Lord and Wilson 1984, p. 32), for instance, using one-point perspective projection. It is consequently simple to prove that:

THEOREM 4: Every graph G can be portrayed in a 3-D diagram as a network of points and (in perspective) noncrossing lines.

Proof. Embed G in Euclidean 3-space. Project the embedding into the plane. \square

The fact that in Autosegmental Phonological Representations, the set of vertices is partitioned into tiers, each of which is totally ordered, does not affect the validity of these theorems. The single axis of vertices required for the construction used in the proof of Theorem 3 may be partitioned into totally ordered subsets of objects without affecting the result.

It is harder to show that a graph is necessarily nonplanar than that a diagram is 3-D. For a diagram to be 3-D it merely has to appear to be 3-D. A necessary and sufficient criterion for the nonplanarity of a graph G is:

THEOREM 5. (Kuratowski 1930)⁶ G is nonplanar if and only if it contains a subgraph which is homeomorphic to either of the two graphs K_5 (the fully connected graph over five vertices) and $K_{3,3}$ (the fully connected bipartite graph over two sets of three vertices), shown in Figure 2.

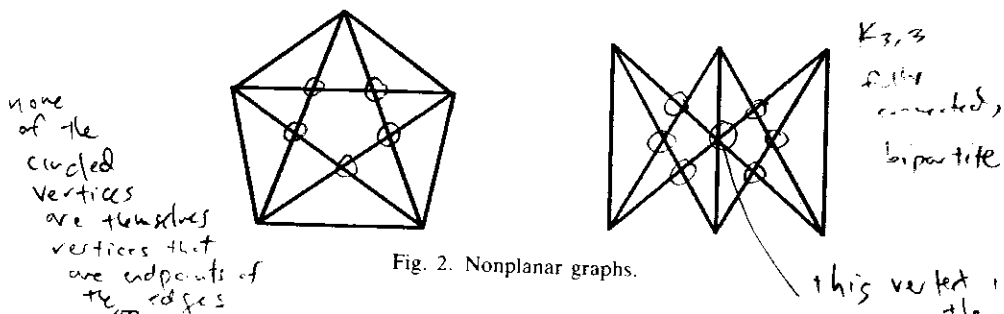


Fig. 2. Nonplanar graphs.

(Two graphs are homeomorphic if they can both be obtained from the same graph by inserting new vertices of degree two into its edges.)

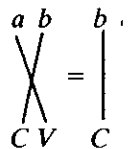
⁶ It is beyond the scope of this paper to present a proof of Kuratowski's theorem, since it requires extensive and advanced familiarity with graph theory. A reasonably approachable presentation of the proof is Gibbons (1985, pp. 77-80).

Note that Kuratov graph is shown to be nonplanar.

3.2. Paddle-wheel A

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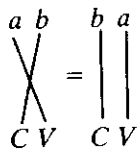
⁷ Kaye (1985, pp. 289, Lowenstamm and Kaye is three-dimensional, d

Note that Kuratowski's Theorem requires only that a subgraph of a graph is shown to be nonplanar in order to show that the whole graph is nonplanar.

3.2. Paddle-wheel Autosegmental Phonological Representations

Pulleyblank (1986, pp. 12-14) considers limitations on the association relation. He argues that if the objects in every tier may only be associated with the objects ('slots') in a distinguished ('skeletal') tier, and not to the objects in any other tier, the theory which results is 'considerably more restrictive'. The APRs yielded by Pulleyblank's proposed restriction have come to be known as 'paddle-wheel' APRs (Archangeli 1985, p. 337) since they consist of a set of planar graphs which intersect along a shared tier, the skeleton (cf. Figures 1 and 15), and consequently look like a paddle-wheel. Since Autosegmental Phonological Representations are graphs, Pulleyblank's claim must mean that versions of Autosegmental Phonology which allow only 'paddle-wheel' graphs are 'considerably more restrictive' than theories which allow general graphs. Yet Theorem 3 shows that this is not correct. For Pulleyblank's claim to hold, there must also be restrictions on the composition of each tier other than linear ordering (in other words, restrictions on the *objects* in each tier), and on the *straightness* of association lines.

A number of Autosegmental phonologists (Clements and Keyser 1983, p. 11; Prince 1984, p. 235; Clements 1986; Pulleyblank 1986, p. 14) who subscribe to the 'paddle-wheel' theory claim that timing relations between Autosegments are dependent on the ordering of objects in the skeleton. In this case, the maximally parsimonious account is one in which autosegmental tiers are not explicitly ordered.⁷ If only elements on the skeletal tier are ordered, then the NCC has no force since



where C and V are on the skeletal tier and the tier {a, b} is unordered.

⁷ Kaye (1985, pp. 289, 301-304) crucially requires nonskeletal tiers to be unordered, as does Lowenstamm and Kaye (1986), although this latter paper explicitly denies that phonology is three-dimensional, despite accepting the basic principles of Autosegmental Phonology.

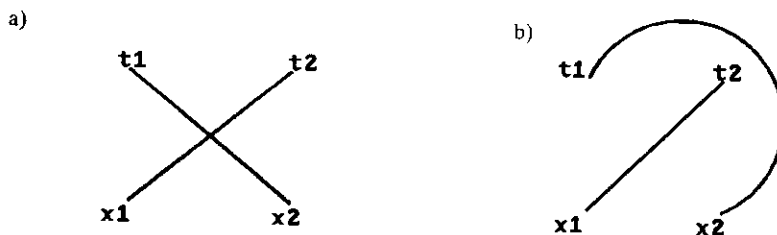


Fig. 3. Two embeddings of a planar graph.

The need for association lines to be straight for the NCC to work can be demonstrated as follows. Consider the graph:⁸

$$(\{t_1, t_2, x_1, x_2\}, \{(t_1, x_2), (t_2, x_1)\})$$

with partition into tiers $L^1 = \{t_1, t_2\}$, $L^2 = \{x_1, x_2\}$ and the order $t_1 < t_2$, $x_1 < x_2$ (Figure 3a). If the NCC requires association lines to be straight, then this Autosegmental Phonological Representation cannot be portrayed without crossing lines (Figure 3a), and it would thus be excluded by the NCC. But if there is no such restriction on the straightness of association lines, this Autosegmental Phonological Representation can be portrayed without crossing lines (Figure 3b), and thus the NCC does not prohibit this APR.

This demonstrates that the No Crossing Constraint is a condition on pictures, not phonological representations, since straightness of lines is a property of pictures, not linguistic representations. The straightness of association lines is conventional rather than formal: it has never been explicitly defended in Autosegmental Phonology, it does not follow from other principles of the theory, and it is sometimes abandoned when it is convenient to do so (see for example McCarthy (1979/1982, p. 140), Archangeli (1985, p. 345), Prince (1987, p. 501), Pulleyblank (1988, pp. 256, 259), Hayes (1989, p. 300), McCarthy and Prince (1990, p. 247)). If the lines denoting the association relation need not be straight, then the NCC will sometimes necessarily hold and at other times only contingently hold. The cases in which the NCC contingently holds are those like Figure 3, in which if the lines need not be straight, the NCC can be circumvented. In such cases, the NCC is nonrestrictive, and therefore cannot be linguistically relevant. However, in the cases in which the NCC necessarily holds, it is indeed restrictive, for it limits the class of Autosegmental Phonological Representations to planar graphs. In these cases adoption of the NCC is

⁸ Recall that graphs have the general form (V, E) . In this instance the set of vertices $V = \{t_1, t_2, x_1, x_2\}$, and the set of edges $E = \{(t_1, x_2), (t_2, x_1)\}$.

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Since the straightness of association lines is a property of Autosegmental diagrams, and not of Autosegmental Phonological Representations, Pulleyblank's position can only be maintained if there are constraints on tier composition which would diminish the force of our criticism. No such constraints have yet been established, although there are several possibilities:

- (1) Each object on a tier is a specific bundle of features, each of which cannot occur on any other tier.
- (2) Each object on a tier is a specific single phonological feature, which cannot occur on any other tier.
- (3) Each tier bears objects consisting of all of the segmental structure dominated by a single node of the Universal segment tree. (Clements 1985)
- (4) Each tier represents a morpheme (the *Morphemic Tier Hypothesis*). (McCarthy 1979/1982, Halle and Vergnaud 1980)

The first position cannot be maintained, since it is necessary in Autosegmental Phonology to allow more than one tier to bear *the same* feature or features. Such proliferation of tiers has been employed in Autosegmental analyses of cases where a single feature (or set of features) has two different morphophonological functions. Prince (1987, p. 499) gives the following illustration of this:

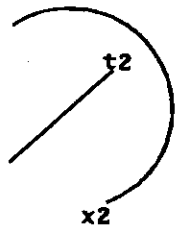
Arabic requires the same features to appear on different planes: for example, the affix /w/ is featurally identical to any other /w/, yet it clearly stands apart, tier-wise, because a root consonant may spread over it without line crossing in form XII . . .

Halle and Vergnaud (1980) contains many similar examples.

Yet without the prohibition against the multiplication of features on different tiers, this position is simply the null hypothesis that Autosegments are (unconstrained) bundles of features.

The second position (the 'single feature hypothesis') has been challenged on the grounds that it is empirically inadequate: it is sometimes desirable to treat two or more features as a single Autosegmental unit (when they have the same distribution, for instance).

McCarthy's widely-supported analysis of Semitic morphology requires entire segmental melodies, not just single features, to be Autosegmental. The single feature hypothesis would not be sufficient to maintain Pulleyblank's claim concerning the restrictiveness of paddle-wheel APRs, unless



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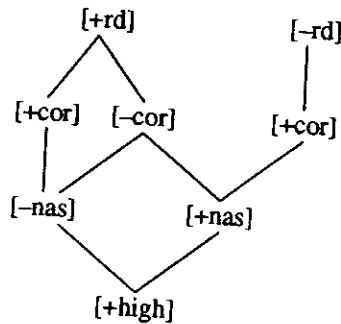


Fig. 4. A planar graph.

multiplication of single features on several tiers (a move which is necessary to Autosegmental Phonology) were also prohibited.

The third hypothesis, proposed in Clements (1985), also falls foul of the need identified by McCarthy (1979/1982) and Prince (1987) for feature-structures to be replicated on several tiers. In every case, such replication undermines the restrictiveness of any proposal regarding tier composition, Clements (1985) included.

The fourth hypothesis, the Morphemic Tier Hypothesis, is not sufficient to maintain Pulleyblank's claim, because it begs the question as to what phonological objects may constitute a morpheme. McCarthy (1989) shows that in the analysis of some languages (e.g., Mayan) it is necessary to represent vowel and consonant features on independent planes, although there is evidence that vowels and consonants constitute separate mor-

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understand the NCC, we are especially interested in the class of properly (i.e., necessarily) nonplanar graphs, which cannot be portrayed in 2-D diagrams without crossing lines.

4. PLANARITY AND THE NCC

In the early days of Autosegmental Phonology (Goldsmith 1976), all Autosegmental diagrams were drawn as if to lie entirely in the plane of the paper. As we showed in the preceding section, however, if the No Crossing Constraint applies to Autosegmental Phonological Representations, not diagrams, it defines a general, topological sense of planarity:

Planarity Condition: a graph is planar if and only if it can be embedded in a plane surface with (by definition of 'embedding') no edges crossing.

Not all graphs can fulfil this requirement, however they are portrayed, and it is therefore necessary to determine whether all Autosegmental Phonological Representations can, if only in principle, be portrayed in the plane. If some cannot, then APRs are in general nonplanar (whether they are portrayed as such or not), and the No Crossing Constraint is not restrictive.

If the NCC applies to diagrams, it is a drawing convention, not a part of linguistic theory. But if it applies to Autosegmental Phonological Representations, the planarity condition and the No Crossing Constraint are equivalent: the No Crossing Constraint has no specifically linguistic status, in that it is the defining characteristic of planarity. We foresee that proponents of the NCC might wish to argue that if the NCC is retained, it is the planarity condition which is vacuous. But this argument is inadequate in two respects. Firstly, in addition to prohibiting Autosegmental Phonological Representations which are necessarily nonplanar, the NCC also excludes Autosegmental Phonological Representations which may be portrayed in diagrams whose lines only contingently cross, even if there is some other way of drawing them in which no lines cross. Thus the NCC prohibits some Autosegmental Phonological Representations merely on the basis of the way in which they might sometimes be portrayed. The planarity condition, on the other hand, is a condition on Autosegmental Phonological Representations, not diagrams of Autosegmental Phonological Representations. It thus constrains linguistic (phonological) representations, not diagrams of linguistic representations. Secondly, to the extent that 'no crossing' is a universal property of planar graphs and not just those planar graphs employed in phonological theory, there is no reason

for the mathematical definition of planarity to be imported into linguistics as a 'principle of Universal Grammar'. The relevant principle is 'Autosegmental Phonological Representations are planar graphs' (if such is the case), not 'association lines must not cross'. For these reasons, given the equivalence of the NCC and the planarity condition, those Autosegmental Phonologists seeking to defend the planarity hypothesis would do better to adopt the planarity condition directly (since it is a constraint on linguistic representations) than the NCC, which is a constraint only on *drawings* of linguistic representations.

Various phonologists (e.g., Archangeli 1985, Goldsmith 1985, Pulleyblank 1986) have found it convenient to generalize the planar representations of early Autosegmental Phonology to 3-D, 'paddle-wheel', representations, in which several independent Autosegmental planes intersect along a distinguished tier of timing units.

There is, however, an important distinction between convenience and necessity. It is widely believed and commonly assumed by Autosegmental Phonologists that the *necessity* of 3-D representations has already been uncontentionally demonstrated. Archangeli (1985, p. 337), for instance, writes

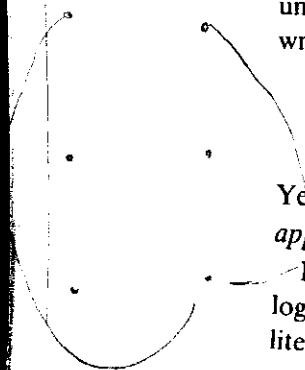
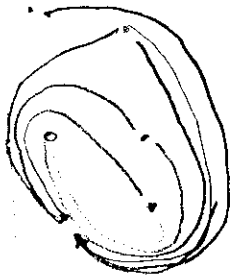
McCarthy's (1979; 1981) analysis of Semitic forced a truly three dimensional phonological representation. [Our emphasis.]

Yet McCarthy (1979, 1981) contain not a single diagram which even appears to be nonplanar, let alone a necessarily nonplanar representation.

Because the belief in the necessity for nonplanar Autosegmental Phonological Representations is widespread, it has rarely been defended in the literature. As far as we are aware, no necessarily nonplanar phonological representation has yet been presented as a proof that Autosegmental Phonological Representations are nonplanar.⁹

We shall attempt to defend our claim that the nonplanarity of Autosegmental Phonological Representations has not yet been proven by establishing a necessary and sufficient criterion for a graph to be (necessarily) nonplanar. We shall then use this criterion to test the logic of the argument and examples adduced in support of the claim that phonological representations have already been shown to be necessarily nonplanar. We shall

⁹ In our examination of the literature, we have discovered one article containing three diagrams of Autosegmental Phonological Representations which are, in fact, demonstrably nonplanar (see Figure 13). But these examples are not offered in defence of the argument that Autosegmental Phonological Representations are nonplanar. They are presented as part of a derivational account of timing in Ancient Greek. We reproduce and discuss these examples in more detail below.



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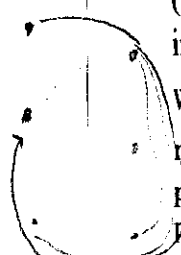
Since the associativity theorem, however, is a condition for an Autosegmental representation to be found in the literature, the paper are cases of

4.1. 3-D Diagrams

Figure 1, taken from the association rule in Archangeli's logic (1985, p. 337): such a consonant melody (a vowel melody) of the paper above below the anchor (syllable template) in the plane of time with more than

nonplanar. This is a paddle, but it is portrayed in the plane embedding of Autosegmental Phonology, the exception of the plane embedding in no way affects *unchanged*, it remains to all the relevant (Hoberman 1988), nothing other than

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argue that the falsity of claims in the literature about 3-dimensionality arise from a failure to distinguish diagram conventions from genuine and uncontentious universal properties of graphs.

Since the association relation is a bipartite graph, according to Kuratowski's theorem, homeomorphism to $K_{3,3}$ is a necessary and sufficient condition for an Autosegmental Phonological Representation to be necessarily nonplanar. In fact, the only instances of homeomorphism to $K_{3,3}$ to be found in the Autosegmental literature and in the remainder of this paper are cases of *identity* to $K_{3,3}$.

4.1. 3-D Diagrams in the Autosegmental Literature

Figure 1, taken from Archangeli (1985), is typical of those diagrams of the association relation that are purported to be necessarily nonplanar. Archangeli's logic is inexplicit, but seems to be as follows (cf. Archangeli, 1985, p. 337): suppose there is a tier above the anchor tier (for instance, a consonant melody), and another tier below the anchor tier (for instance, a vowel melody). Then there are at least two 'paddles', one in the plane of the paper above the anchor tier, the other in the plane of the paper below the anchor tier. Now if yet another independent tier is called for (syllable templates, perhaps), yet another paddle, separate from the two in the plane of the paper, is required. Thus phonological representations with more than two melody tiers on separate paddles are necessarily nonplanar. This argument is erroneous. Figure 1 has three independent paddles, but it is not homeomorphic to either K_5 or $K_{3,3}$, and can be portrayed in the plane with no lines crossing. Figure 5 is one possible plane embedding of Figure 1. All the other examples of three-paddle Autosegmental diagrams that we are aware of from the literature (with the exception of those in Figure 13 which we discuss below) also have plane embeddings. The graph of which Figure 5 is a plane embedding is in no way affected by the manner in which it is portrayed. Since it is *unchanged*, it retains all the structure of Figure 1, still supporting reference to all the relevant notions of locality (i.e., adjacency or association cf. Hoberman 1988) and accessibility as in Figure 1. Such an embedding is nothing other than a different way of looking at the same formal object.

The argument which Archangeli offers is one of only a few published attempts to establish the nonplanarity of Autosegmental Phonological Representations. However, Archangeli's hypothesis has been generally accepted by Autosegmental phonologists, presumably because it is undeniably *convenient* to use 3-D diagrams in Autosegmental Phonology. We will examine some more examples of 3-D diagrams taken from the litera-

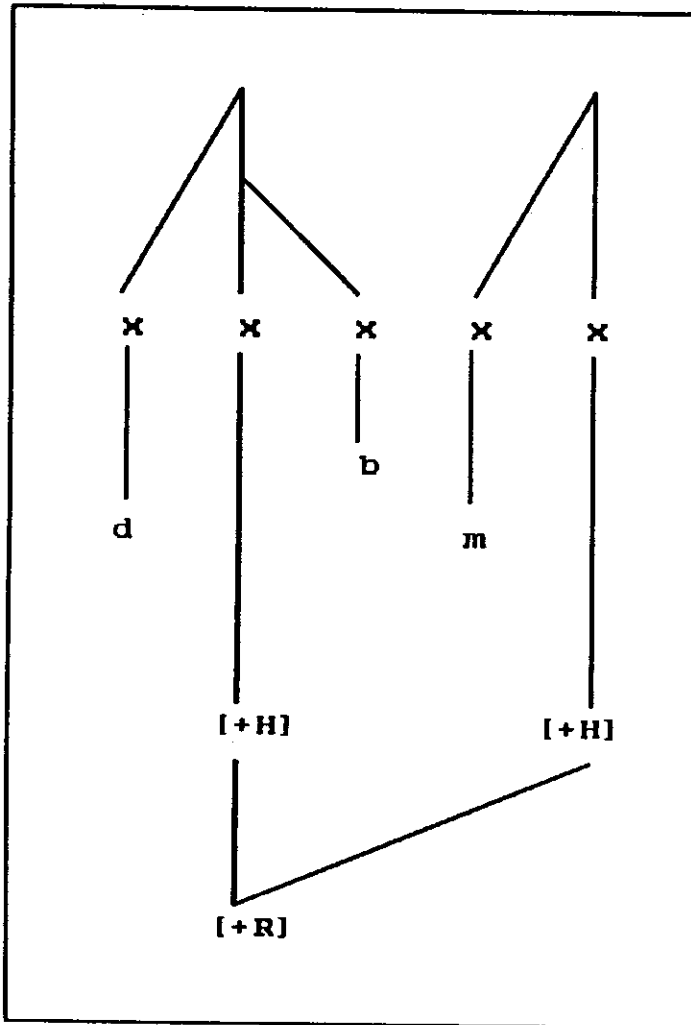


Fig. 5. A planar embedding of a paddlewheel graph.

ture and show why, like Archangeli's example, they do not portray *necessarily* nonplanar APRs.

Pulleyblank (1988). Like Halle and Vergnaud (1980), Pulleyblank (1988) subscribes to the Morphemic Tier Hypothesis, stating:

Following McCarthy (1981; 1986) among others, I assume that the melodic content of different morphemes enters the phonology on distinct tiers (planes).

But Pulleyblank's examples contain at most only two morphemes, and

thus all of them represent. None of Pulleyblank's examples are *necessarily* nonplanar.

Halle and Vergnaud (1980). Halle and Vergnaud (1980) argue that the morphemic tiers are not necessarily parallel planes, and that the morphemes are not necessarily on parallel planes (they are talking about mean nonplanar). Several examples are given in several respects. Below are two examples for the 3-D nature of the tiers explicitly presented. The first is the unsupported contour.

Autosegmental tiers can be represented by parallel lines. For example, in the word 'tune', the tiers for tone, stress, and the morphemes must correspond to the tone.

The conclusion that the tiers are parallel is unjustified. The tiers are not always parallel. It is attempted to demonstrate that two parallel lines

Since two parallel lines can be established, the tone plane is (1). (Halle and Vergnaud 1980: 100)

Two parallel lines can be established that do not establish the tone plane.

The next step in the analysis of tone, is autosegmentation.

We propose that the morphemic tiers are not necessarily parallel planes or call the tiers.

It is not clear and in the representation of the tiers, the tiers are not necessarily parallel.

thus all of them represent graphs which may be embedded in the plane. None of Pulleyblank's 'apparently' 3-D diagrams (44), (50) represent *necessarily* nonplanar graphs.

Halle and Vergnaud (1987). Halle and Vergnaud argue that Autosegmental Phonological Representations consist of several intersecting planes, and that they are therefore 'three dimensional' (by which, since they are talking about linguistic representations, not diagrams, they must mean nonplanar). We shall argue that their reasoning is defective in several respects. Because it is one of the few papers in which an argument for the 3-D nature of Autosegmental Phonological Representations is explicitly presented, we shall work through it very carefully, emphasizing the unsupported conclusions.

Autosegmental Phonology has made it clear that tones must be represented as a sequence of units (segments) that is separate and distinct from the sequence of phonemes – in other words, that in tone languages phonological representations must consist of two *parallel* lines of entities: the phonemes and the tones. (Halle and Vergnaud 1987, p. 45, our emphasis).

The conclusion that the sequence of phonemes and the sequence of tones are parallel is unsupported. It is true that in Autosegmental diagrams, tiers always are parallel, but no Autosegmental phonologist has ever even attempted to demonstrate that 'phonological representations *must* consist of two parallel lines'.

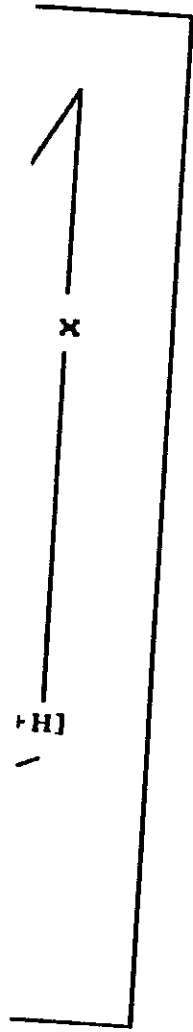
Since two parallel lines define a plane, we shall speak of the *tone plane* when talking about representations such as those in (1). (Halle and Vergnaud 1987, p. 45).

Two parallel lines do indeed define a plane, but Halle and Vergnaud have not established that associated tiers are parallel.

The next step in Halle and Vergnaud's argument is to show that stress, like tone, is autosegmental.

We propose to treat stress by means of the same basic formalism as tone – that is, by setting up a special autosegmental plane on which stress will be represented and which we shall call the *stress plane*. (Halle and Vergnaud 1987, p. 46).

It is not an accident that the bottom line both in the tone plane and in the stress plane is constituted by the string of phonemes representing the words. In fact, all autosegmental planes intersect in a single line, which as a first approximation may



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be viewed as containing the phoneme strings of the words. Autosegmental Phonological Representations are *therefore* three-dimensional objects of a very special type: they consist of a number of autosegmental planes (to be geometrically precise, half-planes) that intersect in a single line, the line of phonemes. (Halle and Vergnaud 1987, p. 46, our emphasis).

This displays the same false reasoning as Archangeli (1985), discussed above. The establishment of several independent tiers linked to a common core is *not* sufficient to prove that APRs are *necessarily* nonplanar. It is sufficient to motivate the use of 3-D diagrams for clarity of presentation, but expository convenience is not a relevant factor in assessing the nature of phonological representations.

We have argued that stress is represented on a separate plane from the rest of the phonological structure. It has been proposed elsewhere that other properties of morphemes, such as tone (Goldsmith 1976) and syllable structure (Halle 1985), are also to be represented on separate planes. Therefore, a morpheme will in general be represented by a family of planes intersecting in a central line. Given this formalization, the combination of morphemes into words will involve a combination of families of planes. (Halle and Vergnaud 1987, p. 54).

Even if we grant that the tiers in an APR are parallel, and therefore do indeed define a family of planes, it does not follow that such a family of planes defines a three-dimensional object. While it is conceptually simple to picture a family of planes as forming a three-dimensional object, it is geometrically quite possible for a family of planes to lie in the same two-dimensional planar space.

Halle and Vergnaud attempt to support their conception of phonological structure with the claim that:

McCarthy (1986) has proposed that the separate autosegmental planes of Semitic morphology are the result of the fact that distinct morphemes must be represented on separate planes – for example, as in (20). (Halle and Vergnaud 1987, p. 54)

But unlike Halle and Vergnaud (1987) and Goldsmith (1985), McCarthy's (1986) article contains no autosegmental representations that are even *apparently* nonplanar (and no 3-D diagrams), let alone *necessarily* nonplanar. The kind of 3-D diagram which Halle and Vergnaud present does not actually occur in any of McCarthy's works, McCarthy (1986) included.

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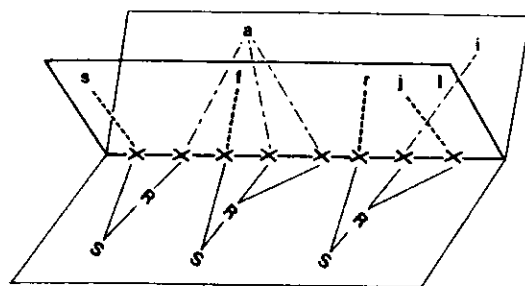


Fig. 6. A 3-D diagram.

The structure of Halle and Vergnaud's argument can be summarized as follows:

- (1) Autosegmental tiers are parallel to the skeleton.
- (2) Therefore, each tier defines a plane.
- (3) An Autosegmental Phonological Representation may contain several autosegmental tiers.
- (4) Therefore, an Autosegmental Phonological Representation consists of a family of intersecting (half-)planes.
- (5) Therefore, an Autosegmental Phonological Representation is a three-dimensional object.

Their argument does not go through, however, since the first proposition is unsupported and the final conclusion does not follow from the premisses.

Halle (1985). Although 3-D diagrams are rare, even in the works of such proponents of '3-D Phonology' as Halle and Vergnaud, Halle (1985) presents a 3-D diagram, a representation of the Arabic word *safaarij* 'quinces' (Figure 6). However, no subgraph of the graph represented in this diagram is homeomorphic to $K_{3,3}$ or K_5 , and thus Figure 6 portrays a planar graph.

Halle (1985) is clear that diagrams such as Figure 6 are not to be confused with the APRs that they denote. He states:

information about the phonic shape of the words is stored in a fluent speaker's memory in the form of a *three-dimensional object that for concreteness one might picture as a spiral-bound notebook*. (Halle 1985, p. 101, our emphasis).

I have tried to present a *picture* of this type of representation in Figure [6-JC & JL]. (Halle 1985, p. 112, our emphasis).

Moreover, Halle is clear that the diagrams of '3-D Phonology' are a

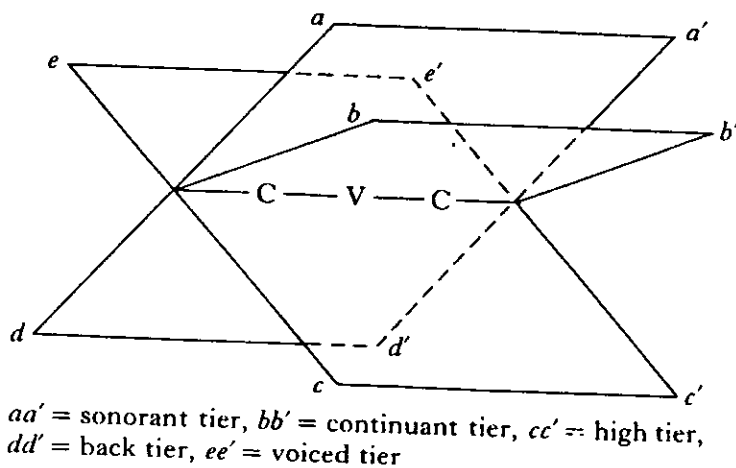


Fig. 7. 3-D diagram with skeletal core.

notation for Autosegmental Phonological Representations, rather than being the representations themselves:

there are no promising alternative *notations* to the multi-tiered autosegmental representation that has been described here. (Halle 1985, p. 112, our emphasis).

Yet, as we have demonstrated throughout this paper, arguments for the felicity or utility of 3-D diagrams, or in other words pictures of APRs, do not constitute evidence for the necessary nonplanarity of those representations.

Clements (1985). Clements (1985) examines two models of segmental organisation.

To clarify our ideas, it would be useful to contrast two possible models of multi-tiered feature representation, representing opposed views of hierarchical organisation. (Clements 1985, p. 227)

In the first model, a segment is a star-graph whose root node is a skeletal object, whose leaf nodes are autosegments, and whose edges are association lines. The sequence of leaf nodes in adjacent segments forms tiers, and the sequence of root nodes forms a skeletal tier (Figure 7). Phonological representations are thus:

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own tiers, and are linked to a common core or 'skeleton'. (Clements 1985, p. 227)

Clements uses the metaphor of an open book to describe such graphs:

In such a conception, a phonological representation resembles an open book, suspended horizontally from its ends and spread open so that its pages flop freely around its spine. The outer edge of each page defines a *tier*; the page itself defines a *plane*, and the spine corresponds to the *skeleton*. (Clements 1985, p. 228)

Each segment in this model is a star-graph consisting of a skeletal slot linked to the features which constitute that segment, each on its particular tier. The linear extension of a star-graph is a 'paddle-wheel' graph.

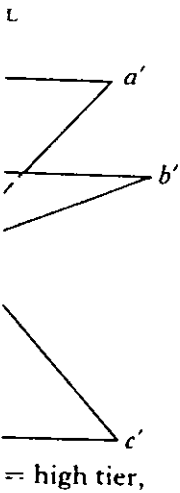
Clements contrasts this view with an alternative model in which each segment is not a star-graph but a tree-graph (Figure 8).

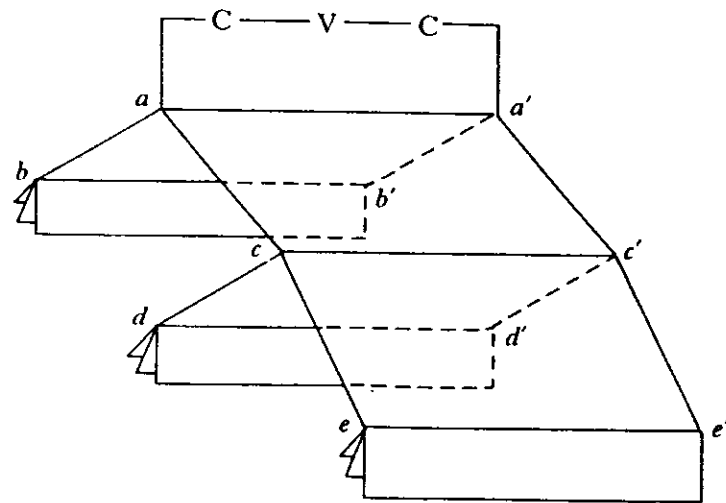
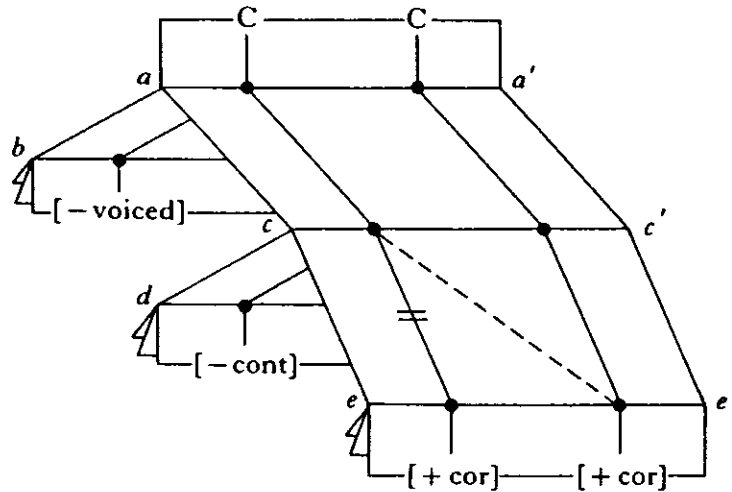
This conception resembles a construction of cut and glued paper, such that each fold is a class tier, the lower edges are feature tiers, and the upper edge is the CV tier. (Clements 1985, p. 229)

Like other writers, Clements provides a number of appealing arguments for using 3-D diagrams, and indeed offers empirical evidence in support of his position. But nowhere does he demonstrate that the evidence he musters explicitly proves that Autosegmental Phonological Representations are necessarily nonplanar graphs. All that he demonstrates is that it is convenient for expository reasons, simplicity etc. for APRs to be portrayed as multiplanar objects of a particular type.

Both of the models which Clements compares are capable of supporting the Single Feature Hypothesis of tier content, but the second, tree-structured model is not capable of supporting the Morphemic Plane Hypothesis, as it is a highly specific theory of tier content. To the extent that morphemes may be phonologically arbitrary, in the manner described by Prince (1987, p. 449) and discussed above, it is over-constrained as a theory of noncatenative morphology.

There are, of course, many theories of segmental organisation consistent with all the principles of Autosegmental Phonology, other than the two which Clements singles out for consideration. Goldsmith (1976, p. 159), in which segments are *chains* of Autosegments, or Pulleyblank (1986, p.



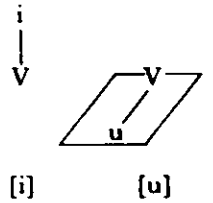


aa' = root tier, *bb'* = laryngeal tier, *cc'* = supralaryngeal tier, *dd'* = manner tier, *ee'* = place tier

Fig. 8. 3-D diagrams with tree-structured segments.

13),¹⁰ in which noncore tiers may be associated to other noncore tiers, are two attested alternatives, and others are possible. For instance, segmental structure might quite plausibly be represented by directed acyclic graphs, as in *Unification Phonology* (Broe 1988, Scobbie 1988, Local 1989, Bird and

¹⁰ Pulleyblank discusses, but does not subscribe to this view.



Klein 1990, Coleman 1990, and Pulleyblank 1985. The leaf association of Goldsmith (1985) was the first to do this. In fact, Goldsmith was first to do this. In fact, Goldsmith was first to do this. In fact, Goldsmith was first to do this. However, for purposes only, and

The seven segments can be created by the association of the skeletal structure of each of the tiers. In fact, Goldsmith (1985) sent four

In his conclusion,

if the segments are not correct, the representation of both of the tiers standing together come to the fore. (1985, p. 13)

But unlike Halle and Pulleyblank (1986), although the theory of Autosegmental Phonology (Vergnaud, Pulleyblank 1985, p. 13)

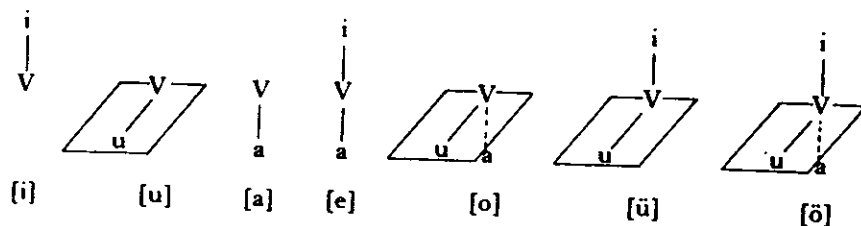


Fig. 9. The vowels of Khalkha Mongolian.

Klein 1990, Coleman 1990), *circuits* or *wheels* (star-graphs with leaf-to-leaf association cf. Wilson, 1985, p. 19).

Goldsmith (1985). In this paper, Goldsmith employs 3-D diagrams for the first time. (It is not clear whether Goldsmith, Halle or Clements was first to do this. In each case, the earliest publication of 3-D diagrams was in 1985.) However, Goldsmith (1985) uses 3-D diagrams for expository purposes only, and makes no theoretical claims about them. He states:

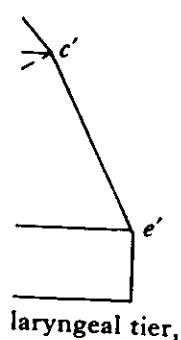
The seven vowels of Mongolian are the seven vowels that can be created by the combinations of the feature [*front*] (represented as [i]), the feature [*round*] ([u]), and the feature [*low*] ([a]). These combinations arise through the association of a skeletal position with segments on three distinct tiers, one for each of these three features. This is illustrated in [Figure 9—JC & JL], where I have attempted to use perspective to represent four distinct tiers. (Goldsmith 1985, p. 257)

In his conclusion, he states:

if the spirit of the analyses of Khalkha Mongolian, Yaka, Finnish, and Hungarian that are presented here is fundamentally correct, then the revisions of our conception of phonological representation that we must adapt to are far-reaching, affecting both our view of autosegmental geometry and our understanding of traditionally segmental features. We will have to come to grips with truly rampant autosegmentalism (Goldsmith 1985, p. 271)

But unlike Halle and Clements, Goldsmith does not claim that APRs are three-dimensional objects.

Pulleyblank (1986). There are no necessarily nonplanar graphs in Pulleyblank (1986), although he does present a few considerations on the topology of Autosegmental Phonological Representations. Like Halle and Vergnaud, Pulleyblank takes the view that:



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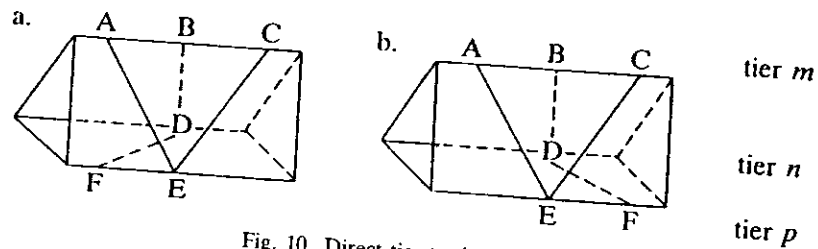


Fig. 10. Direct tier-to-tier linking.

Nasality may be represented on a separate tier, vowel harmony features may be autosegmentalized, etc. This means that a language may require several independent (but parallel) tiers in its phonological representation. (Pulleyblank 1986, p. 12)

Just like Halle and Vergnaud, Pulleyblank slips in the unsupported assertion that if several independent tiers are required, they must be parallel, an assumption which is crucial to the hypothesis that tiers are organised into planes.

Pulleyblank considers two types of nonplanar Autosegmental Phonological Representations. The first possibility which he considers is that each tier may be associated to any other. The only *formal* argument which Pulleyblank gives for rejecting this view is that tier-to-tier association can lead to contradictions in the temporal sequencing of autosegments. Commenting on Figure 10, his example (21), he says:

In (21), segments A and C have the value E on tier *p*; segment B, on the other hand, has the value F by virtue of the transitive linking B - D - F. But note that F *precedes* E in (21a), while it *follows* E in (21b)! In other words, the representation in (21) has as a consequence that the temporally ordered sequence EF is nondistinct from the sequence FE. (Pulleyblank 1986, p. 13)

This argument is flawed because Association *cannot* be a transitive relation. If association were transitive, then the temporal interpretation of contour segments and geminates would be logically paradoxical (Sagey 1988, p. 110).

Furthermore, if temporal sequence is determined by the order of core elements, nonskeletal sequence is redundant. If nonskeletal tiers are unordered, then the apparent problem which Pulleyblank identifies vanishes.

Pulleyblank proposes an alternative type of nonplanar representation, paddle-wheel graphs, by adopting the restriction that Autosegmental tiers can only link to slots in the skeletal tier. He claims that the effect of this constraint is a 'considerably more restrictive multi-tiered theory', a claim

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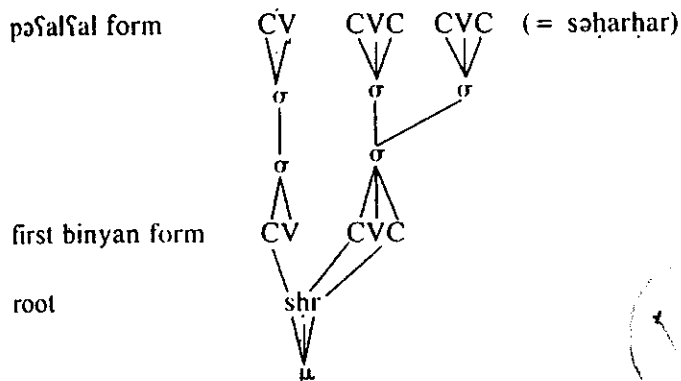
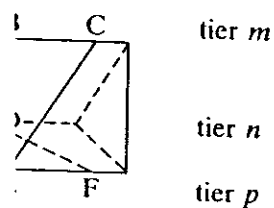


Fig. 11. A diagram with two CV 'cores'.

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which we challenged above.¹¹ The only example of a 3-D diagram of a paddle-wheel graph which Pulleyblank presents includes no association lines at all, and it is therefore (trivially) planar.

McCarthy (1981). McCarthy (1981) includes none of the apparently 3-D diagrams of his earlier thesis (McCarthy 1979/1982), although the material in this paper is an abridged version of parts of that work. The framework is that of *n-tiered autosegmental phonology* without organisation into planes à la Halle and Vergnaud. In fact, quite contrary to Halle and Vergnaud, McCarthy has diagrams such as Figure 11 (McCarthy 1981, p. 409 Figure 53) in which the CV 'core' occurs twice, in order to show the morphological correspondence between the first binyan and pəʕalʕal (a partially reduplicative form) in Hebrew.

Halle and Vergnaud (1980). Although Halle and Vergnaud do not present any 3-D Autosegmental diagrams, they argue that 'the phonological representation is a three dimensional object' (Halle and Vergnaud 1980, p. 101) in the following manner.

Its core is constituted by a linear sequence of slots – the skeleton. Each morpheme of the word is represented by a sequence of distinctive feature complexes . . . the MELODY. (Halle and Vergnaud 1980, p. 101)

They accept the proposals of Autosegmental phonologists concerning the

¹¹ The theory which results is considerably more restricted, but that is a different matter. Generative grammars of a particular type are certainly made considerably more restricted if their nonterminal symbols must all be words over the Cyrillic alphabet, but no more restrictive for all that.

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conditions which govern the linking of melody tiers to the skeleton, and claim that:

The lines that link the melody with skeleton define a plane. Thus, the phonological representation of a word contains as many planes as there are morphemes in the word. (Halle and Vergnaud 1980, p. 101)

This argument suffers from the same logical fallacy as that of Archangeli (1985): the (undisputed) clarity of presentation afforded by drawing subsets of the association relation over each individual morpheme's melody and the skeleton does not amount to a *proof* that planar Autosegmental Phonological Representations are formally inadequate. Furthermore, as we argued above, without restrictions on what phonological material can constitute a morpheme, the departure from planar representations which Halle and Vergnaud support diminishes the force of the NCC to the extent that it ceases to be restrictive.

McCarthy (1979/1982). McCarthy's (1979/1982) thesis extended Goldsmith's (1976) Autosegmental theory of tonal phenomena to the nonconcatenative morphology of Semitic languages. There are no diagrams in this thesis which are even apparently 3-D, and nowhere in the text is the possibility of multiplanar (as opposed to multi-tiered) representations raised, although five of McCarthy's examples might, with generosity, be taken as attempting to portray Autosegmental Phonological Representations using perspective. These are reproduced in Figure 12. Even if these examples are taken to be 3-D diagrams, they do not portray nonplanar graphs. Since they are all drawn on a plane surface with no crossing lines, they all portray planar graphs.

Goldsmith (1976). Goldsmith (1976) concentrates on two-tier Autosegmental Phonological Representations, those with just a phoneme tier and a tone tier. He considers extending this formalism to multi-tiered Autosegmental Phonological Representations (of which he presents a planar example portrayed in a 2-D diagram), but does not raise the possibility of 3-D diagrams or nonplanar Autosegmental Phonological Representations.

Our examination of the Autosegmental Phonology literature has shown that the common belief that a necessarily nonplanar Autosegmental Phonological Representation already exists is mistaken. This is surprising, for under the conventional assumption concerning the universality and homogeneity of language, demonstration that just one Autosegmental Phonological Representation is necessarily nonplanar is necessary and sufficient

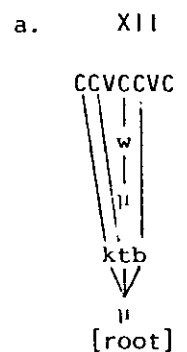


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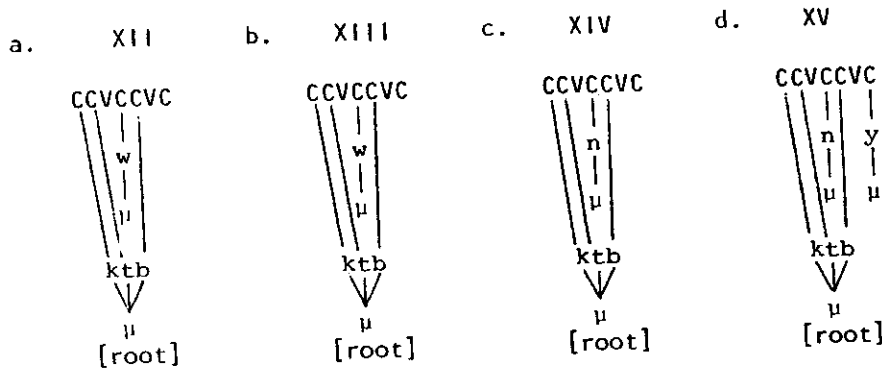
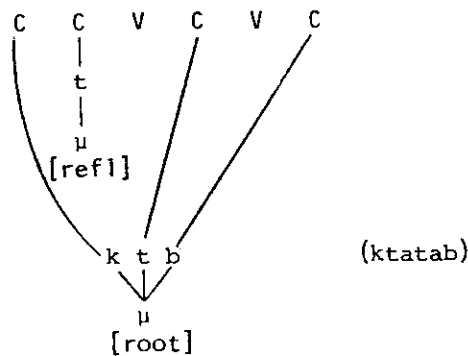


Fig. 12. Autosegmental Phonological Representations of Arabic.

for rejection of planar Autosegmental Phonology and 2-D diagrams as inherently too restrictive.

One such APR has in fact been portrayed in the Autosegmental literature (there are perhaps others too), in Wetzels (1986). In the course of an Autosegmental derivation Wetzels gives a few diagrams of Autosegmental Phonological Representations which contain $K_{3,3}$ as a subgraph. The $K_{3,3}$ subgraphs of Figure 13 are shown in Figure 14. However, Wetzels's example is not presented as a proof that Autosegmental Phonological Representations are nonplanar, and he does not remark on the fact that his examples are nonplanar graphs. Furthermore, since Wetzels's example is from Classical Greek, and is therefore not amenable to verification, there being no living speakers of Classical Greek, and since his analysis may therefore be questioned, as a demonstration of nonplanarity it is not as uncontentious as is desired for a result to be established. In the next section, we shall present a contemporary (i.e., falsifiable) example of an Autosegmental Phonological Representation which is necessarily non-

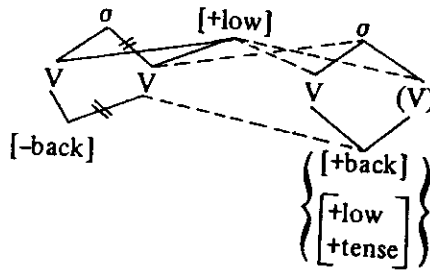
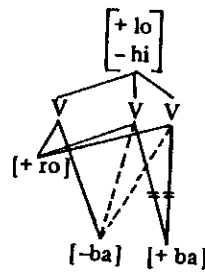
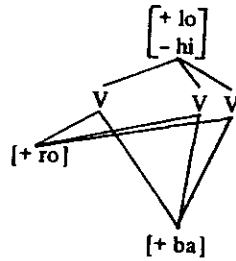


Fig. 13. Nonplanar Autosegmental Phonological Representations.

planar. We shall establish the necessity of 3-D diagrams in Autosegmental Phonology by presenting an Autosegmental Phonological Representation which is homeomorphic to $K_{3,3}$. Whether we are successful or not in advancing this example does not affect the general argument concerning the vacuity or nonrestrictiveness of the NCC. And even if our example is falsified by subsequent evidence, the nonplanarity of APRs will remain to be proved.

Consider a path between three autosegmental nodes portrayed with $K_{3,3}$.

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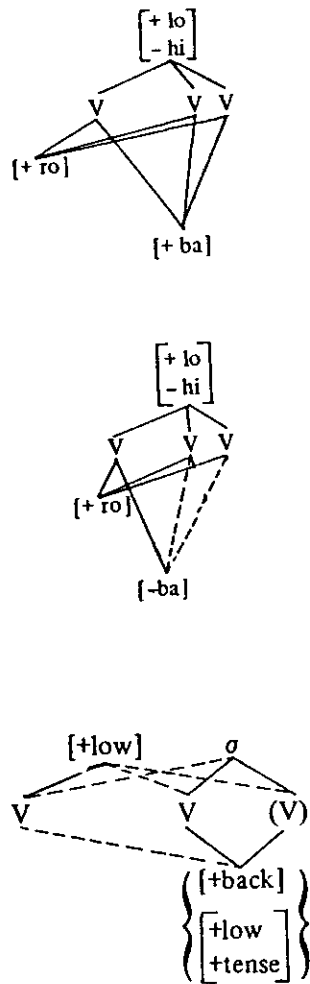


Fig. 14. $K_{3,3}$ subgraphs of Figure 13.

5. A NECESSARILY 3-D DIAGRAM

Consider a phonological representation with three anchor units on one tier, three autosegments on one or more other tiers, and a line of association between each anchor and each autosegment. Such a graph cannot be portrayed without crossings on a plane surface, since it is homeomorphic to $K_{3,3}$.

There is no linguistic reason why such a representation might not be motivated in certain cases. Wetzels's example (Figure 13) is one such case.

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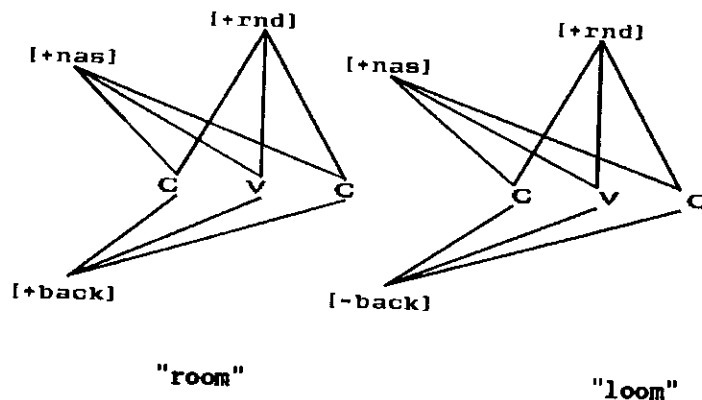


Fig. 15. Necessarily nonplanar Autosegmental Phonological Representations.

Two more are illustrated in Figure 15, which shows the distribution of backness, rounding and nasality over three timing units in the pronunciation of the words *room* and *loom* by a Guyanese English speaker. Both of the graphs portrayed in Figure 15 are homeomorphic to $K_{3,3}$, and thus they are necessarily nonplanar. In order to demonstrate that the graphs portrayed in Figure 15 are the correct Autosegmental Phonological Representations of the two words, we must establish that the features [back], [round], and [nasal] are indeed independent autosegmental features. We shall demonstrate that this is so by showing that they are lexically associated with independent segments, and therefore spread independently. For this to be the case, they must lie on independent tiers. Before we demonstrate this, we shall briefly explain the way in which derivational steps are notated in Autosegmental Phonology.

The two basic representation-altering operations of Autosegmental Phonology are the addition of association lines to Autosegmental Phonological Representations and the deletion of association lines from Autosegmental Phonological Representations. Association lines which are added to a representation are portrayed as dotted lines. Thus Figures 16f and 18c denote representations to which a single association line has been added, and Figures 16ae, 18a,b and 19 denote representations to which two association lines have been added. Where the addition of association lines to a representation incrementally 'links' a single item on one tier to successive objects on another tier, the single item is said to *spread*. There are no instances of deletion in this example, so we shall not discuss it further.

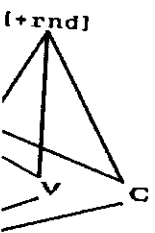
We shall argue in detail that comparison of similar words with the same general phonological shape, such as *tomb*, *root*, *loot* and so on, shows

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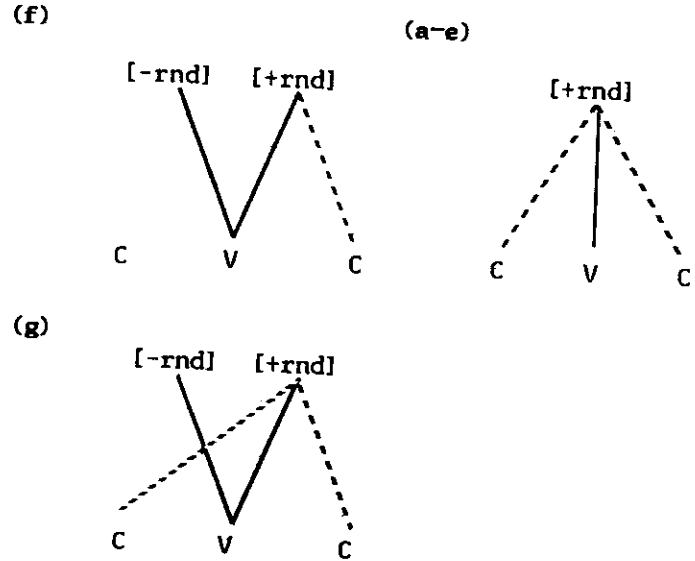


Fig. 16. Rounding is autosegmental in Guyanese.

that the spread of backness, rounding and nasality is clearly phonologically distinctive, and cannot simply be attributed to automatic phonetic coarticulation effects. Consider the transcriptions in Figure 17.¹²

5.1. Rounding

Along with the proponents of Autosegmental Phonology, we regard it as uncontentious that there is a feature of liprounding (under whatever name) which is a primary articulation of vowels and a secondary articulation of consonants. Comparison of the unrounded onset consonant in Figure 17f with the rounded onset consonants in Figure 17a-e and of the rounded coda consonants in Figure 17a-f with the unrounded coda consonants in Figure 17g-i shows that the spread of rounding from rounded vowels to neighbouring consonants is not an automatic coarticulatory effect in this language, but is a phonologically principled phenomenon.

The nucleus /u/ has four variants [u], [yu], [œy] and [ey]. Of these, the

¹² The original observations were made by John Local and John Kelly of the speech of Kean Gibson, a native speaker of Guyanese English. Their transcriptions were made using the symbols of the IPA before its revision in 1989. We have changed the way in which velarization is transcribed to accord with the revised IPA (International Phonetic Association 1989). For readability, we have removed some diacritics denoting contextual devoicing of [j] in *feud* and *cube*, slight implosion of final voiced stops, and coarticulatory advancement of the [k] in *cube*. Further details of the phonology of such words and a great many more examples are to be found in Kelly and Local (1989, pp. 218-241).

a) room	$[\bar{v}^{\gamma} \bar{u}^{\gamma} m^{\gamma}]$
b) loom	$[l^{\gamma} \bar{y}^{\gamma} m^{\gamma}]$
c) zoom	$[z \bar{y}^{\gamma} \bar{u}^{\gamma} m^{\gamma}]$
d) root	$[v^{\gamma} \gamma u^{\gamma} r^{\gamma}]$
e) loot	$[l^{\gamma} \alpha y^{\gamma} r^{\gamma}]$
f) lute	$[l^{\gamma} e y^{\gamma} r^{\gamma}]$
g) rule	$[\bar{v}^{\gamma} \bar{u}^{\gamma} l^{\gamma}]$
h) feud	$[f^{\gamma} j e y d^{\gamma}]$
i) cube	$[k j e y b^{\gamma}]$
j) rip	$[\bar{v}^{\gamma} r^{\gamma} p^{\gamma}]$
k) red	$[\bar{v}^{\gamma} e d^{\gamma}]$

Superscript $\bar{\quad}$ and γ denote palatalized and velarized articulations respectively. γ denotes a close-mid, back, unrounded vowel. Superscript $\bar{\quad}$ denotes nasality. Subscript ω denotes lip-rounding. v denotes a voiced labiodental frictionless continuant, this speaker's version of /r/.

Fig. 17. Transcriptions of Guyanese English words.

first two are [+back] and the second two are [-back]. Three of these variants are diphthongs: mid-vowels changing to close-vowels. However, we shall not attend to the representation of vowel-height here. The back diphthong [γu] also changes from unrounded to rounded. The front diphthong [$e y$] also changes from unrounded to rounded, whereas the front diphthong [$\alpha e y$] is rounded throughout.

Comparison of Figure 17a, in which the coda is rounded, with Figure 17g, 17h and 17i, in which the codas are not rounded even though a rounded nucleus precedes, demonstrates that coda rounding is not an automatic coarticulatory effect, but is a phonologically principled phenomenon. In accordance with Autosegmental Phonology's preference for autosegmental analyses of feature-spreading, the perseverative rounding of the

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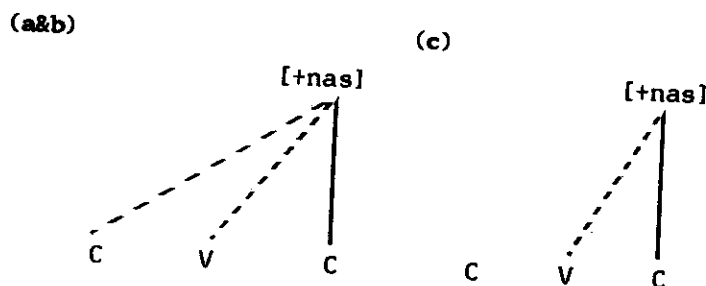


Fig. 18. Nasality is autosegmental in Guyanese.

coda in Figure 17a must be attributed to the spreading of the autosegmental feature [rnd] from the nucleus.

There is no *phonetic* reason why rounding should not spread from the second vocalic element in Figure 17f to the first vocalic element and thence to the initial consonant. Consequently, something must block the spread of rounding to the onset of Figure 17f. There is no reason to regard the onset itself as the locus of this blocking: Figure 17b and 17e show that rounding is sometimes found with palatalized lateral onsets. Thus it must be the first vocalic element which blocks the forward spread of rounding in Figure 17f.¹³ The only nonarbitrary way of blocking such spreading within Autosegmental Phonology is to propose the presence of an adjacent autosegment which is associated to the skeletal tier in such a way that the NCC would be violated if the spreading continued further. Thus it is not possible to derive Figure 16g from Figure 16f. (This analysis also demonstrates that [rnd] is Autosegmental even if there are two V units in the Autosegmental representation of Figure 17f.)

5.2. Nasality

A comparison of Figure 17c with Figure 17a and 17b shows that nasality spreads from nasal coda consonants to vowels (an uncontroversial analysis) and thence to 'liquid' sonorants /l/ and /r/ (i.e., [ɹ]). The absence of nasality in the onset of *zoom* (as well as *soon*, which behaves similarly) shows that this spreading is phonologically conditioned (Figure 18).

5.3. Backness

The feature [\pm back] is used to denote the tongue-body opposition of front vs. back vocalic resonance. With respect to consonants, the feature

¹³ Thus, our Autosegmental analysis of *loot* and *lute* parallels the standard segmental analysis of English, in which the nucleus /u/ is rounded throughout, whereas the nucleus /ju/ is inherently diphthongal: first unrounded, then rounded.

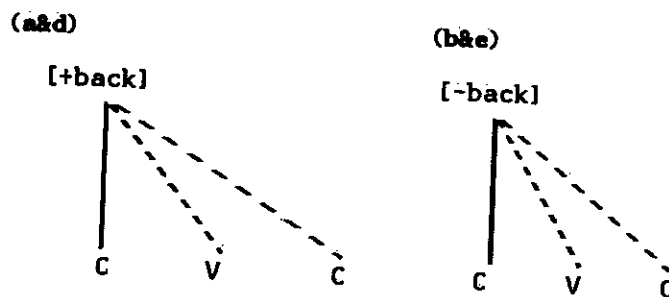


Fig. 19. Backness is autosegmental in Guyanese.

$[\pm\textit{back}]$ denotes the *secondary articulation* of the consonant, i.e., “palatalized” or “clear” $[-\textit{back}]$ vs. “velarized” or “dark” $[\textit{+back}]$.

A comparison of Figure 17a with 17b and 17d with 17e shows that the ‘liquid’ onsets /l/ and /r/ are systematically associated with the feature $[\pm\textit{back}]$, as is characteristic of many varieties of English (cf. Kelly and Local 1986; 1989, pp. 74, 218–241). In this variety, /l/ is $[-\textit{back}]$ and /r/ is $[\textit{+back}]$. Although the secondary articulation of onset consonants (notably obstruents) in English is attributable to the features of the vocalic nucleus, this is not the case with liquid onsets. The $[-\textit{back}]$ liquid remains $[-\textit{back}]$ before systemically $[\textit{+back}]$ vowels (Figure 17b) and the $[\textit{+back}]$ liquid remains $[\textit{+back}]$ before systemically $[-\textit{back}]$ vowels (Figure 17j, k). The distinctive association of liquid onsets with $[\pm\textit{back}]$ affects the nucleus and coda too, resulting in advanced vowel qualities and palatalized codas with the ‘clear’ liquid (Figure 17b), and retracted vowel qualities and velarized codas with the ‘dark’ liquid (Figure 17a). The spread of $[\pm\textit{back}]$ as far as the coda only applies in the case of coda consonants which are not themselves lexically associated to $[\pm\textit{back}]$. In the case of liquid codas, of course, spreading of $[\pm\textit{back}]$ from the nucleus is sometimes blocked (Figure 17g) viz. when the liquid’s value of $[\pm\textit{back}]$ is contrary to that of the nucleus. Thus $[\pm\textit{back}]$ is an autosegmental feature of liquids which in Figure 17a and 17b spreads from the onset to the nucleus and thence to the coda (Figure 19). In the terms of Autosegmental Phonology, it is clear in the analysis of Figure 17a and 17b that

- $[\pm\textit{rnd}]$, $[\pm\textit{back}]$ and $[\pm\textit{nas}]$ must be autosegments on separate tiers;
- liquid onsets are lexically associated with $[\pm\textit{back}]$, the nucleus with $[\pm\textit{rnd}]$, and the coda with $[\pm\textit{nas}]$; and
- these three autosegments then spread to each of the other syllable terminals, as in Figure 17a-e, Figure 18a, b, and Figure 19 to produce the Autosegmental Phonological Representations portrayed in Figure 15.

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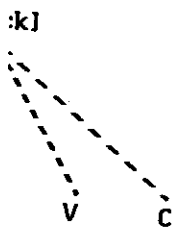
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These interacting principles are each widely exemplified in several varieties of English, and although we present only a handful of critical examples here, many more may be found in Kelly and Local (1986, 1989).

We have thus shown that planar graphs are not in general adequate for Autosegmental Phonological Representations of Guyanese English, because the Autosegmental Phonological Representations of *room* and *loom* cannot be planar. Given that the principles which interact to produce this result are not particularly special and are individually attested elsewhere, we have no reason to believe that Guyanese English is either unnatural or special in this respect, and thus planarity (i.e., the No Crossing Constraint) is too severe to be a universal constraint on Autosegmental graphs.

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6. CONCLUSION

Since Autosegmental Phonology is necessarily nonplanar, the No Crossing Constraint has no force, because all graphs, however complex, can be portrayed in 3-D diagrams without edges crossing. The fact that some versions of Autosegmental Phonology employs 'paddle wheel' graphs, rather than unrestricted (i.e., Euclidean) graphs, does not affect this result. We conclude that the No Crossing Constraint is not a constraint at all, since it either incorrectly restricts the class of phonological graphs to planar graphs, or else it carries no force.

We have demonstrated that APRs are graphs, and concluded that the NCC does not restrict the class of such graphs. The conclusion follows from the definition of APRs advanced by Autosegmental Phonologists from the inception of Autosegmental Phonology. To avoid this conclusion, those fundamental definitions would have to be changed.

It could be argued that Goldsmith's formalization of APRs on which our argument is based is wrong-headed from the start, as APRs are not formal objects at all, but diagrams, metaphors, merely pictures of 'convenient fictions', and that we take Autosegmental Phonologists too literally in their use of the terms "point", "line", "sequence", "set", "parallel", "plane", that such terms have been borrowed from mathematical vocabulary as suggestive, rather than literal, terms. According to that view, our argument does not hold because its premises (that APRs are graphs of the Association relation) are unsound. We accept that this is a possible position to adopt, but if this position is adopted, Autosegmental Phonology is beyond the reach of rigorous investigation.

Other formalizations than Goldsmith's are possible, of course, but any that holds Association to be a relation (i.e., a graph) and tiers to be

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sets of arbitrary phonological objects must necessarily recapitulate our conclusion. In order to advance this line of enquiry further, we make the following observations:

- (1) Although the NCC is not a "syntactic" constraint, being a necessary consequence of the temporal interpretation of APRs, it is a constraint on the semantic interpretation of APRs.
- (2) The NCC is an informal way of stating that the mapping between two tiers induced by the Association relation is sequence-preserving. This goes back to Goldsmith's original formal definition, in which it is \triangleleft which ensures sequence-preservation.
- (3) Autosegmental Phonology therefore makes a twofold claim: firstly that phonological representations are divided into tiers, secondly that the tiers are sequenced. The status of the timeline thus deserves further investigation.
- (4) The NCC could be revised as follows: "Within a chart, Association lines may not cross." (cf. Kitagawa 1987). This alteration to Autosegmental Phonology is not sufficient by itself to rescue the NCC, however, since Plane Reduction¹⁴ (Halle and Vergnaud 1987, p. 55) may still derive a chart which violates the revised NCC from two charts which independently adhere to it.

~~WITHIN 2-TIERS~~
 WITHIN 2-TIERS
 (A CHART) ~~...~~
 A₁ A₂ A₃ A₄ A₅ A₆ A₇ A₈ A₉ A₁₀
 A₁ A₂ A₃ A₄ A₅ A₆ A₇ A₈ A₉ A₁₀
 - x x x i
 - v d j
 - b k k
 + h i l

the NCC holds

Our result concerning the non-restrictiveness of the NCC does not condemn the Autosegmental approach to phonological representation. However, it does mean that the NCC cannot be invoked as a constraining "principle" of Autosegmental theories.

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¹⁴ This operation, originally called *Tier Conflation* by McCarthy (1986), is chart conflation, according to Goldsmith's terminology (cf. p. 8).

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The purpose of this paper is to show that the condition on Pullum's (1985) of Higginbotham's constructions in I be interested in the case for such an on whether such noncontext-free, the conditions or nontrivial and co

In the paper, intersecting English

$$(1) \quad R = \{ \bar{1} \}$$

The result was clear show that English under intersection

$$(2) \quad B = \{ \bar{1} \}$$

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The idea behind in R must meet enforced by a cc with the man m and vice versa (c not belong to a perplexing condi

* I thank Daniel Ra