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# A Silicon-based Nuclear Spin Quantum Computer

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**An architecture for a quantum mechanical computer is presented in which information is encoded onto nuclear spins situated on donors in silicon electronic devices. Logical operations on individual spins are performed using externally applied electric fields, and spin measurement is accomplished using currents of spin polarised electrons. A Si quantum computer, built using technologies being developed for future conventional electronics, can potentially be scaled to sizes large enough that the extraordinary promise of quantum computation can be realised.**

While the concept of information underlying all modern computer technology is essentially classical, physicists know that nature obeys the laws of quantum mechanics. The idea of a quantum computer has been developed theoretically over several decades to elucidate fundamental questions concerning the capabilities and limitations of machines in which information is treated quantum mechanically [1][2]. Specifically, in quantum computers the 1's and 0's of classical digital computers are replaced by the quantum state of a two level system (a qubit). Logical operations carried out on the qubits and their measurement to determine the result of the computation must obey quantum mechanical laws. Quantum computation can in principle only occur in systems which are almost completely isolated from their surrounding environment and which consequently must dissipate no energy during the process of computation, conditions which are extraordinarily difficult to fulfill in practise.

Interest in quantum computation has increased dramatically in the past four years because of two important insights: firstly, quantum algorithms (most notably for prime factorisation [3][4] and for exhaustive search [5]) have been developed that outperform the best known algorithms doing the same tasks on a classical computer. These algorithms require that the internal state of the quantum computer be controlled with extraordinary precision, so that the coherent quantum state upon which the quantum algorithms rely is not destroyed. Because completely preventing decoherence (uncontrolled interaction of a quantum system with its surrounding environment) is impossible, the existence of quantum algorithms does not prove that they can ever be implemented in a real machine.

The second critical insight has been the discovery of quantum error correcting codes which enable quantum computers to operate despite some degree of decoherence and which may make quantum computers experimentally realisable [6][7]. The tasks that lay ahead to create an actual quantum computer are formidable: Preskill [8] has estimated that a quantum computer operating on $10^6$ qubits with a $10^{-6}$ probability of error in each operation would exceed the capabilities of contemperary conventional computers on the prime factorisation problem. To make use of error correcting codes, logical operations and measurement must be able to proceed in parallel on qubits throughout the computer.

The states of spin 1/2 particles are two level systems that can potentially be used for quantum computation. Nuclear spins have been incorporated into several quantum computer proposals [9][10][11], because they are extremely well isolated from their surrounding environment, and consequently operations on nuclear spin qubits could have low error rates. The primary challenge in using nuclear spins in quantum computers lies in measuring the spins. The bulk spin resonance approach to quantum computation proposed by Gershenfeld and Chuang [11] circumvents the single spin detection problem essentially by performing quantum calculations in parallel in a large number of molecules and determining the result from macroscopic magnetisation measurements. The measurable signal decreases exponentially with the number of qubits, however, and is unlikely to be scalable beyond several tens of qubits.

To attain the goal of a $10^6$ qubit quantum computer, several authors have suggested that a "solid state" approach [12] might eventually replicate the enormous success of modern electronics fabrication technology. An attractive alternative approach to nuclear spin quantum computation is to incorporate nuclear spins into an electronic device and to detect the spins and control their interactions electronically [13]. Electron and nuclear spins are coupled by the hyperfine interaction [14]. Under appropriate circumstances polarisation is transferred between the two spin systems and nuclear spin polarisation is detectable by its effect on the electronic properties of a sample [15][16]. Electronic devices for both generating and detecting nuclear spin polarisation, implemented at low temperatures in GaAs/$Al_xGa_{1-x}As$ heterostuctures, have been developed by the author [17], and similar devices have been incorporated into nanostructures [18][19]. While the number of spins probed in the nanostructure experiments is still large ($\sim 10^{11}$ [18]), sensitivity will improve in optimised devices and in systems with larger hyperfine interactions.

Presented here is a architecture for a quantum computer implemented on an array of nuclear spins located on donors in silicon: the semiconductor used in most conventional computer electronics. Logical operations and measurements can in principle be performed independently and in parallel on each spin in the array. I describe specific electronic devices for the manipulation and measurement of nuclear spins, fabrication of which will require significant advances in the rapidly moving field of nanotechnology. While it is likely that scaling the devices proposed here into a computer of the size envisioned by Preskill will be an extraordinary challenge, a Si-based quantum computer is in a unique position to benefit from the resources and ingenuity being directed towards making conventional electronics of ever smaller size and increasing complexity.

## I. Quantum Computation with a $^{31}$P Array in Silicon

The strength of the hyperfine interaction is proportional to the probability density of the electron wave function at the nucleus. In semiconductors, the electron wave function extends over large distances through the crystal lattice. Two nuclear spins can consequently interact with the same electron, leading to electron mediated or indirect nuclear spin coupling [14]. Because the electron is sensitive to externally applied electric fields, the hyperfine interaction and electron mediated nuclear spin interaction can be controlled by voltages applied to metallic gates in a semiconductor device, enabling the external manipulation of nuclear spin dynamics that is necessary for quantum computation.

The conditions required for electron-coupled nuclear spin computation and single nuclear spin detection can arise if the nuclear spin is located on a positively charged donor in a semiconductor host. The electron wave function is then concentrated at the donor nucleus (for $s$-orbitals and energy bands composed primarily of them), yielding a large hyperfine interaction energy. For shallow level donors, however, the electron wave function extends tens or hundreds of Å away from the donor nucleus, allowing electron-mediated nuclear spin coupling to occur over comparable distances. The quantum computer proposed here is comprised of an array of such donors positioned beneath the surface of a semiconductor host (Fig. 1). A quantum mechanical calculation proceeds by the precise control of three external parameters: (1) gates above the donors control the strength of the hyperfine interactions and hence the resonance frequency of the nuclear spins beneath them; (2) gates between the donors turn on and off electron mediated coupling between the nuclear spins [12]; (3) a globally applied AC magnetic field $B_{AC}$ flips nuclear spins resonant with the field. Custom adjustment of the coupling of each spin to its neighbors and to $B_{AC}$ enables different operations to be performed on each of the spins simultaneously. Finally, measurements are performed by transferring nuclear spin polarisation to the electrons and determining the electron spin state by its effect on the orbital wave function of the electrons, which can be probed using capacitance measurements between adjacent gates.

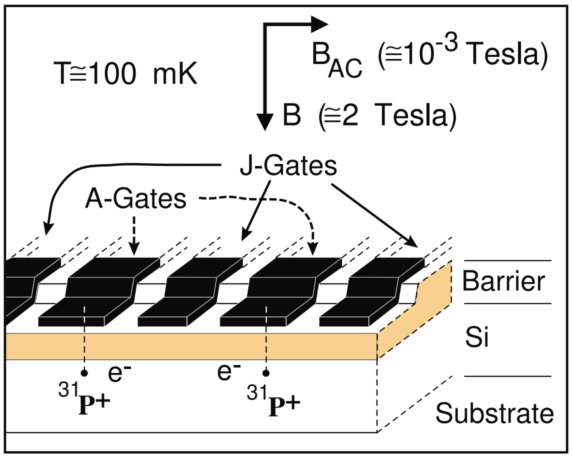


**Fig. 1: Illustration of two cells in a 1D array containing $^{31}$P donors and electrons in a Si host, separated by a barrier from metal gates on the surface. "$A$-gates" control the resonance frequency of the nuclear spin qubits, while "$J$-gates" control the electron-mediated coupling between adjacent nuclear spins. The ledge over which the gates cross localizes the gate electric field in the vicinity of the donors.**

An important requirement for a quantum computer is to isolate the qubits from any degrees of freedom that may lead to decoherence. If the qubits are spins on a donor in a semiconductor, nuclear spins in the host are a large reservoir with which the donor spins can interact. Consequently, the host should contain only nuclei with spin $I=0$. This simple requirement unfortunately eliminates all III-V semiconductors as host candidates, since none of their constituent elements possess stable $I=0$ isotopes [20]. Group IV semiconductors are composed primarily of $I=0$ isotopes and can in principle be purified to contain only $I=0$

isotopes. Because of the advanced state of Si materials technology and the tremendous effort currently underway in Si nanofabrication, Si is the obvious choice for the semiconductor host.

The only $I=1/2$ shallow (Group V) donor in Si is $^{31}$P. The Si:$^{31}$P system was exhaustively studied forty years ago by Feher in the first electron-nuclear double resonance experiments [21][22][23]. At sufficiently low $^{31}$P concentrations at temperature $T$=1.5 K, Feher observed that the *electron* spin relaxation time was thousands of seconds and the $^{31}$P nuclear spin relaxation time exceeded 10 hours. It is likely that at millikelvin temperatures the phonon limited $^{31}$P relaxation time is of order $10^{18}$ seconds [24], making this system ideal for quantum computation.

The purpose of the electrons in the computer is to mediate nuclear spin interactions and to facilitate measurement of the nuclear spins. Irreversible interactions between electron and nuclear spins must not occur as the computation proceeds: the electrons must be in a nondegenerate ground state throughout the computation. At sufficiently low temperatures, electrons only occupy the lowest energy bound state at the donor, whose two-fold spin degeneracy is broken by an applied magnetic field $B$. (The valley degeneracy of the Si conduction band is broken in the vicinity of the donor [25]. The lowest donor excited state is approximately 15 meV above the ground state [23].) The electrons will only occupy the lowest energy spin level when $2\mu_B B \gg kT$, where $\mu_B$ is the Bohr magneton. (In Si, the Landé g-factor is very close to +2, so g=2 is used throughout this discussion.) The electrons will be completely spin polarised ($n_\uparrow/n_\downarrow < 10^{-6}$) when $T \leq$100 mK and $B \geq 2$ Tesla. A quantum mechanical computer is non-dissipative and can consequently operate at low temperatures. Dissipation will arise external to the computer from gate biasing and from eddy currents caused by $B_{AC}$, and during polarisation and measurement of the nuclear spins. These effects will determine the minimum operable temperature of the computer. For this discussion, I will assume $T$=100 mK and $B$=2 T. Note that these conditions do not fully polarise the nuclear spins, which are instead aligned by interactions with the polarised electrons.

## II. Magnitude of Spin Interactions in Si:$^{31}$P

The size of the interactions between spins determines both the time required to do elementary operations on the qubits and the separation necessary between donors in the array. The Hamiltonian for a nuclear spin-electron system in Si, applicable for an $I$=1/2 donor nucleus and with $B \parallel z$ is:

$$H_{en} = \mu_B B\sigma_z^e - g_n\mu_n B\sigma_z^n + A\sigma^e \cdot \sigma^n, \tag{1}$$

where $\sigma$ are the Pauli spin matrices (with eigenvalues $\pm$ 1), $\mu_n$ is the nuclear magneton, $g_n$ is the nuclear g-factor (=1.13 for $^{31}$P [20]), and $A = \frac{8}{3}\pi\mu_B g_n\mu_n|\Psi(0)|^2$ is the contact hyperfine interaction energy, with $|\Psi(0)|^2$ the probability density of the electron wave function evaluated at the nucleus. If the electron is in its ground state, the frequency separation of the nuclear levels is, to second order:

$$h\nu_A = 2g_n\mu_n B + 2A + \frac{2A^2}{\mu_B B}. \tag{2}$$

In Si:$^{31}$P, $2A/h$= 58 MHz, and the second term in Eq. (2) exceeds the first term for $B < 3.5$ T.

An electric field applied to the electron-donor system shifts the electron wave function envelope away from the nucleus and reduces the hyperfine interaction. The size of this shift, following estimates of Kohn [25] of shallow donor Stark shifts in Si, is shown in Fig. 2 for a donor 200 Å beneath a gate. A donor nuclear spin-electron system close to an "$A$-gate" functions as a voltage controlled oscillator: the precession frequency of the nuclear spin is controllable externally, and spins can be selectively brought into resonance with $B_{AC}$, allowing arbitrary rotations to be performed on each nuclear spin.

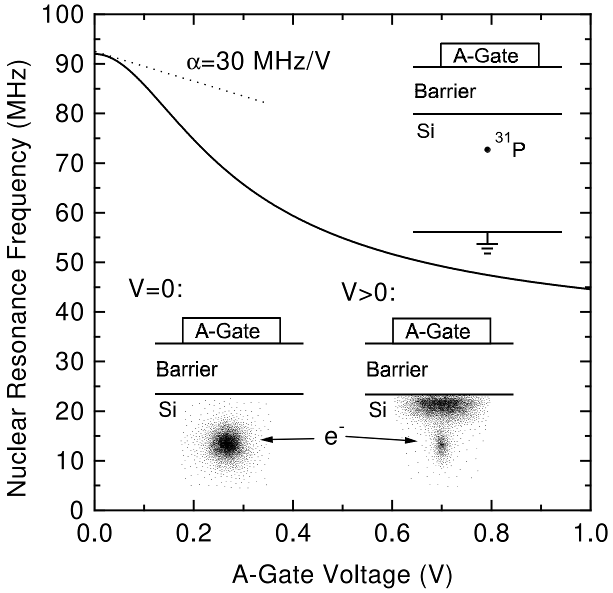


**Fig. 2: An electric field applied to an $A$-gate pulls the electron wave function away from the donor and toward the barrier, reducing the hyperfine interaction and the resonance frequency of the nucleus. The donor nucleus-electron system is a voltage controlled oscillator with a tuning parameter $\alpha$ of order 30 MHz/V.**

Quantum mechanical computation requires, in addition to single spin rotations, the two qubit "controlled rotation" operation, which rotates the spin of a target qubit through a prescribed angle if and only if the control qubit is oriented in a specified direction, and leaves the orientation of the control qubit unchanged [26][27]. Performing the controlled rotation operation requires nuclear spin exchange between two donor-electron spin systems [12], which will arise from electron-mediated interactions when the donors are sufficiently close to each other. The Hamiltonian of two coupled donor nucleus-electron systems, valid at energy scales small compared to the donor-electron binding energy, is:

$$H = H(B) + A_1\sigma^{1n} \cdot \sigma^{1e} + A_2\sigma^{2n} \cdot \sigma^{2e} + J\sigma^{1e} \cdot \sigma^{2e}, \tag{3}$$

where $H(B)$ are the magnetic field interaction terms for the spins. $A_1$ and $A_2$ are the hyperfine interaction energies of the respective nucleus-electron systems. $4J$, the exchange energy, depends on the overlap of the electron wave functions. For well separated donors [28]:

$$4J(r) \cong 1.6\frac{e^2}{\epsilon a_B}(\frac{r}{a_B})^{\frac{5}{2}} \exp(\frac{-2r}{a_B}), \tag{4}$$

where $r$ is the distance between donors, $\epsilon$ is the dielectric constant of the semiconductor, and $a_B$ is the semiconductor Bohr radius. This function, with values appropriate for Si, is plotted in Fig. 3. Eq. 4, originally derived for H atoms, is complicated in Si by its valley degenerate anisotropic band structure [29]. Exchange coupling terms from each valley interfere, leading to oscillatory behavior of $J(r)$. In this discussion the complications introduced by Si band structure will be neglected. In determining $J(r)$ in Fig. 3, the transverse mass for Si ($\cong 0.2m_e$) has been used, and $a_B$=30 Å. Because $J$ is proportional to the electron wave function overlap, it can be varied by an electrostatic potential imposed by a "$J$-gate" positioned between the donors [12]. As shall be seen below, significant coupling between nuclei will occur when $4J \approx \mu_B B$, and this condition approximates the necessary separation between donors of 100-200 Å. While actual separations may be considerably larger than this value, since the $J$-gate can be biased positively to reduce the barrier between donors, the gate sizes required for the quantum computer are near the limit of current electronics fabrication technology.

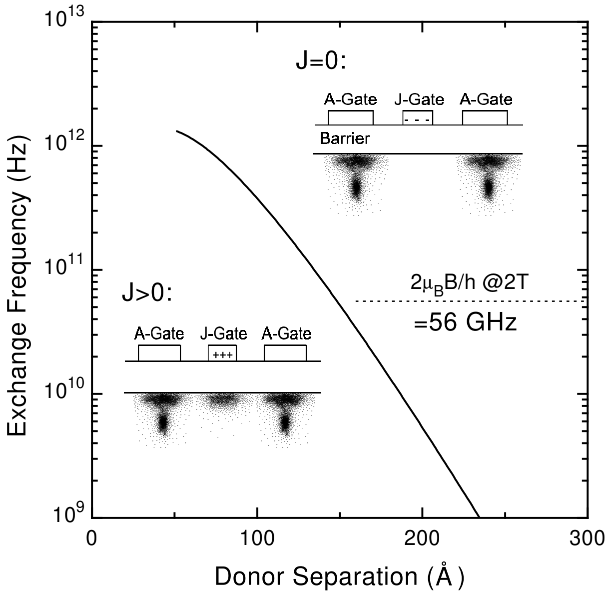


**Fig. 3: $J$-gates vary the electrostatic potential barrier $V$ between the donors to enhance or reduce the exchange coupling, proportional to the electron wave function overlap. The exchange frequency ($= 4J/h$) when $V$=0 is plotted for Si.**

For two electron systems the exchange interaction lowers the electron singlet ($|\uparrow\downarrow - \downarrow\uparrow\rangle$) energy with respect to the triplets [30]. (The $|\uparrow\downarrow\rangle$ notation here is used to represent the electron spin state, and the $|01\rangle$ notation the nuclear state; in the $|\downarrow\downarrow 11\rangle$ state, all spins point in the same direction. For simplicity normalisation constants are omitted.) In a magnetic field, however, $|\downarrow\downarrow\rangle$ will be the electron ground state if $J < \mu_B B/2$ (Fig. 4a). In the $|\downarrow\downarrow\rangle$ state, the energies of the nuclear states can be calculated to second order in $A$ using perturbation theory. When $A_1 = A_2 = A$, the $|10 - 01\rangle$ state is lowered in energy with respect to $|10 + 01\rangle$ by:

$$h\nu_J = 2A^2\left(\frac{1}{\mu_B B - 2J} - \frac{1}{\mu_B B}\right). \tag{5}$$

The $|11\rangle$ state is above the $|10 + 01\rangle$ state and the $|00\rangle$ state below the $|10 - 01\rangle$ state by an energy $h\nu_A$, given in Eq. 2. For the Si:$^{31}$P system at $B$=2 T and for $4J/h$=30 GHz, Eq. 5 yields $\nu_J$=75 kHz. This nuclear spin exchange frequency approximates the rate at which binary operations can be performed on the computer. ($\nu_J$ can be increased by increasing $J$, at the expense, however, of also increasing the relaxation rate of the coupled nuclear-electron spin excitations.) The speed of single spin operations is determined by the size of $B_{AC}$ and is comparable to 75 kHz when $B_{AC}$=$10^{-3}$ T.

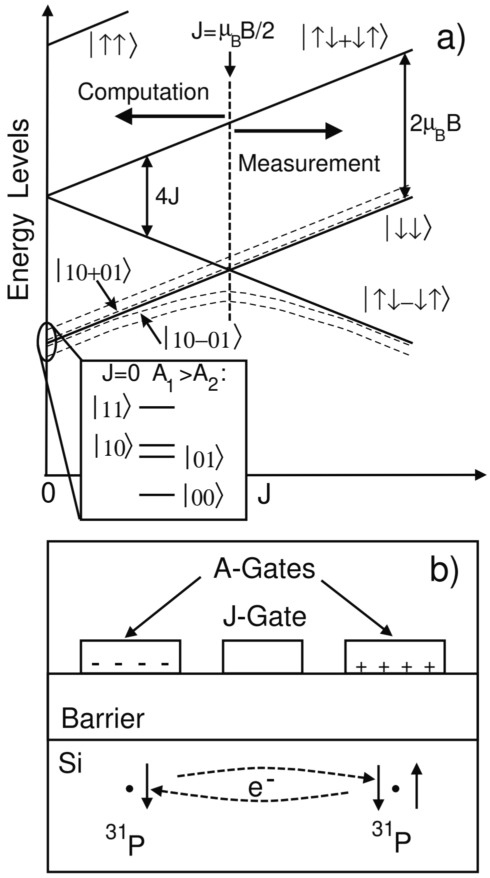


**Fig. 4: (a) Electron (solid lines) and lowest energy coupled electron-nuclear (dashed lines) energy levels as a function of $J$. When $J < \mu_B B/2$, two qubit computations are performed by controlling the $|10{-}01\rangle$-$|10{+}01\rangle$ level splitting with a $J$-gate. Above $J = \mu_B B/2$ the states of the coupled system evolve into states of differing electron polarisation. The state of the nucleus at $J = 0$ with the larger energy splitting (controllable by the $A$-gate bias) determines the final electron spin state after an adiabatic increase in $J$. (b) Only $|\uparrow\downarrow - \downarrow\uparrow\rangle$ electrons can make transitions into states in which electrons are bound to the same donor ($D^-$ states). Electron current during these transitions is measurable using capacitive techniques, enabling the underlying spin states of the electrons and nuclei to be determined.**

### III. Spin Measurement

Measurement of nuclear spins in the proposed quantum computer is accomplished in a two step process: nuclear spin states are adiabatically converted into states with different *electron* polarisation, and the electron spin is determined by its effect on the symmetry of the orbital electron wave function. A procedure for accomplishing this conversion is shown in Fig. 4. While all computation is performed when $J < \mu_B B/2$, and the electrons are fully polarised, measurements are made when $J > \mu_B B/2$, and $|\uparrow\downarrow - \downarrow\uparrow\rangle$ states have the lowest energy (Fig. 4a). As the electron levels cross, the $|\downarrow\downarrow\rangle$ and $|\uparrow\downarrow - \downarrow\uparrow\rangle$ states are coupled by hyperfine interactions with the nuclei. During an adiabatic increase in $J$ the two lower energy nuclear spin states at $J = 0$ evolve into $|\uparrow\downarrow - \downarrow\uparrow\rangle$ states when $J > \mu_B B/2$, while the two higher energy nuclear states remain $|\downarrow\downarrow\rangle$. If, at $J = 0$, $A_1 > A_2$, the orientation of nuclear spin 1 alone will determine whether the system evolves into the $|\uparrow\downarrow - \downarrow\uparrow\rangle$ or the $|\downarrow\downarrow\rangle$ state during an adiabatic increase in $J$.

A method to detect the electron spin state using electronic means is shown in Fig. 4b. Both electrons can become bound to the same donor (a $D^-$ state) if the $A$-gates above the donors are biased appropriately. In Si:P the $D^-$ state is always a singlet with a second electron binding energy of 1.7 meV [31][32]. Consequently, a differential voltage applied to the $A$-gates can result in charge motion between the donors that only occurs if the electrons are in a singlet state. This charge motion is measurable using sensitive single electron capacitance techniques [33]. This approach to spin measurement produces a signal which persists until the electron spin relaxes, a time which, as noted above, can be thousands of seconds in Si:P.

The spin measurement process can also be used to prepare nuclear spins in a prescribed state by first determining the state of a spin and flipping it if necessary so that it ends up in the desired spin state. As with the spin computation procedures discussed above, spin measurement and preparation can in principle be performed in parallel throughout the computer.

## IV. Initialising the Computer

Prior to any computation, the computer must be initialised by calibrating the $A$-gates and the $J$-gates. Fluctuations from cell to cell in the gate biases necessary to perform logical operations are an inevitable consequence of variations in the positions of the donors and in the sizes of the gates. The parameters of each cell, however, can be determined individually using the measurement capabilities of the computer, since the measurement technique discussed above does not require precise knowledge of the $J$ and $A$ couplings. The $A$-gate voltage at which the underlying nuclear spin is resonant with an applied $B_{AC}$ can be determined using the technique of adiabatic fast passage [34]: when $B_{AC}$ =0 the nuclear spin is measured and the $A$-gate is biased at a voltage known to be off resonance. $B_{AC}$ is then switched on, and the $A$-gate bias is swept through a prescribed voltage interval. $B_{AC}$ is then switched off and the nuclear spin is measured again. The spin will have flipped if and only if resonance occurred within the prescribed $A$-gate voltage range. Testing for spin flips in increasingly small voltage ranges leads to the determination of the resonance voltage. Once adjacent $A$-gates have been calibrated, the $J$-gates can be calibrated in a similar manner by sweeping $J$-gate biases across resonances of two coupled cells.

This calibration procedure can be performed in parallel on many cells, so calibration is not a fundamental impediment to scaling the computer to large sizes. Calibration voltages can be stored on capacitors located on the Si chip adjacent to the quantum computer. External controlling circuitry would thus need to control only the timing of gate biases, and not their magnitudes.

## V. Spin Decoherence Introduced by Gates

In the quantum computer architecture outlined above, biasing of $A$-gates and $J$-gates enables custom control of the qubits and their mutual interactions. The presence of the gates, however, will lead to decoherence of the spins if the gate biases fluctuate away from their desired values. These effects need to be considered to evaluate the performance of any gate-controlled quantum computer. During computation the largest source of decoherence is likely to arise from voltage fluctuations on the $A$-gates. (When $J < \mu_B B/2$ modulation of the state energies by the $J$-gates is much smaller than by the $A$-gates. $J$ exceeds $\mu_B B/2$ only during the measurement process, when decoherence will inevitably occur.) The precession frequencies of two spins in phase at $t = 0$ depends on the potentials on their respective $A$-gates. Differential fluctuations of the potentials produce differences in the precession frequency. At some later time $t = t_\phi$ the spins will be 180° out of phase. $t_\phi$ can be estimated by determining the transition rate between $|10+01\rangle$ (spins in phase) and $|10-01\rangle$ (spins 180° out of phase) of a two spin system. The Hamiltonian that couples these states is:

$$H_\phi = \frac{1}{4} h\Delta(\sigma_z^{1n} - \sigma_z^{2n}), \tag{6}$$

where $\Delta$ is the fluctuating differential precession frequency of the spins. Standard treatment of fluctuating Hamiltonians [34] predicts: $t_\phi^{-1} = \pi^2 S_\Delta(\nu_{st})$, where $S_\Delta$ is the spectral density of the frequency fluctuations, and $\nu_{st}$ is the frequency difference between the $|10-01\rangle$ and $|10+01\rangle$ states. At a particular bias voltage, the $A$-gates have a frequency tuning parameter $\alpha = d\Delta/dV$. Thus:

$$t_\phi^{-1} = \pi^2\alpha^2(V)S_V(\nu_{st}), \tag{7}$$

where $S_V$ is the spectral density of the gate voltage fluctuations.

$S_V$ for good room temperature electronics is of order $10^{-18}$ V$^2$/Hz, comparable to the room temperature Johnson noise of a 50 $\Omega$ resistor. $\alpha$, estimated from Fig. 2, is 10-100 MHz/V, yielding $t_\phi$ = 10-1000 sec. $\alpha$ is determined by the size of the donor array cells and cannot readily be reduced (to increase $t_\phi$) without reducing the exchange interaction between cells. Because $\alpha$ is a function of the gate bias (Fig. 2), $t_\phi$ can be increased by minimising the voltage applied to the $A$-gates.

While Eq. 7 is valid for white noise, at low frequencies it is likely that materials dependent fluctuations ($1/f$ noise) will be the dominant cause of spin dephasing. Consequently, it is difficult to give hard estimates of $t_\phi$ for the computer. Charge fluctuations within the computer (arising from fluctuating occupancies of traps and surface states, for example) are likely to be particularly important, and minimising them will place great demands on computer fabrication.

While materials-dependent fluctuations are difficult to estimate, the low temperature operation of the computer and the dissipationless nature of quantum computing mean that in principle fluctuations can be kept extremely small: using low temperature electronics to bias the gates (for instance by using on chip capacitors as discussed above) could produce $t_\phi \approx 10^6$ sec. Electronically controlled nuclear spin quantum computers thus have the theoretical capability to perform at least $10^5$ to perhaps $10^{10}$ logical operations during $t_\phi$, and can probably meet Preskill's criterion for an error prabability of $10^{-6}$ per qubit operation.

## VI. Constructing the Computer

Building the computer presented above will obviously be an extraordinary challenge: the materials must be almost completely free of spin ($I \neq 0$ isotopes) and charge impurities in order to prevent dephasing fluctuations from arising within the computer. Donors must be introduced into the material in an ordered array hundreds of Å beneath the surface. Finally, gates with lateral dimensions and separations $\sim$100 Å must be patterned on the surface, registered to the donors beneath them. Although it is possible that the computer can use $SiO_2$ as the barrier material (the standard MOS technology used in most current conventional electronics), the need to reduce disorder and fluctuations to a minimum means that heteroepitaxial materials, such as Si/SiGe, may ultimately be preferable to $Si/SiO_2$.

The most obvious obstacle to building the quantum computer presented above is the incorporation of the donor array into the Si layer beneath the barrier layer. Currently, semiconductor structures are deposited layer by layer. The $\delta$-doping technique produces donors lying on a plane in the material, with the donors randomly distributed within the plane. The quantum computer envisioned here requires that the donors be placed into an ordered 1D or 2D array; furthermore, precisely one donor must be placed into each array cell, making it extremely difficult to create the array by using lithography and ion implantation or by focused deposition. Methods currently under development to place single atoms on surfaces using ultra high vacuum scanning tunneling microscopy [35] or atom optics techniques [36] are likely candidates to be used to position the donor array. A challenge will be to grow high quality Si layers on the surface subsequent to placement of the donors.

While fabricating large arrays of donors may prove to be difficult, two spin devices, which can be used to test the logical operations and measurement techniques presented above, can be made using random doping techniques. While only a small fraction of such devices will work properly, adjacent conventional Si electronic multiplexing circuitry can be used to separately examine many devices. The relative ease of fabricating such "hybrid" (quantum-conventional) circuits is a particularly attractive feature of Si based quantum computation.

In a Si-based nuclear spin quantum computer, the highly coherent quantum states necessary for quantum computation are incorporated into a material in which the ability to implement complex computer architectures is well established. The substantial challenges facing the realisation of the computer, particularly in fabricating 100 Å-scale gated devices, are similar to those facing the next generation of conventional electronics; consequently, new manufacturing technologies being developed for conventional electronics will bear directly on efforts to develop a quantum computer in Si. Quantum computers sufficiently complex that they can achieve their theoretical potential may thus one day be built using the same technology that is used to produce conventional computers.

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