Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

**6.894 Legged Locomotion in Robots and Animals**

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# Dimensional Analysis

## Terms

**Physical Variables** Variables most relevant to problem.

**Fundamental Quantities** Mass(M), Length(L), Time(T), Force(F), temperature($\Phi$). Choice of groups MLT$\phi$, FLT$\phi$, etc. is arbitrary but must be independent.

**Dimensionless Groups** Groups of relevant variables whose dimensions cancel.

**Buckingham's Theorem** If there are $m$ physical variables defined in terms of $n$ independent fundamental quantities, there are $p = m - n$ independent dimensionless groups.

## Method

1. List Physical variables and identify dimensional formulas.
2. Use Buckingham's Theorem to determine number of independent dimensionless groups, $p = m - n$.
3. Form a product group with arbitrary exponents.
4. Solve for exponents. There will an infinite amount of solutions, but only $p$ independent solutions. Choose those which are most convienient.

## Examples

### Pendulum

**period** $T_p \equiv T$

**length** $l \equiv L$

**gravity** $g \equiv \frac{L}{T^2}$

$$T_p\ l^a\ g^b \equiv [T]\ [L]^a\ \left[\frac{L}{T^2}\right]^b$$

$$b = \frac{1}{2},\ a = -\frac{1}{2},\ T_p\sqrt{\frac{g}{l}} = C$$

**Atomic Bomb Explosion**

**density** $\rho \equiv \frac{FT^2}{L^4}$

**radius** $r \equiv L$

**Energy** $E \equiv FL$

**Time** $t \equiv T$

$$\rho\ r^a\ E^b\ t^c \equiv \left[\frac{FT^2}{L^4}\right]\ [L]^a\ [FL]^b\ [T]^c$$

$$b = -1,\ c = -2\ a = 5,\ \frac{r^5\ \rho}{E\ t^2} = C$$

**Submarine**

**density** $\rho \equiv \frac{FT^2}{L^4}$

**viscosity** $\mu \equiv \frac{FT}{L^2}$

**length** $r \equiv L$

**velocity** $v \equiv \frac{L}{T}$

**Drag** $D \equiv F$

**gravity** $g \equiv \frac{L}{T^2}$

$$\rho^a\ \mu^b\ l^c\ v^d\ D^e\ g^f$$

$$F : a + b + e = 0,\ L : -4a - 2b + c + d + f = 0\ T : 2a + b - d - 2f = 0$$

$$\frac{\rho\ lv}{\mu}, \frac{D}{\rho\ v^2 l^2}, \frac{v^2}{gl}$$

## Scale Models

**Dynamic Similarity** All dimensionless groups must be kept the same in the model and prototype to achieve perfect dynamic similarity.

### Scale Model Questions and Examples

1. We want to know the period of a pendulum of arbitrary length in arbitrary gravity. How many models and measurements must we make?

2. The US performs an atomic test of a 10KTon bomb and films the results. They find that $r = Kt^{0.4}$. After examinging films of a Chinese detonation they find that the Chinese explosion had $r = 2Kt^{0.4}$. What was the size of the Chinese weapon in KTon?

3. We want to find the drag force, D, on a proposed submarine with all other variables known. Using a $\frac{1}{2}$ scale model how can we do this if we a) assume the Reynolds number matters but Froude number doesn't? b) assume the Froude number matters but the Reynolds number doesn't c) assume both Reynolds and Froude number matter?