Input/Output Hidden Markov Models for Modeling Stock Order Flows

Adlar Jeewook Kim

Artificial Intelligence Laboratory and
The Center for Biological and Computational Learning
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

http://www.ai.mit.edu

The Problem: The project aims to develop a trainable system, which generates a sequence of orders. For the same market conditions as the training data, the generated orders will form a similar trading process (price and volume). The market conditions will be determined by the orders that are already generated, and by the market maker's actions.

Motivation: In the past few years, CBCL has developed an adaptive learning model for market-making by using reinforcement learning (Chan and Shelton, 2001[4]). The model was tested under a simply simulated market environment. However, testing under such simplified environment is inadequate to argue how well the market- making model will perform under the real market. In this project, we focus on designing a sophisticated and dynamic market environment using Input / Output Hidden Markov Models[2] .

Previous Work: Previously, the research on the information content of the trading process has been carried out (Easley, Kiefer, and O'Hara, 1997[6]). In this work, they fit a model of the trade process, which allowed to explain degrees of information content of the trading process of a particular stock. The model was fitted by maximum likelihood using transactions data on six stocks over 60 days. They showed that the trade process provided wealth of information as the following:

1. The large and small trades have different information content, but this varies across stocks.
2. The uninformed trades are history dependent.
3. The large buys and large sells are equally informative.

Although this is not directly related to the order flow generation process, it helps defining the structure of our IOHMMs (particularly, input variables).

Approach: In this work, we propose to construct an IOHMM to generate a sequence of orders given the market conditions, which will form a trading process. Each state will emit an order based on their estimated conditional Gauissian distributions. The inputs will affect the transition probabilities distribution of the next following states, as well as the emission probabilities of the outputs (orders). The inputs will consist of variables, which can describe the market conditions. Some of the possible inputs are bid / ask prices, bid / ask sizes, and information content of the trading process. As a result, the IOHMM will generate orders that react to the changes in market conditions of a particular stock used for training. The IOHMM is defined by

- Number of hidden states m.
- Initial state distribution $P_0(s_0)$.
- State transition probabilities $P_1(s_{t+1}|s_t, \text{market condition}_t)$.
- Output probabilities $P_0(O_t|s_t, \text{market condition}_t)$.

Before using IOHMMs, we preliminarily tried Hidden Markov Models [8] to model the order generating process. Considering HMMs, we are ignoring the market conditions that might have various impacts on the order generating process. Since we are ignoring the market conditions, the orders that are generated by the model will form a similar trading process as the training orders. Table 1. shows volume, average bid/ask spread, and volume weighted average price[1] of trained and generated orders of three trading days of IBM stock.

	Nov. 2, 1990		Nov. 6, 1990		Nov. 9, 1990	
Address	**Training**	**Generated**	**Training**	**Generated**	**Training**	**Generated**
Volume	356,800	465,200	294,400	440,000	576,900	515,600
Avg. Bid/Ask	\$0.83	\$1.19	\$0.43	\$0.52	\$1.08	\$0.82
VWAP	\$107.55	\$107.55	\$107.20	\$107.22	\$108.74	\$108.91

Table 1: HMM Training Results of IBM Using 8 Hidden States

Impact: Previously, modeling a financial market environment involved a number of unrealistic assumptions such as the existence of a true price process, or differentiating the informed and uninformed traders. And these assumptions over-simplified the model. The new approach will empirically model the aggregated behavior of the trading crowd. This will make the adaptive learning model for market-making possible to be tested under a more realistic market environment.

Future Work: Clearly, the next step to this project is to test the market-making model under IOHMM market environment. Also, the results of the IOHMM model will be extensively studied by applying various scenarios of market conditions.

Research Support: Research at CBCL is sponsored by grants from: Office of Naval Research (DARPA) under contract No. N00014-00-1-0907, National Science Foundation (ITR) under contract No. IIS-0085836, National Science Foundation (KDI) under contract No. DMS-9872936, and National Science Foundation under contract No. IIS-9800032

Additional support was provided by: Central Research Institute of Electric Power Industry, Center for e-Business (MIT), Eastman Kodak Company, DaimlerChrysler AG, Compaq, Honda R&D Co., Ltd., Komatsu Ltd., Merrill-Lynch, NEC Fund, Nippon Telegraph & Telephone, Siemens Corporate Research, Inc., Toyota Motor Corporation and The Whitaker Foundation.

References:

[1] Y. Bengio. Markovian models for sequential data. *Neural Computing Surveys*, 2:129–162, 1999.

[2] Y. Bengio and P. Frasconi. Input/output hmms for sequence processing. *IEEE Transactions on Neural Networks*, 7(5):1231–1249, 1996.

[3] Y. Bengio, V. Lauzon, and R. Ducharme. Experiments on the application of iohmms to model financial return series. *IEEE Transactions on Neural Networks*, 12(1):113–123, 2001.

[4] N. T. Chan and C. R. Shelton. An Electronic Market Maker. Technical Report 200-005, MIT Artificial Intelligence Laboratory, 2001.

[5] T. Chan. *Artificial Markets and Intelligent Agents.* PhD thesis, Massachusetts Institute of Technology, 2001.

[6] D. Easley, N. Kiefer, and M. O'Hara. The information content of the trading process. *Journal of Empirical Finance*, 4:159–186, 1997.

[7] D. Easley, N. Kiefer, and M. O'Hara. One day in the life of a very common stock. *Review of Financial Studies*, 10(3):805–835, 1997.

[8] L. Rabiner. A tutorial on hidden markov models and selected applications in speech recognition. In *Proceedings of IEEE, Vol. 77, No. 2*, pages 257–286, 1989.

[1] $(\Sigma_{t=1}^{T} P_t \cdot S_t)/(\Sigma_{t=1}^{T} S_t)$ where P_t = last price at time t, S_t = traded size at time t