6.825 Techniques in Artificial Intelligence

Logic Miscellanea

- Completeness and Incompleteness
- Equality
- Paramodulation

Completeness and Decidability

Complete: If $KB \vdash \alpha$ then $KB \vdash \alpha$
- If it's entailed, there is a proof

Semi-decidable:
- If there's a proof, we'll halt with it
- If not, maybe halt, maybe not

Gödel's Completeness Theorem: There exists a complete proof system for FOL

Robinson's Completeness Theorem: Resolution refutation is a complete proof system for FOL

FOL is semi-decidable: if the desired conclusion follows from the premises then eventually resolution refutation will find a contradiction.

Adding Arithmetic

Gödel's Incompleteness Theorem: There is no consistent, complete proof system for FOL + Arithmetic.

Either there are sentences that are true, but not provable or there are sentences that are provable, but not true.

Arithmetic gives you the ability to construct code-names for sentences within the logic.
- $P = "P is not provable."
- If $P$ is true: it's not provable (incomplete)
- If $P$ is false: it's provable (inconsistent)

Equality

8. $x, x = x$
8. $xy, x = y \iff y = x$
8. $xyz, x = y \iff y = z \land x = z$
8. $xy. x = y \iff (P(x) \land P(y))$

etc., etc., etc., ...

Paramodulation

Need one more rule to deal with resolution and equality.

\[
\begin{align*}
\alpha \lor (x = t) \\
\beta \lor \gamma[r] \\
\frac{(\alpha \lor \beta) \lor \gamma[r]}{(\alpha \lor \beta) \lor \gamma[r]} \\
\end{align*}
\]

$\gamma[r]$ is a literal containing term $r$

\[
\begin{align*}
\delta(x) = B \\
(\delta(y) \lor W(y, F(y))) \\
(\delta(y) \lor W(y, B)) \\
\frac{P(x) \lor F(x) = B}{P(y) \lor W(y, F(y))} \\
\frac{P(y) \lor W(y, B)}{\delta(y) \lor W(y, B)} \\
\end{align*}
\]

where
- $x = F(x)$
- $t = B$
- $\gamma[1] = W(y, )$
- $r = F(y)$
- $\theta = \{x/y\}$

Recitation Problems

Formalize each group of sentences (using the given function and predicate symbols), then prove the last from the others using resolution and paramodulation.

Jane's lover drives a red car.
Fred is the only person who drives a red car.
Therefore, Fred is Jane's lover.
($L(x)$ = the lover of $x$; $D(x) = x$ drives a red car)

Mrs. Abbot only teaches good students.
John and Mary have the same teacher.
Mrs. Abbot is Mary's teacher.
Therefore, John is a good student.
($T(x)$ = the teacher of $x$; $G(x) = x$ is a good student)
More Recitation Problems

- Every part is either made by FooCorp or BarCorp.
- All fragile parts are stored in the warehouse of their manufacturer.
- BarCorp can't manufacture titanium parts.
- The part I need is fragile and made of titanium.
- Therefore, the part I need is the FooCorp’s warehouse.

(M(x) = the manufacturer of part x; W(x,y) = part x is stored in the warehouse of company y; T(x) = part x is made of titanium; F(x) = part x is fragile; use a constant for "the part I need").