1 Bayesian Networks

(22 points) Consider the network shown below:

1. (2 point) Is this a polytree?
2. (3 points) Assuming the nodes are binary, how many parameters are required to specify the CPTs?
3. (2 points) Is $F$ independent of $A$ given $B$?
4. (2 points) Is $G$ independent of $E$ given $A$ and $F$?
5. (2 points) Is $B$ independent of $F$ given $C$, $D$, and $E$?
6. (3 points) Give an expression for $\Pr (d|c)$ (where $d$ and $c$ are specific values of variables $D$ and $C$) in terms of parameters stored in the network?
7. (2 point) Which variables are irrelevant to the query $\Pr (d|c)$?
8. (4 points) What factors are created by variable elimination using order $A, B, E, F, G$?
9. (2 point) Is there another elimination order with a smaller largest factor?

2 Decision Theory

(18 points) Dr. No has a patient who is very sick. Without further treatment, this patient will die in about 3 months. The only treatment alternative is a risky operation. The patient is expected to live about 1 year if he survives the operation; however, the probability that the patient will not survive the operation is 0.3.
1. (3 points) Draw a decision tree for this simple decision problem. Show all the probabilities and outcome values.

2. (3 points) Let $U(x)$ denote the patient’s utility function, where $x$ is the number of months to live. Assuming that $U(12) = 1.0$ and $U(0) = 0$, how low can the patient’s utility for living 3 months be and still have the operation be preferred?
   For the rest of the problem, assume that $U(3) = 0.8$.

3. (4 points)
   Dr. No finds out that there is a less risky test procedure that will provide uncertain information that predicts whether or not the patient will survive the operation. When this test is positive, the probability that the patient will survive the operation is increased. The test has the following characteristics:
   - True-positive rate: The probability that the results of this test will be positive if the patient will survive the operation is 0.90.
   - False-positive rate: The probability that the results of this test will be positive if the patient will not survive the operation is 0.10.

   What is the patient’s probability of surviving the operation if the test is positive?

4. (2 points) Assuming the patient has the test done, at no cost, and the result is positive, should Dr. No perform the operation?

5. (4 points) It turns out that the test may have some fatal complications, i.e., the patient may die during the test. Draw a decision tree showing all the options and consequences of Dr. No’s problem.

6. (2 points) Suppose that the probability of death during the test is 0.005 for the patient. Should Dr. No advise the patient to have the test prior to deciding on the operation?

3 Markov Decision Processes

(15 points) You are playing a game at a carnival, in which you are trying to throw balls through a hoop. You are allowed to play this game for a total of $k$ steps. If, at the end of $k$ steps, you have gotten at least one ball through the hoop, then you win $10. If not, you win nothing. On each step, you are allowed to buy, for $1 each, as many balls as you would like, which you will try to throw simultaneously through the hoop. Each ball has a probability of $p$ of going through the hoop, and each ball’s success is independent of the successes of the other balls and of the number of balls being thrown. After you’ve thrown one set of balls, you can observe whether or not any of them went through the hoop.

1. (3 points) If you throw $n$ balls at once, what is the probability of getting at least one ball through the net? Write an expression in terms of $p$ and $n$. Call this quantity $f(p, n)$ in future parts of this problem.
2. (5 points) If \( k = 1 \), that is, you can only play one round of this game, what is the optimal number, \( n \), of balls to buy and throw? (You only need to write down an expression involving \( n \) and \( p \); don’t worry about getting a closed form).

3. (2 points) Let \( s_1 \) be the state of not having gotten a ball through the hoop and \( s_2 \) be the state of having gotten one through. Let \( V^k(s) \) be the value of being in state \( s \) with \( k \) steps remaining in the game. Then \( V^0(s_1) = 0 \) and \( V^0(s_2) = 10 \). What is \( V^k(s_2) \), assuming there is no discounting.

4. (5 points) Write an expression for \( V^k(s_1) \), in terms of \( V^{k-1}(s) \).

4 Network Structure
Show a Bayesian network structure that encodes the following relationships:

- A is independent of B
- A is dependent on B given C
- A is dependent on D
- A is independent of D given C.

5 At The Races
You go to the racetrack. You can:

- Decline to place any bets at all.
- Bet on Belle. It costs $1 to place a bet; you will be paid $2 if she wins (for a net profit of $1).
- Bet on Jeb. It costs $1 to place a bet; you will be paid $11 if he wins (for a net profit of $10).

You believe that Belle has probability 0.7 of winning and that Jeb has probability 0.1 of winning.

1. Your goal is to maximize the expected value of your actions. What, if any, bet should you place, and what is your expected value? Draw the decision tree that supports your conclusion.

2. Someone comes and offers you gambler’s anti-insurance. If you agree to it,
   - they pay you $2 up front
   - you agree to pay them 50% of any winnings (that is, $.50 if Belle wins, and $5 if Jeb wins).

   How would it affect the expected value of each of your courses of action? What would be the best action to take now? Draw the new decision tree.
6 Still At The Races

A shady character comes to offer you a free tip (a tip is a piece of information): he says Belle did not eat her breakfast. Assume that:

- The probability that a horse will win is dependent on the horse’s health and its speed.
- A horse’s health and its speed are independent.
- A healthy horse has a higher probability of eating breakfast than does a sick horse.
- Your informant is known to be accurate 80% of the time.

1. Draw a Bayesian network with 5 variables (T = you got this tip; B = Belle ate her breakfast; H = Belle is healthy; W = Belle will win; F = Belle is fast). The relationships between the variables should reflect the problem description.

2. What is $P(W)$ (in terms of values stored in the network’s conditional probability tables)?

3. What is $P(W|T)$ (in terms of values stored in the network’s conditional probability tables)?

7 Bayesian Network Structure

Consider a Bayesian network with the following structure:
Does computing $P(M|A)$ depend on:

- $P(L|J)$?
- $P(K|I)$?
- $P(D|B)$?
- $P(H|G)$?

In the network above, if we decided not to include G in our network, but still wanted to model the joint distribution of all the other variables, what is the smallest network structure we could use?

## 8 Conditional Probability

(8) We would like to compute $Pr(a,b|c,d)$ but we only have available to us the following quantities: $Pr(a)$, $Pr(b)$, $Pr(c)$, $Pr(a|d)$, $Pr(b|d)$, $Pr(c|d)$, $Pr(d|a)$, $Pr(a,b)$, $Pr(c,d)$, $Pr(a|c,d)$, $Pr(b|c,d)$, $Pr(c|a,b)$, $Pr(d|a,b)$. 
For each of the assumptions below, give a set of terms that is sufficient to compute the desired probability, or “none” if it can’t be determined from the given quantities.

1. A and B are conditionally independent given C and D
2. C and D are conditionally independent given A and B
3. A and B are independent
4. A, B, and C are all conditionally independent given D

9 Network Structures

(12) Following is a list of conditional independence statements. For each statement, name all of the graph structures, G1 – G4, or “none” that imply it.

1. A is conditionally independent of B given C
2. A is conditionally independent of B given D
3. B is conditionally independent of D given A
4. B is conditionally independent of D given C
5. B is independent of C
6. B is conditionally independent of C given A

10  Counting Parameters

(4) How many independent parameters are required to specify a Bayesian network given each of the graph structures G1 – G4? Assume the nodes are binary.

11  Variable Elimination

(5)

1. In this network, what is the size of the biggest factor that gets generated if we do variable elimination with elimination order A, B, C, D, E, F, G?
2. Give an elimination order that has a smaller largest factor.

12 Decision Theory

(13) You’re an Olympic skier. In practice today, you fell down and hurt your ankle. Based on the results of an x-ray, the doctor thinks that it’s broken with probability 0.2. So, the question is, should you ski in the race tomorrow?

If you ski, you think you’ll win with probability 0.1. If your leg is broken and you ski on it, then you’ll damage it further. So, your utilities are as follows: if you win the race and your leg isn’t broken, +100; if you win and your leg is broken, +50; if you lose and your leg isn’t broken 0; if you lose and your leg is broken -50.

If you don’t ski, then if your leg is broken your utility is -10, and if it isn’t, it’s 0.

1. Draw the decision tree for this problem.

2. Evaluate the tree, indicating the best action choice and its expected utility.

You might be able to gather some more information about the state of your leg by having more tests. You might be able to gather more information about whether you’ll win the race by talking to your coach or the TV sports commentators.

3. Compute the expected value of perfect information about the state of your leg.

4. Compute the expected value of perfect information about whether you’ll win the race.

In the original statement of the problem, the probability that your leg is broken and the probability that you’ll win the race are independent. That’s a pretty unreasonable assumption.

6. Is it possible to use a decision tree in the case that the probability that you’ll win the race depends on whether your leg is broken?

13 Markov Decision Processes

(10) What are the values of the states in the following MDP, assuming $\gamma = 0.9$? In order to keep the diagram from being too complicated, we’ve drawn the transition probabilities for action 1 in one figure and the transition probabilities for action 2 in another. The rewards for the states are the same in both cases.
14 Problems from AIMA2E

- 14.2
- 14.12.a, 14.12.b
- 15.2
- 15.9
- 15.10
- 17.4