Total: 50 points

1. (20 points)
   Consider the following Bayesian network:

   (a) (1 pt) Is it a polytree?
   
   No

   (b) (1 pt) Is \(A\) independent of \(C\)?
   
   Yes

   (c) (1 pt) Is \(C\) independent of \(E\)?
   
   No

   (d) (1 pt) Is \(D\) independent of \(C\)?
   
   No

   (e) (1 pt) Name a variable that, if it were an evidence variable, your answer to the question in part (b) would be different, or say that there is no such variable. (So, if your answer to (b) was that they are independent, then name a variable \(X\) for which \(A\) is not conditionally independent of \(C\) given \(X\).)
B (makes A and C dependent)

(f) (1 pt) Name a variable that, if it were an evidence variable, your answer to part (c) would be different, or say that there is none.

No such (single) variable (2 variables B,F)

(g) (1 pt) Name a variable that, if it were an evidence variable, your answer to part (d) would be different, or say that there is none.

B

(h) (2 pts) If all the nodes are binary, how many parameters would be required to specify all the CPTs in this network? (Remember that if $p$ is specified then it is not necessary to specify $1 - p$ as well.)

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(i) (3 pts) Give an expression for $\Pr(D|C)$ given probabilities that are stored in the CPTs. Don’t include any unnecessary terms.

$$\Pr(D|C) = \sum_b \sum_a \Pr(D|b) \times \Pr(b|a, C) \times \Pr(a)$$

(j) (3 pts) What factor is created if we eliminate $B$ first in the course of using variable elimination to compute $\Pr(A|G)$?

$f\{A, C, D, E, F\}$

There are many correct answers to this problem because A is independent of G.

(k) (2 pts) What is the Markov blanket of $B$?

$A, C, D, E, F$

(l) (3 pts) Imagine that you’re doing likelihood weighting to compute $\Pr(E = e|A = a)$. What weight would you have to assign to sample $\langle a, b, c, d, f, g \rangle$?

$P(A = a)$

2. (10 points)

You are performing surveillance, trying to decide which destination in a harbor a particular submarine is headed toward. At each time step, the situation can be characterized by the following variables:
**destination** Which destination the submarine is headed to.

**location** The submarine’s current location

**observation** Your noisy observation (via underwater sensing) of the submarine’s location

**action** The submarine’s actions (speed and steering)

In addition, there is a variable, **type**, which encodes the type of the submarine (which is useful to know, because it affects the sub’s speed and maneuverability).

(a) (6 pts) Draw a dynamic Bayesian network diagram that describes this system. Only the **observation** variable is directly observable. Show how the values of the variables at the current time step depend on their values in the previous time step. Your model should be able to encode these relationships:

- Submarines tend to stick with the same destination, and don’t frequently change which one they’re aiming at.
- The choice of action depends on relative position of the submarine and its destination.

\[
\begin{align*}
\text{Parents}(destination_{t+1}) &= \{destination_t\} \\
\text{Parents}(location_{t+1}) &= \{location_t, action_t\} \\
\text{Parents}(observation_{t+1}) &= \{location_{t+1}\} \\
\text{Parents}(action_{t+1}) &= \{destination_{t+1}, location_{t+1}\}
\end{align*}
\]

(b) (2 pts) What would you change in your diagram in order to model the idea that some types of submarines have different ranges and that the distance to a location might affect its selection as a destination?

\[
\text{Parents}(destination_{t+1}) = \{destination_t, type, location_{t+1}\}
\]

(c) (3 pts) If you have made three observations, \(O_1, O_2,\) and \(O_3\), give an expression for the probability distribution over the destination at step 3:

\[
\Pr(D_3|O_1, O_2, O_3)
\]

using only probabilities stored in the CPTs of the network (in part (a)).
3. (10 points) Now we’ll consider modeling a problem using probabilistic relational models. Here is our world:

- A car’s speed depends on the size of its engine and the mood of its driver.
- A person’s mood depends on his bank balance.
- A person’s bank balance depends on his employer.

(a) (3 pts) Draw a relational probabilistic model (or show in rules) the structure of this model. What are the classes? What are their simple attributes? What are their complex attributes? How are they related?

<table>
<thead>
<tr>
<th>Simple</th>
<th>Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed(car)</td>
<td>Owner(car):person</td>
</tr>
<tr>
<td>EngineSize(car)</td>
<td>Employer(person):person</td>
</tr>
<tr>
<td>Mood(person)</td>
<td></td>
</tr>
<tr>
<td>BankBalance(person)</td>
<td>Parents(Speed(car)) = {EngineSize(car), Mood(Owner(car))}</td>
</tr>
<tr>
<td></td>
<td>Parents(Mood(person)) = {BankBalance(person)}</td>
</tr>
<tr>
<td></td>
<td>Parents(BankBalance(person)) = {Employer(person)}</td>
</tr>
</tbody>
</table>

(b) (3 pts) Assume the following world (Camaro1 and Pinto2 are cars):

- owner(Camaro1) = John
- owner(Pinto2) = Mary

Draw the Bayesian network associated with the instantiation of your model in this world.
Parents(Speed(Camaro1)) = \{\text{EngineSize(Camaro1), Mood(John)}\}
Parents(Speed(Pinto2)) = \{\text{EngineSize(Pinto2), Mood(Mary)}\}

Parents(Mood(John)) = \{\text{BankBalance(John)}\}
Parents(Mood(Mary)) = \{\text{BankBalance(Mary)}\}

Parents(BankBalance(John)) = \{\text{Employer(John)}\}
Parents(BankBalance(Mary)) = \{\text{Employer(Mary)}\}

(c) (2 pts) Say what would have to change in your model if the mood of the car’s owner also depended on how comfortable the seats were?

Parents(Mood(person)) = \{\text{BankBalance(person), Seats(CarOf(person))}\}

(d) (1 pt) Are the speeds of Camaro1 and Pinto2 independent, assuming we don’t know who John and Mary work for?

Yes

(e) (1 pt) Are the speeds of Camaro1 and Pinto2 independent given that they both work for Yoyodyne?

No

4. (10 points)
Consider a house-cleaning robot. It can be either in the living room or at its charging station. The living room can be clean or dirty. So there are four states: LD (in the living room, dirty), LC (in the living room, clean), CD (at the charger, dirty), and CC (at the charger, clean). The robot can either choose to suck up dirt or return to its charger. Reward for being in the charging station when the living room is clean is 0; reward for being in the charging station when the living room is dirty is -10; reward for other states is -1. Assume also that after the robot has gotten a -10 penalty for entering the charging station when the living room is still dirty, it will get rewards of 0 thereafter, no matter what it does.

Assume that if the robot decides to suck up dirt while it is in the living room, then the probability of going from a dirty to a clean floor is 0.5. The return action always takes the robot to the charging station, leaving the dirtiness of the room unchanged. The discount factor is 0.8.

(a) (1 pt) What is $V^*(CC)$ (the value of being in the CC state)?

0
(b) (1 pt) What is $V^*(CD)$?
-10

(c) (2 pts) Write the Bellman equation for $V^*(LC)$.

\[ V^*(LC) = -1 + 0.8 \max_a \sum_s T(LC, a, s)V(s) \]

(d) (2 pts) What is the value of $V^*(LC)$?
-1

(e) (2 pts) Write the Bellman equation for $V^*(LD)$ and simplify it as much as possible.

\[
V^*(LD) = -1 + 0.8 \max_a \sum_s T(LD, a, s)V(s) \\
V^*(LD) = -1 + 0.8 \max_a \{\text{action\_go\_Back, action\_suck\_Up}\} \\
V^*(LD) = -1 + 0.8 \max_a \{-10, 0.5 * V(LC) + 0.5 * V(LD)\} \\
V^*(LD) = -1 + 0.8 \max_a \{-10, 0.5 * -1 + 0.5 * V(LD)\} \\
V^*(LD) = -1 + 0.8 \max_a \{-10, -0.5 + 0.5 * V(LD)\} \\
V^*(LD) = -1 + 0.8 \max_a \{-10, -0.5 + 0.5 * V(LD)\} \\
\]

(f) (2 pts) If $V_0(LD) = 0$ (that is, the initial value assigned to this state is 0), what is $V_1(LD)$, the value of LD with one step to go (computed via one iteration of value iteration)?

\[
V_0(LD) = 0 \\
V_1(LD) = -1 + 0.8 \max_a \{-10, -0.5 + 0.5 * V_0(LD)\} \\
V_2(LD) = -1 + 0.8 \max_a \{-10, -0.5\} \\
V_3(LD) = -1 - 0.8 * 0.5 \\
V_4(LD) = -1 - 0.4 \\
V_5(LD) = -1.4 \\
\]